

PH2213 : Examples from Chapter 4 : Newton's Laws of Motion

Key Concepts

Newton's First and Second Laws (basically $\sum \vec{F} = m\vec{a}$) allow us to relate the forces acting on an object (left-hand side) to the motion of that object, through its acceleration (right-hand side).

Third Law: all forces are **interaction** forces, with $\vec{F}_{ab} = -\vec{F}_{ba}$. (Equal magnitude, opposite direction.)

Statics : if an object is not moving, or is moving at a constant speed, then $\vec{a} = 0$ which means that all the forces must cancel out, as vectors. That means that (separately) $\sum F_x = 0$ and $\sum F_y = 0$ (and $\sum F_z = 0$). This, coupled with the geometry of the situation (the various lengths, angles, etc) allows us to determine information about the forces (such as tension in cables).

Dynamics : if an object is accelerating, then the forces are not balanced (and vice versa). If we have an acceleration, then all our machinery from chapters 2 and 3 on equations of motion can be brought in to help analyze the situation.

Common Errors

- The normal force (\vec{F}_N or sometimes seen as just the symbol \vec{n}) is what keeps an object from moving through another object, and is always **perpendicular** to that surface.
- The normal force is not necessarily just mg . When you include all the various vector forces acting on an object, n is what is left over to prevent the object from moving through the surface it is on.
- Since Newton's laws are vector equations, many (most?) of the problems we encounter involve resolving these vectors into components, which relies on being able to propagate angles around. Detailed step-by-step examples are included on the course website under the **Resolving Vectors** link.
- trig : resolving vectors into components
- Remember: $\sum \vec{F} = m\vec{a}$ means that the (vector) forces acting **on AN object** produce the acceleration of **THAT** object. Be careful to designate the object under consideration and then look only at forces acting on THAT object.

1. Mythbusters : Bullet Stopping in Water

Suppose we fire a 20 gm bullet into the water with an initial speed of 300 m/s at some angle. We find that it comes to a stop very rapidly, after travelling just 1 m through the water. Find the force the water is exerting on the bullet during this interval. How long did it take to bring the bullet to a stop? Initially, assume the bullet is travelling in a straight line and use 1-D equations of motion, then show why that assumption is ok to use.

Let the X coordinate be in the direction of the bullet's motion. Then we have an object with an initial velocity of $v_o = 300 \text{ m/s}$, a final velocity of $v = 0$, and this occurs over a displacement of $\Delta x = 1.0 \text{ m}$ so we can find the acceleration from: $v^2 = v_o^2 + 2a\Delta x$ or $(0)^2 = (300)^2 + (2)(a)(1)$ or $a = -45,000 \text{ m/s}^2$. In terms of g's, this would be $a = -(45,000/9.8)g$ or $a = -4592g$.

How long did it take to come to a stop? $v = v_o + at$ so $0 = 300 + (-45,000)(t)$ and $t = 300/45000 = \frac{1}{150} \text{ s}$ (or $t = 6.67 \times 10^{-3} \text{ s}$ which would also be written as 6.67 ms).

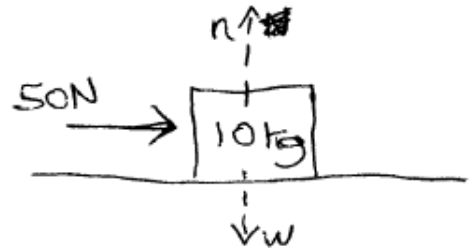
During this time, the water is exerting a force on the bullet bringing it to a stop, but gravity is also exerting a force downward. From above, though, we see that the deceleration due to the water has a magnitude of 4592g which will completely overwhelm the acceleration due to gravity. At least to 4 significant figures, we can completely ignore gravity here...

How much force did the water exert on the bullet? (The units of force, Newtons, are kg m/s² so we need to make sure everything is in standard metric units first. In this case, the mass was given in grams instead of kilograms, so we need to convert it first.) $F = ma$ so $F = (0.02 \text{ kg})(-45,000 \text{ m/s}^2) = -900 \text{ N}$ (i.e. 900 N in the direction opposite the bullet, causing it to slow down).

This is a pretty small amount of force, but the mass is so tiny that it can produce a huge acceleration.

Compare this to the acceleration of the Bugatti Veyron car, which has a mass of about $m = 1900 \text{ kg}$ and a braking acceleration of about 12.75 m/s^2 . That requires a force of $F = ma = (1900 \text{ kg})(12.75 \text{ m/s}^2) = 24,225 \text{ N}$. The force here is a few orders of magnitude larger than what we found for the bullet stopping in the water but the mass of the car is many orders of magnitudes larger, resulting in a much lower acceleration.

2. **Sliding Box (1)** : Let's apply Newton's Laws to a simple scenario of a box being pushed from the side. Suppose we have a 10 kg box sitting on the floor (no friction here yet) and we push horizontally with 50 N . First, we need to consider **all** the forces acting on the box. We have the 50 N of pushing force, plus the force of gravity pulling the box down towards the center of the earth $\vec{F}_g = m\vec{g}$, and we have some normal force between the box and the floor that's preventing the box from passing through the floor.



$\sum \vec{F} = m\vec{a}$ so let's break that into components. We'll use a coordinate system with $+X$ to the right, and $+Y$ upward.

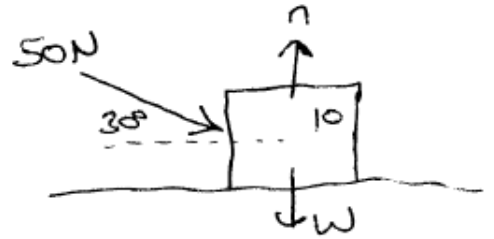
$\sum F_x = ma_x$: here, the weight and normal force are vertical, so they have no X components. The only X component of force I see here is the 50 N we're applying, so $\sum F_x = ma_x$ becomes: $50 = (10)(a_x)$ or $a_x = +5\text{ m/s}^2$, so the box will accelerate to the right at that rate.

$\sum F_y = ma_y$: we know the box doesn't pass through the floor or jump upwards, so it has no acceleration in the Y direction which means $a_y = 0$, which implies that $\sum F_y = 0$. In the Y direction, we have n upward and mg downward, so $n - mg = 0$ or $n = mg$. Here, the normal force is apparently equal to the weight of the box.

Once we have the components of the acceleration, we can bring in all the machinery on motion from chapters 2 and 3:

- If the box starts from rest, how fast is it moving 4 seconds later? $v_x = v_{ox} + a_x t$ so $v_x = (0.0\text{ m/s}) + (5.0\text{ m/s}^2)(4\text{ s}) = 20\text{ m/s}$.
- If the box starts from rest, how fast is it moving after it's displaced 4 m to the right? $v^2 = v_o^2 + 2a_x \Delta x$ so $v^2 = (0)^2 + (2)(5)(4) = 40$ or $v = \sqrt{40} = 6.32\text{ m/s}$.

3. **Sliding Box (2)** Suppose we have the same scenario as in the previous example, but now the force is directed at an angle of 30° below the horizontal. What changes? What acceleration will the box have now?



We analyze this the same way as above, but here since the force is at an angle, we need to **resolve** that vector into X and Y components. The vector \vec{F} is aimed at an angle of 30° below the horizontal, so it has an X component of $F \cos 30$ and a Y component of magnitude $F \sin 30$ and directed downward, so $F_y = -F \sin 30$ here.

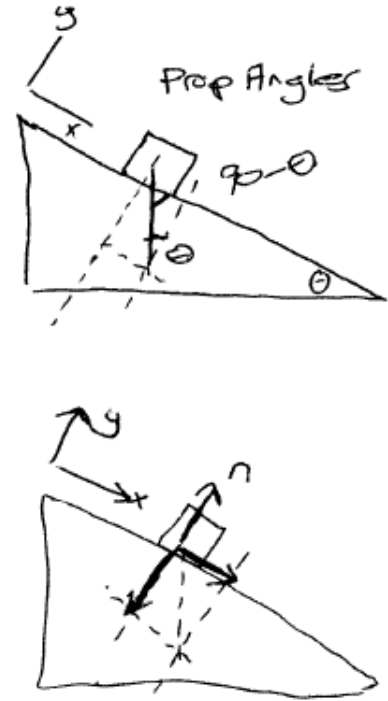
$\sum F_x = ma_x$ so $F \cos 30 = ma_x$ or $(50 \text{ N}) \cos 30 = (10 \text{ kg})a_x$ or $43.30 = 10a_x$ and $a_x = 4.33 \text{ m/s}^2$. We're still pushing with the same magnitude of force (50 N) but the box isn't accelerating as rapidly as before. Since we're pushing down at an angle, some of the force is being 'wasted' trying to push the box into the floor; only part of the force is actually accelerating the box.

If we look at the Y components of the force: $\sum F_y = 0$. Our pushing force has a magnitude of $F \sin 30 = (50)(0.5) = 25 \text{ N}$ and is directed downward. $\sum F_y = n - 25 - mg = 0$ so $n = 25 + mg = 25 + (10)(9.8) = 123 \text{ N}$. The normal force is larger here, so we're putting more stress on the floor.

4. **Box Sliding Down Incline** : Here we have a box of mass m sitting on a frictionless ramp of angle θ . What will be the acceleration of the box down the ramp, and how strong is the normal force?

If we pick a coordinate system with +X to the right and +Y upward, then this looks like 2-dimensional motion: the box is changing both it's X and Y coordinates as it moves.; We 'know' the box will slide down the incline, so it's almost always simpler to switch to a new coordinate system where +X is aligned with the motion along the incline, with +Y perpendicular to that motion, as noted in the top figure.

The 'cost' of doing this is that we now need to convert all our forces into components aligned with this new rotated coordinate system.



See the course website section titled **Resolving Vectors** for the detailed steps in this process.

In our (rotated) Y coordinates, $\sum F_y = 0$ since the object is not moving at all in the Y direction. The normal force is entirely in this direction, and we have a component of the weight also in the -Y direction, so $\sum F_y = 0$ becomes $n - mg \cos \theta = 0$ or $n = mg \cos \theta$. The normal force is now a bit less than what it was when the box was just sitting on a horizontal floor.

In our (rotated) X coordinates, $\sum F_x = ma_x$ so let's pull out all the X components of any forces present. The normal force is perpendicular to the incline, which means it's entirely in the Y direction and therefore has no X components. The only other force present is the force of gravity on the box which will have an X component of $mg \sin \theta$.

$\sum F_x = ma_x$ becomes: $mg \sin \theta = ma_x$ and the mass cancels from both sides, leaving $a_x = g \sin \theta$.

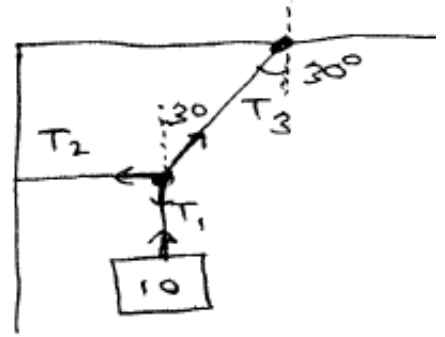
Let's look at this equation at a couple of extreme limits and see if it makes sense:

- In the limit of $\theta = 0$ (the ramp is gone and we're just on a flat floor), then $a_x = g \sin 0 = 0$ and the box will just sit there (or will just continue to slide at a constant velocity).
- In the limit of $\theta = 90$, we basically have the object moving straight down under the influence of gravity and $a_x = g \sin 90 = g$ as expected. (Remember, we have no friction here (yet)).

Wait : in most of the figures I've drawn in class, we have a vertical Y axis, so why did we end up with $a_x = 9.8$ instead of $a_y = -9.8$ in the second case above?

Remember, we defined our +X axis to be pointing along the ramp from the top towards the bottom. That means as the angle of the ramp gets steeper, finally reaching 90° , we end up with a +X axis that is pointing straight downward. The acceleration due to gravity is 9.8 m/s^2 'down' and that direction has become our +X coordinate, so ending up with $a_x = +9.8 \text{ m/s}^2$ is exactly right.

5. **Hanging Lamp** : Suppose we hang a lamp from the ceiling and then connect another wire that pulls the lamp over to the side a bit, resulting in the geometry in the figure shown. What are tensions in all cables?)



As we discussed in class, when we solidly attach cables like this, they can support different tensions in each segment. Thus we have three unknowns here - the three tensions.

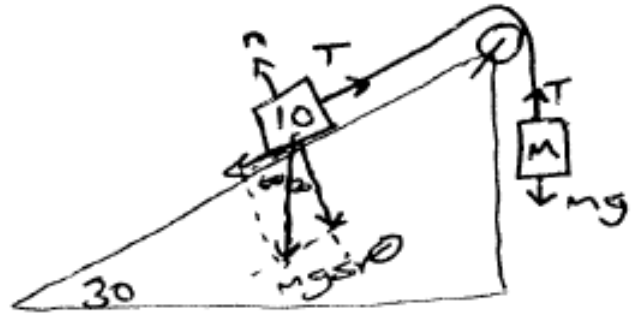
First, we can determine the tension in section 1 easily. Looking at just the lamp itself, the sum of all the vector forces acting on the lamp are what produces the acceleration of the lamp. The lamp is not accelerating, so the sum of all the forces on it must be zero. Here we have T_1 upward and mg downward, so $T_1 - mg = 0$ or $T_1 = mg = (10 \text{ kg})(9.8 \text{ m/s}^2) = 98 \text{ N}$.

At the point where the three sections are connected, the sum of the vector forces acting right there has to be zero since that point isn't accelerating (or moving at all). We'll use coordinates where +X is horizontal to the right, and +Y is vertically upward. Then $\sum F_x = 0$ and $\sum F_y = 0$ separately at that point. That will give us two equations we can use to solve for our two unknowns.

Looking just at the Y components of the forces at that point, we have T_1 pulling downward (with a known magnitude of 98 N) and we have a **component** of T_3 pulling upward. From the geometry of the figure (again, see the **Resolving Vectors** section on the website), the Y component of this force will be $T_3 \cos 30$ and it will be in the +Y direction. So $\sum F_y = 0$ becomes: $-T_1 + T_3 \cos 30 = 0$ or $T_3 = T_1 / \cos 30 = (98 \text{ N}) / 0.8660 = 113.2 \text{ N}$.

Looking at just the X components of the forces acting at that point where the three cables are connected, we have T_2 pulling to the left, and a **component** of T_3 pulling to the right. $\sum F_x = 0$ becomes: $-T_2 + T_3 \sin 30 = 0$ so $T_2 = T_3 \sin 30$ but we just found that $T_3 = 113.2 \text{ N}$ so $T_2 = (113.2 \text{ N})(0.5) = 56.6 \text{ N}$.

6. **Hanging Boxes (1)** : Suppose we have a 10 kg box on a 30° incline that's connected to a box of unknown mass M hanging vertically over the edge as shown in the figure. What does M have to be for the boxes to remain stationary and not move at all?



Nothing is accelerating here (or moving at all), so $\sum \vec{F} = 0$ anywhere we want to look in the figure.

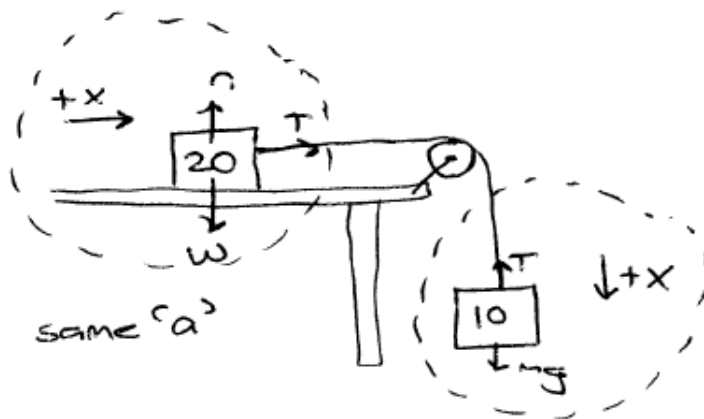
Looking at the hanging block, we have T acting vertically upward and its weight acting downward, so $T - Mg = 0$ and $T = Mg$ but we don't know the mass yet. At least this gives us an equation we can use to **find** the mass, once we know the tension in the cable though.

Moving to the box on the incline, let's use a coordinate system with $+X$ running along the slope towards the top, and $+Y$ perpendicular to the slope. Then in this coordinate system, $\sum F_x = 0$. In this rotated X direction, we have T in the $+X$ direction and a **component** of the weight of the 10 kg box in the $-X$ direction. Resolving the force of gravity on that box into components, we see that the X component will have a magnitude of $mg \sin 30$ or $(10\text{ kg})(9.8\text{ m/s}^2)(0.5) = 49\text{ N}$. It's directed downslope, so $\sum F_x$ becomes $-49 + T = 0$ or $T = 49\text{ N}$.

But $T = Mg$ so $M = T/g = (49\text{ N})/(9.8\text{ m/s}^2) = 5\text{ kg}$.

Apparently at this ramp angle, the hanging block only needs half the mass of the block on the incline for things to remain stationary.

7. **Hanging Boxes (2)** : Suppose we have a 20 kg box on a table connected to 10 kg box hanging off edge. We have **no** friction present yet. What happens to the boxes?



If we assume **nothing is moving**, we can find the tension in the cable by looking at the hanging box. There we have T upward and it's weight of $mg = (10)(9.8) = 98 \text{ N}$ downward, so $T - 98 = 0$ and $T = 98 \text{ N}$. The block sitting on the table is connected to that same rope, so it is being pulled to the right with a 98 N force and there's nothing else to stop it from moving, so it will accelerate to the right. The boxes are connected together, so the hanging block will accelerate downward too. But that's opposite of what we assumed (that the hanging block was not accelerating) so we've arrived at a contradiction. Our initial assumption must be wrong: the hanging block **does** accelerate...

Well **that changes everything**. If the hanging block is accelerating, then $\sum F$ is **not** zero on it. That means the tension in that cable **cannot** be just equal to the weight of the hanging block. So we have **two** unknowns now: the tension in the rope, and the mutual acceleration that the two blocks are undergoing.

The blocks do move together: the hanging block will accelerate downward, while the box on the table will accelerate to the right. Let's create **two** coordinate systems here: one for each block. For the hanging block, we'll have $+X$ be pointing downward; for the sliding block, we'll have $+X$ be pointing to the right.

Hanging block: $\sum F_x = ma_x$. We have T acting upward (which is in our $-X$ direction here) and it's weight acting downward (which is in the $+X$ direction) so we have $-T + mg = ma$ or $-T + (10)(9.8) = (10)(a)$ or $\boxed{-T + 98 = 10a}$.

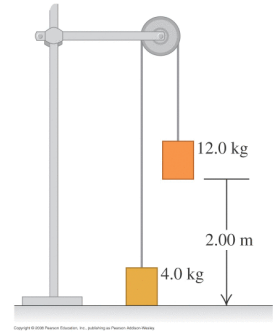
Sliding block: $\sum F_x = ma_x$ and here we have T acting to the right (the $+X$ direction for **this block**) so $\sum F_x = ma_x$ becomes: $\boxed{T = (20)(a)}$.

That gives us two equations and two unknowns, which we can solve in various ways. In this case, since the second equation gave us simply $T = 20a$ we can substitute $20a$ for the symbol T in the first equation, so $-T + 98 = 20a$ becomes $-(20a) + 98 = 10a$ which we can rearrange to $30a = 98$ or $a = 3.2677 \text{ m/s}^2$.

Once we have a , we can find T since $T = 20a = (20)(3.27) = 65.3 \text{ N}$.

Note that the tension in the cable is **no longer** equal to the weight of the hanging box - it's less.

8. **Hanging Boxes (3) : The Atwood Machine** A system of two paint buckets connected by a lightweight rope is released from rest with the 12 kg bucket initially 2.00 m above the floor. Find the speed of the buckets the instant before the 12 kg one hits the floor. (Ignore friction and the mass of the pulley.)



The heavier block will fall downwards and will pull the lighter block upwards. The weight of the heavier block is larger, so there will be a net force causing these blocks to accelerate (and not just move at a constant speed). The heavier block will be accelerating downward, and the lighter block will be accelerating upward but since they're tied together, the **magnitudes** of these accelerations will be the same.

Coordinates : Let's use a common coordinate system for each of the two objects, where the +Y axis is always pointing upward. The two objects are tied together so they will have a common tension, speed, and magnitude of acceleration, although the directions of these quantities may be different for each object: one moving up, and one moving down, so we'll have to be careful with signs.

Light Block : here we have the tension acting upward, and it's own weight acting downward. Our generic Newton's law $\sum F = ma$ becomes: $T - (4)(9.8) = (4)(a)$ or $T - 39.2 = 4a$.

Heavy Block : here we have tension acting upward, it's own weight acting downward, and we also know it will be accelerating with the same magnitude as the lighter block (since they're tied together), but **downward**. The generic Newton's law $\sum F = ma$ becomes: $T - (12)(9.8) = (12)(-a)$. Note: since this block is accelerating downward and we defined our positive direction to be upward, this is $-a$. This becomes: $T - 117.6 = -12a$.

This gives us two equations with two unknowns, but we have a couple of common ways to solve those.

Here, it looks like if we subtract the second equation from the first one, that will result in cancelling out the variable T , so we'll just do that:

$$(T - 39.2) - (T - 117.6) = 4a - (-12a) \text{ from which } 78.4 = 16a \text{ or } a = 4.9 \text{ m/s}^2.$$

Looking at the equations of motion for the lighter block, we can use $v^2 = v_o^2 + 2a\Delta y$ to find the speed after this block has moved upward by 2 m: $v^2 = 0 + (2)(4.9)(2) = 19.6$ or again $v = 4.43 \text{ m/s}$.

We can also find the tension in the cable using either of our original equations: $T - 39.2 = 4a$ for example gives $T = 39.2 + (4)(4.9) = 58.8 \text{ N}$ or $T - 117.6 = -12a$ gives $T = 117.6 - 12a = 117.6 - 12(4.9) = 58.8$ (same result, so that's a good check).