

PH2213 : Examples from Chapter 2 : One Dimensional Motion

Key Concepts

Kinematics deals with the description of how objects move. The description of the motion of any object must always be given relative to some particular **reference frame**. In this chapter, we start with simple 1-D motion, then extend this to 2 and 3 dimensional motion in chapter 3.

The **displacement** of an object is the change in position of the object (typically Δx or $x - x_o$ or similar).

The **average speed** is the **distance** travelled (regardless of direction or sign) divided by the time interval Δt . The **average velocity** is the displacement divided by the time interval.

A good approach to solving problems is to pick a reference frame (i.e. choose a coordinate system) which simplifies the description of the motion.

Key Equations

Average velocity: $\bar{v} = \frac{x_B - x_A}{t_B - t_A} = \frac{\Delta x}{\Delta t}$

Instantaneous velocity $v = dx/dt$.

Average acceleration: $\bar{a} = \frac{v_B - v_A}{t_B - t_A} = \frac{\Delta v}{\Delta t}$

Instantaneous acceleration $a = dv/dt$.

Straight-line motion with constant acceleration.

$$\begin{aligned} v &= v_o + at & \bar{v} &= \Delta x / \Delta t & \bar{v} &= \frac{1}{2}(v_o + v) & \bar{v} &= v_o + \frac{1}{2}at & v^2 &= v_o^2 + 2a(x - x_o) \\ x &= x_o + v_o t + \frac{1}{2}at^2 & x - x_o &= \left(\frac{v_o + v}{2}\right)t \end{aligned}$$

Freely-falling bodies: special case where the acceleration a is due to gravity. The **magnitude** of the acceleration due to gravity varies slightly around the surface of the earth, but for purposes of this class (and particularly the online homework), assume $g = 9.80 \text{ m/s}^2$. The sign of the **acceleration** will depend on your choice of coordinate system. On earth, if our axis is pointing up, then $a = -g = -9.80 \text{ m/s}^2$.

Common Errors

- Frequently, equations involving gravity have already presumed an axis pointing vertically upward, in which case $a = -g$ and this negative sign has **already** been incorporated into the equations. **DO NOT** ever use $g = -9.8 \text{ m/s}^2$. g by itself **always** represents a positive number.
- distance vs displacement
- average vs instantaneous quantities
- the equations of motion presume that the acceleration is a constant. If the acceleration is **not** constant over the interval, it may still be possible to break the scenario into separate intervals with different but still constant accelerations (example: car speeds up, then cruises at a constant speed, then comes to a stop: here we have three intervals where acceleration is constant: but a different value in each interval).

1. Cheetah

A cheetah is being observed by a photographer. When the flash goes off, the cat darts directly away from the photographer. Suppose we set up an axis with an origin at the location of the photographer, with the cat initially 20 m away, with $t = 0$ starting when the cat starts moving. We now observe that its coordinate is given by: $x = (20\text{ m}) + (5\text{ m/s}^2)t^2$.

(a) Find the displacement of the cheetah between $t=1$ and $t=2$ seconds.

At $t = 1$, the cheetah is located at $x_1 = 20 + (5)(1)^2 = 25\text{ m}$. At $t = 2$, the cheetah is located at $x_2 = 20 + (5)(2)^2 = 45\text{ m}$.

The displacement Δx then is: $\Delta x = x(t = 2) - x(t = 1) = x_2 - x_1 = 40\text{m} - 25\text{m} = 15\text{m}$.

(b) Find the average velocity between $t=1$ and $t=2$ sec.

The average velocity is given by $\bar{v} = \Delta x / \Delta t$ and we computed Δx for this same time interval in part (a), so:

$$\bar{v} = \Delta x / \Delta t = \frac{15\text{ m}}{(2\text{ s}) - (1\text{ s})} = (15\text{m}) / (1\text{s}) = 15\text{m/s}.$$

(c) Find the instantaneous velocity of the cheetah at $t = 1$ and $t = 2$.

The instantaneous velocity $v = dx/dt$ and here $x = 20 + 5t^2$ so we can differentiate this expression and get a closed-form expression for v : $v = dx/dt = 0 + 10t$. (All the units in every term and every constant in the equation was already in standard metric units, so the velocity must come out in proper metric units of m/s if I supply all the variables in proper metric units (the time in seconds, for example).

At $t = 1$, the instantaneous velocity of the cheetah will be $v = 10t = (10)(1) = 10\text{ m/s}$.

At $t = 2$, the instantaneous velocity of the cheetah will be $v = 10t = (10)(2) = 20\text{ m/s}$.

(d) Find the acceleration of the cheetah at $t = 1$ and $t = 2$; also find the average acceleration between those times.

$a = dv/dt$ but we have a closed-form expression for $v = 10t$ so $a = 10$ (or putting the units back in: $a = 10\text{ m/s}^2$). For this particular motion, apparently the cheetah has a constant acceleration, so the answers for all three parts of this problem would be the same.

2. Car Accelerating from Rest

Suppose I am waiting to get onto the highway. An opening appears, and I hit the gas, accelerating from rest to 60 *miles/hr* over a time interval of 10 *s*. What acceleration did the car have, and assuming the acceleration is constant how far did I travel?

First, let's make sure everything is in standard metric units. Converting the final speed of the car: $(60 \frac{\text{miles}}{\text{hr}}) \times \frac{1609 \text{ m}}{1 \text{ mile}} \times \frac{1 \text{ hr}}{3600 \text{ s}} = 26.82 \text{ m/s}$.

The average acceleration is given by $\bar{a} = \Delta v / \Delta t$, so here we have $\bar{a} = \frac{26.82 - 0}{10 - 0} = 2.682 \text{ m/s}^2$.

Assuming this remains constant, how far did I travel? We have several ways of solving this:

(a) We know the initial and final velocities, so $\bar{v} = \frac{1}{2}(v_o + v) = (0.5)(26.82 + 0) = 13.41 \text{ m/s}$ but $\bar{v} = \Delta x / \Delta t$ so $\Delta x = \bar{v} \Delta t = (13.41 \text{ m/s})(10 \text{ s}) = 134.1 \text{ m}$.

(b) We computed the acceleration to be 2.682 m/s^2 so we could use that to solve the problem as well. $x = x_o + v_o t + \frac{1}{2} a t^2$ or $x = 0 + 0 + \frac{1}{2}(2.682)(10)^2 = 134.1 \text{ m}$ as well.

(c) Yet another option: $v^2 = v_o^2 + 2a\Delta x$ so in our case: $(26.82)^2 = (0)^2 + (2)(2.682)\Delta x$ which also produces $\Delta x = 134.1 \text{ m}$.

Any path through our equations of motion results in the 'right' answer. Note that (b) and (c) required us to use a **computed** value (the acceleration) during the solution. Recall the discussion about significant figures and round-off errors in chapter 1 though. Unless we keep extra digits, any intermediate computed numbers are going to be less accurate, so using a calculated value in a later step will increase the final error even more. We kept enough digits in the solutions above so that it didn't show up, but imagine if you had rounded off everything to 3 or even just 2 significant figures. In that case, the final answers could all be quite different.

Rule of thumb for this particular course: if you have to write down an intermediate result, retain at least 4 significant figures. (Even better, try to arrange the calculation so the numbers never leave your calculator, which internally does its calculations using 6 or 7 significant figures.)

3. Car Coming to a Stop

Suppose the car in the previous example is moving along at 60 *miles/hr* and slams on the brakes, screeching to a halt over a distance of 50 *m*. What acceleration does this represent? How many seconds did this take?

Here we have a car with an initial velocity of 26.82 *m/s* and a final velocity of 0.0 *m/s*, and this ‘event’ took place over a displacement of 50 *m*.

We have multiple ways to solve this, but we can get the acceleration in a single step using $v^2 = v_o^2 + 2a\Delta x$, so let’s do so: $(0)^2 = (26.82)^2 + (2)(a)(50)$. Rearranging this: $a = -\frac{(26.82)^2}{100} = -7.193 \text{ m/s}^2$.

Notice that this came out as a negative acceleration, which is what we expect since the velocity is decreasing here.

Now that we have the acceleration, we could use $v = v_o + at$ to find the time interval: $0 = 26.82 + (-7.193)(t)$ or $t = 3.73 \text{ s}$.

4. Designing Highway On-ramp

Suppose traffic will be entering the on-ramp at 30 *miles/hr* and will need to accelerate up to 60 *miles/hr*. Assume that an average car can manage an acceleration of at least 5 m/s^2 and determine how long the on-ramp needs to be. Also how long (seconds) will the driver be on the ramp?

(Let's convert everything to metric first. From an earlier problem, we already know that 60 *miles/hr* is 26.82 m/s , so 30 *miles/hr* would be exactly half that or 13.41 m/s .)

Here we have the initial and final velocities, and an acceleration, and we're looking for how far the car will travel during this scenario (i.e. the car's displacement), so we can use: $v^2 = v_o^2 + 2a\Delta x$. Rearranging to solve for the length of the ramp:

$$\Delta x = \frac{v^2 - v_o^2}{2a}$$

From this expression, we see that the required length of the ramp is varying inversely with the acceleration. A car with a given acceleration will require a ramp of a certain length. A car with a higher acceleration (sports car, e.g.) would not need to use the full length of the ramp. A car with a lower acceleration would need a longer distance to reach highway speed. We can't please everyone, so we've picked a particular acceleration to target, and we'll just have to hope that traffic makes room for the slower cars...

Our particular assumption about acceleration ($a = 5 \text{ m/s}^2$) implies a ramp length of $\Delta x = \frac{v^2 - v_o^2}{2a} = \frac{(26.82)^2 - (13.41)^2}{(2)(5)} = 53.9 \text{ m}$

(1 meter is about 3.281 feet, so this is about 177 feet.)

(b) What time interval was needed for the car to accelerate to highway speed?

We have the initial and final speeds and the acceleration, so the time interval is easy: $v = v_o + at$ so $26.82 = 13.41 + 5t$ or $t = 2.68 \text{ s}$

5. Free-fall: Water Balloon 1

Suppose we drop a water balloon from the top of Hilbun. How long does it take to fall, and how fast is it moving when it hits the ground? Assume the distance from the top of Hilbun to the ground is 15 m .

We have some choices about coordinates here. Technically we could use anything, but two obvious choices are to put the origin at the top of Hilbun, or on the ground where the balloon hits. We also have choices about the direction of our coordinate axis: up or down.

For this version of the problem, let's choose the origin to be where I initially release the water balloon, and let's have the positive direction be vertically upward. Then in these coordinates, the initial position will be at $y = 0$, the final position will be $y = -15\text{ m}$. The acceleration is due to gravity so will have a **magnitude** of 9.80 m/s^2 but this acceleration is directed **downward** towards the center of the earth, so in **our** coordinate system this represents an acceleration of $a = -9.80\text{ m/s}^2$.

Now **finally** we can pull in whatever equations of motion we need to solve the problem.

We can find the speed at the ground from $v^2 = v_o^2 + 2a\Delta x$. Our initial velocity is $v_o = 0$, the displacement $\Delta x = x_{final} - x_{initial} = (-15) - 0 = -15\text{ m}$. The acceleration is $a = -9.80\text{ m/s}^2$, so: $v^2 = (0)^2 + (2)(-9.80)(-15) = +294$ so $v = \sqrt{294} = \pm 17.15\text{ m/s}$.

Notice that our equation told us nothing about the direction (sign) of this velocity, but our real-world experience tells us that this velocity is actually in the downward direction at this point, so $v = -17.15\text{ m/s}$ would be the velocity of the object.

(Note: the question just asked for the **speed** when the object hit the ground, so $|v| = 17.15\text{ m/s}$ would technically be the correct answer here...)

How long did the object take to fall? $v = v_o + at$ and here we **do** have to take into account all the signs properly. At the point where the water balloon touches the ground: $(-17.15\text{ m/s}) = (0\text{ m/s}) + (-9.80\text{ m/s}^2)(t)$ so $t = \frac{-17.15}{-9.80} = +1.75\text{ s}$.

Alternate solution: When the acceleration is constant (which it is here), the position as a function of time is given by: $x = x_o + v_o t + \frac{1}{2}at^2$. Here, with our choice of coordinate system, $x_o = 0$ and $v_o = 0$. The acceleration is $a = -9.80\text{ m/s}^2$, and when the balloon hits the ground, it has an x coordinate of $x = -15\text{ m}$. We know everything in the equation except for the time then: $(-15) = (0) + (0)(t) + \frac{1}{2}(-9.8)t^2$ or $-15 = -4.9t^2$ which leads to $t^2 = (-15)/(-4.9) = +3.061$ or $t = \pm 1.75\text{ s}$.

Again, we have to bring reality into the picture: clearly the balloon didn't hit the ground before it was released, so $t = +1.75\text{ s}$ would be the correct solution.

And now that we know how long it took to fall, we could compute the velocity at the ground: $v = v_o + at = (0) + (-9.80)(1.75) = -17.15\text{ m/s}$.

6. Free-fall: Water Balloon 2

Let's change up the previous problem and instead of just letting the balloon go, we **throw** the water balloon **straight down** with an initial speed of 10 m/s . How fast does it hit the ground now and how long does it take to reach the ground?

We'll use the same coordinate system as in the previous problem, which means our initial position is at the origin, our initial velocity is downward which makes $v_o = -10\text{ m/s}$ (negative). The ground level will be at $y = -15$ again.

(a) How fast will it be going when it hits the ground?

$v^2 = v_o^2 + 2a\Delta y$ where $\Delta y = y_{final} - y_{initial} = -15 - 0 = -15\text{ m}$, so:

$(v^2) = (-10\text{ m/s})^2 + (2)(-9.8\text{ m/s}^2)(-15\text{m})$ or $(v^2) = 100\text{m}^2/\text{s}^2 + 294\text{m}^2/\text{s}^2 = 394\text{m}^2/\text{s}^2$
so $v = \sqrt{394} = \pm 19.85\text{ m/s}$. As before, we know from reality that the balloon is moving downward when it hits the ground and 'down' is the negative direction in our choice of coordinates, so $v = -19.85\text{ m/s}$ would be the correct value for the velocity of the balloon. (The question just asked for 'how fast' it was moving, so 19.85 m/s would be the correct answer for this part.)

(b) How long will it take to hit the ground?

$v = v_o + at$ so $(-19.85\text{ m/s}) = (-10\text{ m/s}) + (-9.8\text{ m/s}^2)(t)$ so $(-9.85\text{ m/s}) = (-9.8\text{ m/s}^2)(t)$
or $t = 1.005\text{ sec}$.

Note how careful we had to be with signs here. Also part (a) just asked for the **speed** of the balloon when it hit the ground, which is the absolute value of the velocity, **but** we needed to use the correct **signed** velocity to solve part (b).

7. Free-fall: Model Rocket

A model rocket is fired vertically upward. It accelerates upwards at 3 m/s^2 for 10 seconds (i.e. the rocket is overcoming gravity and then some, so that in the net, it accelerates upward.) Then the engine cuts off. How far up does the rocket go? How long is it in flight? How fast does it hit the ground?

Discussion : The acceleration of the rocket is **not** constant if we look at the complete trip (from launch to its return to the ground). It does consist of two intervals over which the acceleration **is** constant though: during the initial 10 seconds, the acceleration is 3 m/s^2 upward; during the rest of its flight, the acceleration is 9.8 m/s^2 downward.

After the initial 10 seconds, the rocket will be located at some height above the ground, and will be shooting upward at some considerable velocity. At this point, it will be accelerating downward due to gravity, which will cause the velocity to decrease, eventually causing the rocket to come to a (momentary) stop at some height, at which point the rocket will accelerate towards the earth.

Let's break the entire flight into several segments:

- (A) represents the point when the rocket is initially fired
- (B) will be the point where the rocket cuts off (at $t = 10 \text{ s}$)
- (C) is the point where the rocket has stopped coasting upward and has come to a stop (this 'maximum height' is called the apogee)
- (D) the point where the rocket has returned to the ground

Coordinate System

Since the rocket starts at ground level, flies upward to some height, then falls back to the ground, let's choose a vertical coordinate (which we'll call the Y axis) that has its origin on the ground, with +Y pointing vertically upward. (Note: it doesn't matter what we call the axis but I'll follow the common convention here.)

Interval from A to B (the first 10 seconds)

During this interval, the rocket is moving upwards at the given acceleration so $y = y_o + v_o t + \frac{1}{2}at^2$ so $y = (0) + (0)(10) + \frac{1}{2}(3 \text{ m/s}^2)(10 \text{ s})^2$ so $y = 150 \text{ m}$.

How fast is the rocket moving at this time? $v = v_o + at$ so $v = (0) + (3)(10) = 30 \text{ m/s}$.

Interval from B to C (coasting upward) : Determining the apogee

What happens at point B now? The rocket engine has just cut off, but the rocket does not suddenly come to a stop. Instead, we now have an object located 150 m above the ground, moving upward at a velocity of 30 m/s , but now being acted on by gravity resulting in an acceleration of -9.8 m/s^2 . Just like a ball thrown upward into the air, the rocket will continue to fly upward, but will be slowing down. Eventually the velocity will reach zero, and then start becoming more and more negative as the rocket falls back towards the ground. The point where it stops moving upward and (momentarily) come to a stop will be the maximum height the rocket reaches above the ground, a point called the **apogee**.

How much farther upward will the rocket coast before coming to a stop?

For this interval, we have an initial y coordinate of 150 m and an initial velocity of $v = +30 \text{ m/s}$, and an acceleration of $a = -9.8 \text{ m/s}^2$.

$v^2 = v_o^2 + 2a\Delta y$ so: between points B and C we have:

$(0)^2 = (30 \text{ m/s})^2 + (2)(-9.8 \text{ m/s}^2)\Delta y$ or $0 = 900 - 19.6\Delta y$. Rearranging: $(19.6)\Delta y = 900$ or $\Delta y = 900/19.6 = 45.9 \text{ m}$. Thus during this interval, the rocket continues to move up another 45.9 m before reaching its apogee and finally coming to a stop.

This displacement is on top of the height that the rocket had already reached during the burn phase, which means that it is now located at $y = 150 + 45.9$ or $y = 195.9 \text{ m}$ (about 630 feet above the ground).

How much time did it take to reach this apogee point?

Looking at just the interval from B to C, $v = v_o + at$ so $0 = (+30) + (-9.8)t$ or $t = (-30)/(-9.8) = 3.06 \text{ s}$. This is the time interval for **just** this little part of the motion: from when the rocket cut out, to the point where it has reached its maximum height. So starting from the ground, our stop watch would read 10 seconds plus this additional amount of time, or $t = 10 + 3.06 = 13.06 \text{ s}$.

Interval from B to D

We'd like to know how fast the rocket hits the ground, and how much total time has elapsed from when we initially fired it off.

We have to look at 'constant acceleration' segments so that we can use our equations of motion. From A to B, we had an acceleration of $+3 \text{ m/s}^2$. From that point all the way up to the apogee and then all the way back down to the ground, the acceleration has the same constant value of -9.8 m/s^2 , so we can actually go all the way from B (the point when the rocket cut out) to the ground level (point D).

During this interval, our initial condition is: rocket located 150 m above the ground, moving upward at $v_o = +30 \text{ m/s}$. Our final condition is the rocket hitting the ground (i.e. it has a y coordinate of $y = 0$) with some velocity v .

$$v^2 = v_o^2 + 2a\Delta y$$

$$\Delta y = y_{final} - y_{initial} \text{ so } \Delta y = (0_m) - (150 \text{ m}) = -150 \text{ m}.$$

Substituting in what we know:

$$v^2 = (+30)^2 + (2)(-9.8)(-150) = 900 + 2940 = 3840 \text{ so } v = \sqrt{3840} = \pm 61.97 \text{ m/s}$$

We know the rocket is moving **downward** when it hits the ground, so we can discard the positive solution there and say that $v = -61.97 \text{ m/s}$ when it returns to the ground level. (This is almost 140 miles/hr, which explains why these toy rockets often end up in pieces when they hit the ground...)

How long did this part of the flight take? Looking again all the way from B to D: $v = v_o + at$ so $(-61.97) = (+30) + (-9.8)(t)$ or $t = (-91.97)/(-9.8) = 9.38 \text{ s}$.

That's the time just from B to D though, so if we want the entire time of flight, from launch to 'landing', we need to add the time during the interval from A to B (the 10 seconds during which the rocket was firing), so our stop watch for the entire trip would read $10 + 9.38$ or 19.38 s .

8. Car Chase (1)

Suppose a speeding car is travelling down a straight road at 30 m/s . A police car on the side of the road sees the speeder coming and at the instant the speeder passes the police car, the police car starts accelerating after the speeder with an acceleration of 4 m/s^2 . The police car eventually catches up to the speeder: when and where does that happen?

We have two objects here, each with their own set of equations of motion. Each has a constant acceleration (zero for the speeder, 4 m/s^2 for the police car). One way of solving this would be to use a single coordinate system to describe each vehicle and then their meeting spot would correspond to the point where they both have the same X coordinate.

Let's define our coordinate system so that $x = 0$ where the police car is initially sitting at rest. And we'll let $t = 0$ be the instant the speeder passes the police car. We'll let the $+X$ direction be along the road in the direction the speeder is travelling.

The position of an object (when acceleration is constant) is given by $x = x_o + v_o t + \frac{1}{2}at^2$.

For the speeder, at $t = 0$ they're located at $x = 0$ (this is the point where the speeder is passing the stationary police car). It's moving at a constant speed of 30 m/s and has an acceleration of 0 m/s^2 so it's equation of motion would be: $x_s = 0 + (30 \text{ m/s})(t) + 0$.

To simplify writing all these equations, let's make sure everything is in standard metric units and then drop writing them. Then: $x_s = 30t$ gives the position of the speeder as a function of time.

For the police car, they're starting at $x = 0$ at rest and accelerating at 4 m/s^2 so this vehicle's equation of motion would be $x_p = 0 + 0 + \frac{1}{2}(4 \text{ m/s}^2)t^2$ or $x_p = 2t^2$.

When the police car catches up to the speeder, they'll both have the same value of x , so setting the equations equal to one another: $x_p = x_s$ or $2t^2 = 30t$.

We can write this as $2(t - 15)(t) = 0$ which has **two** solutions: $t = 0 \text{ sec}$ and $t = 15 \text{ sec}$.

If we plug $t = 0$ into the equations of motion, we get $x_s = x_p = 0$ and this solution corresponds to the point where the speeder is just passing by the stationary police car at the beginning of the problem.

The $t = 15$ solution then is the later time when the police car has caught up to the speeder.

At this time, $x_p = 2t^2 = 2(15)^2 = 450 \text{ meters}$ and $x_s = 30t = (30)(15) = 450 \text{ meters}$ (did both of them to make sure they actually did come out to the same value).

How fast are the vehicles moving at this time? The speeder is moving at a constant 30 m/s . The police car is accelerating, so $v = v_o + at = 0 + 4t = (4)(15) = 60 \text{ m/s}$ (a bit over 130 miles/hour).

9. Car Chase (2)

Let's modify the previous problem slightly by adding a delay. Suppose the police car doesn't start accelerating right away; instead let's say it takes 2 seconds for the driver to react before it starts accelerating after the speeder.

This causes a bit of a problem because we want to use the same coordinate system to describe both vehicles but we can't use our equations of motion unless the acceleration of each object is constant. If we try to use the same coordinate system from the previous version of the problem, then the police car has an acceleration of **zero** for two seconds and then 4 m/s^2 thereafter. We have two different accelerations for the police car.

All is not lost though, suppose we just wait for two seconds and start $t = 0$ at that point. Now the speeder still has their zero acceleration, and the police car has a constant 4 m/s^2 acceleration from that point on.

The equation of motion for the police car will be the same then. It's still starting at $x = 0$ with an initial velocity of 0, so $x_p = 0 + 0 + \frac{1}{2}(4 \text{ m/s}^2)t^2$ or $x_p = 2t^2$ just as before.

What about the speeder though? We've waited two seconds before defining the $t = 0$ point but the speeder kept travelling during that time so is now some distance down the road. $x_s = x_o + v_o t + \frac{1}{2}at^2$ but their x_o is no longer zero. Where is the speeder located at this point? It's been travelling down the road at 30 m/s for 2 seconds, so apparently it's 60 m along the road now. It's still moving at a constant 30 m/s (acceleration of zero) so the speeder's location will be $x_s = (60 \text{ m}) + (30 \text{ m/s})(t) + 0$ or $x_s = 60 + 30t$.

When the police car catches up to the speeder, we have $x_p = x_s$ or $2t^2 = 60 + 30t$ which we can rearrange into the usual quadratic form of: $2t^2 - 30t - 60 = 0$ or $t^2 - 15t - 30 = 0$.

That quadratic equation has two solutions, $t = -1.787 \text{ s}$ and $t = +16.787 \text{ s}$.

Only one of these is the actual physical solution though; the police car obviously doesn't catch up to the speeder until some time **after** they start accelerating, so the true solution here must be $t = +16.787 \text{ s}$.

Where are the vehicles at this time? $x_p = 2t^2 = 563.6 \text{ m}$ or $x_s = 60 + 30t = 563.6 \text{ m}$ also.

How fast are they moving? $v_s = 30 \text{ m/s}$ (constant) and $v_p = v_o + at = 0 + (4)(16.787) = 67.15 \text{ m/s}$ (about 150 miles/hr).

In the rocket problem and these two chase problems, it was important to remember that all our equations of motion are only valid when acceleration is a constant value over the entire interval of interest. We can usually deal with this by breaking the problem up into intervals where the acceleration is constant and solving each of these segments separately (or in the case of the second version of the chase example, choosing a coordinate system where the acceleration is constant over the interval of interest).