

## PH2213 : Examples from Chapter 5 : Applying Newton's Laws

### Key Concepts

Newton's Laws (basically  $\sum \vec{F} = m\vec{a}$ ) allow us to relate the forces acting on an object (left-hand side) to the motion of the object, through its acceleration (right-hand side).

**Statics** : if an object is not moving, or is moving at a constant speed, then  $\vec{a} = 0$  which means that all the forces must cancel out, as vectors. That means that (separately)  $\sum F_x = 0$  and  $\sum F_y = 0$  (and  $\sum F_z = 0$ ). This, coupled with the geometry of the situation (the various lengths, angles, etc) allows us to determine information about the forces (such as tension in cables).

**Dynamics** : if an object is accelerating, then the forces are not balanced (and vice versa). If we have an acceleration, then all our machinery from chapters 2 and 3 on equations of motion can be brought in to help analyze the situation.

**Kinetic Friction** : If the object is **moving**, the friction between it and the surface it is sliding on is given by  $f_k = \mu_k n$ .  $\mu_k$  is the coefficient of kinetic friction and depends on the two materials involved.

**Static Friction** : If the object is **not yet moving**, then a different kind of friction exists that changes its magnitude as needed to keep the object from moving. It can only do this up to some maximum value though, so we write this type of friction as:

$$f_s \leq f_{s,max} \text{ where } f_{s,max} = \mu_s n$$

**Circular Motion** : In uniform circular motion, the acceleration vector is directed toward the center of the circle and has a magnitude of  $a_c = v^2/r$ . Since there is an acceleration, there must be some force causing the object to move in this circle (perhaps the tension in a string, or static friction between the tires of a car and the road).

### Common Errors

- SEE the list given for chapter 04 where Newton's laws were introduced.
- Only one type of friction is present at a time. If the object is not yet moving, only static friction is present. Once it starts moving, only kinetic friction is present.

1. Friction : Box Sliding to a Stop : Suppose we have a box of mass  $10 \text{ kg}$  sliding across a horizontal floor with an initial speed of  $5 \text{ m/s}$ . The coefficient of kinetic friction between the box and the floor is  $\mu_k = 0.6$ . How long does it take the box to come to a stop? How far did it travel?

**Coordinates** : Let  $+Y$  be vertically upward, and  $+X$  be in the direction the box is moving.

**What are all the forces acting on the block?** We have its own weight  $mg$  downward, some normal force  $n$  upward keeping the box from going through the floor, and some force of friction, acting to oppose the motion, so friction will be pointing in the direction opposite the motion.

$\sum F_x = ma_x$  : we have  $f_k$  as the only force in the X direction. The **magnitude** of the force of kinetic friction is  $f_k = \mu_k n$  and it is in the direction opposite motion, so  $\sum F_x = ma_x$  becomes  $-\mu_k n = ma_x$ . That's as far as we can go at this point since we don't know the normal force yet.

$\sum F_y = 0$  here since the box isn't accelerating (or moving at all) in the Y direction. Taking all the Y components of the forces present, we have:  $n - mg = 0$ , so in this simple case we have  $n = mg = 98.0N$ .

Now that we know the normal force, we can find the force of kinetic friction:  $f_k = (0.6)(98.0) = 58.8 \text{ N}$ .  $-f_k = ma_x$  leads to  $-58.8 = (10)(a)$  or  $a = -5.88 \text{ m/s}^2$ .

Now that we know the acceleration of the box, we can apply all the machinery from the earlier chapters related to equations of motion to determine how far the box slides before coming to a stop and how long, in seconds, that took:

$v^2 = v_o^2 + 2a\Delta x$  becomes:  $0^2 = (5 \text{ m/s})^2 + (2)(-5.88)\Delta x$  or  $\Delta x = (-25)/(-11.76) = +2.13 \text{ m}$ .  
 $v = v_o + at$  so  $(0) = (5 \text{ m/s}) + (-5.88 \text{ m/s}^2)(t)$  or  $t = 0.85 \text{ s}$ .

**Generic Version** : Let's look at this problem generically where we have a box of mass  $m$  sliding across the floor with an initial speed of  $v_o$ , and a coefficient of kinetic friction of  $\mu_k$ . Then the magnitude of the force of kinetic friction will be  $f_k = \mu_k n$ .

$\sum F_x = ma_x$  becomes:  $-f_k = ma_x$  or  $-\mu_k n = ma_x$ .

$\sum F_y = 0$  becomes:  $n - mg = 0$  or  $n = mg$  so replacing  $n$  in the  $\sum F_x$  equation:

$-\mu_k(mg) = ma_x$  and dividing the equation by  $m$  yields:  $a_x = -\mu_k g$ .

Substituting this value for  $a_x$  into our  $v^2$  equation:  $v^2 = v_o^2 + 2a_x\Delta x$  so when the box comes to a stop:  $(0)^2 + v_o^2 + (2)(-\mu_k g)(\Delta x)$  which we can rearrange (careful with signs) into:  $\mu_k = \frac{v_o^2}{2g\Delta x}$

which gives us a simple way to determine the value of  $\mu_k$  between two objects. If we can get the object moving with some known initial speed and then measure how far it slides before coming to a stop, we can easily compute the value of  $\mu_k$  (for that pair of materials).

We can also rearrange the equation to solve for  $\Delta x$  and find that:  $\Delta x = \frac{v_o^2}{2\mu_k g}$  which gives us the 'stopping distance' for an object with an initial speed and coefficient of friction.

Note that the stopping distance is **inversely** proportional to the coefficient, which means that the object will slide longer if the interface is more slippery. The stopping distance is also proportional to the **square** of the initial speed. If you are in a car and slam on the brakes so that the tires slide against the road, driving twice as fast means the stopping distance is  $2^2$  or 4 times longer.

2. Pushing Object Initially at Rest Suppose we have the same 10 kg box as before but it is now initially at rest.

(a) How much (horizontal) force do we need to apply to start the box moving?

In this case, the box is not initially moving, so static friction is present between the box and the floor. The harder I push on the box, the stronger static friction holds the box back, but static friction can only provide up to  $f_{s,max} = \mu_s n$  before giving up.

The geometry of this problem is the same as the previous, so looking in the vertical direction we have  $\sum F_y = ma_y = 0$  which implies that  $+n - mg = 0$  or  $n = mg$ . In the horizontal direction, we have the pushing force  $F$  being opposed by the frictional force. I can keep increasing the pushing force right up until it reaches  $f_{s,max}$  before the box will start moving, so this will occur when  $F = f_{s,max} = \mu_s n = \mu_s mg = (0.8)(10 \text{ kg})(9.8 \text{ m/s}^2) = 78.4 \text{ N}$ .

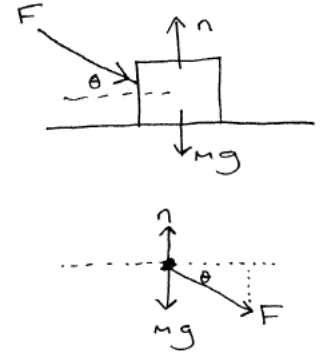
(b) If we continue to push with that same force, what acceleration will the box have?

The box now breaks free of the floor and starts sliding in the +X direction. Now we have me pushing with 78.4 N and being opposed by kinetic friction of magnitude  $f_k = \mu_k n = (0.6)(10 \text{ kg})(9.8 \text{ m/s}^2) = 58.8 \text{ N}$ . The net force in the X direction on the box is now  $\sum F_x = +78.4 - 58.8 = 19.6 \text{ N}$  and the crate will start accelerating:  $\sum F_x = ma_x$  so  $19.6 = 10a_x$  or  $a_x = 1.96 \text{ m/s}^2$ .

10 kg box on an interface with  $\mu_s = 0.8$  and  $\mu_k = 0.6$ . Then  $n = mg = 98.1 \text{ N}$  and  $f_{s,max} = \mu_s n = (0.8)(98.1 \text{ N}) = 78.48 \text{ N}$ .

This is a 'real world' problem we often encounter: it takes more force to start something moving than it does to keep it moving. This is particularly a problem with certain combinations of materials. A sheet of glass laying on top of another sheet of glass has a very high coefficient of static friction, but a very low coefficient of kinetic friction, so it takes a lot of force to start one of the sheets moving, but once it starts moving it accelerates dangerously rapidly.

3. **Force an an Angle (1)** : Suppose we try to move our 10 kg box by **pushing** on it at an angle. Let  $m = 10\text{ kg}$ , the coefficients of static and kinetic friction between the box and floor are  $\mu_s = 0.8$  and  $\mu_k = 0.6$ . We found earlier that it took about 78 N to get the crate moving. Suppose we push with  $F = 100\text{ N}$  but directed  $30^\circ$  below the horizontal. Does the box move? If so, what is its acceleration?  
Coordinates: let +Y be vertically upward, and +X be to the right.



We have friction here, which means we're going to need to determine the normal force, so let's deal with that direction first:

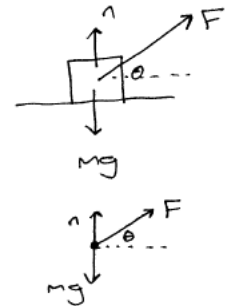
$$\sum F_y = ma_y = 0 \text{ so } -mg + n - F \sin \theta = 0 \text{ or: } -(10)(9.8) + n - (100) \sin 30 = 0 \text{ or } -98 + n - 50 = 0 \text{ or finally } n = 50 + 98 = 148.$$

$\sum F_x = ma_x$  but is it moving or not? Is the force applied enough to cause it to move? Let's look at the X forces. We have  $F \cos \theta = (100)(0.866) = 86.6\text{ N}$  to the right. How much force is friction providing though?  $f_{s,max} = \mu_s n = (0.8)(148.1) = 118.5\text{ N}$ . Hm... that means that the 86.6 N force I am applying is **not** enough to cause the crate to move. Even though I'm providing **more force** than before, some of it is going into the **normal force** and causing static friction to be higher, and the box will just sit there.

4. **Force an an Angle (2)** : Suppose we try **pulling** the crate along with a force angled upward this time. When we were pushing the crate with a perfectly horizontal force, it took about 78 N to get the box moving, so this time suppose we're pulling with just 70 N of force, but at the  $30^\circ$  angle shown.

$$\sum F_y = ma_y = 0 \text{ so } -mg + n + F \sin \theta = 0 \text{ so: } -(10)(9.8) + n + (70) \sin 30 = 0 \text{ or } -98 + n + 35 = 0 \text{ or finally } n = 98 - 35 = 63\text{ N}.$$

$\sum F_x = ma_x$  but is it moving or not? Is the force applied enough to cause it to move? Let's look at the X forces. We have  $F \cos \theta = (70)(0.866) = 60.6\text{ N}$  to the right. How much force is friction providing?  $f_{s,max} = \mu_s n = (0.8)(63) = 50.4$ , so this time around, friction is **not** providing enough force to keep the crate from moving.



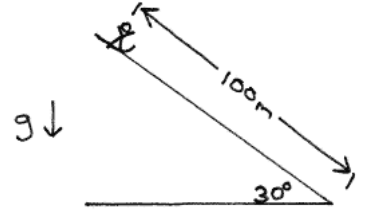
Now that we know that the crate **will** move, we need to redo the problem using kinetic friction instead of static friction. Nothing else has changed in the geometry of the problem - all the angles are the same, etc. The only thing that is different now is that we have kinetic friction  $f_k$  instead of static friction. Our computation for  $f_k$  is identical to what we did for  $f_s$  but now we have to use  $\mu_k$  instead of  $\mu_s$ .

$$f_k = \mu_k n = (0.6)(63) = 37.8\text{ N}. \text{ Thus } \sum F_x = (F \cos 30) - f_k = (70)(0.866) - (37.8) = 22.82\text{ N}. \text{ Finally, } \sum F_x = ma_x \text{ becomes: } 22.82 = 10a \text{ or } a = 2.282\text{ m/s}^2.$$

**General Comment** : Pushing downward on an object increases the normal force and thus increases the frictional force, making it harder to move the object. Pulling upward had the opposite effect, reducing the normal force and thereby reducing friction, making it easier to move the object.

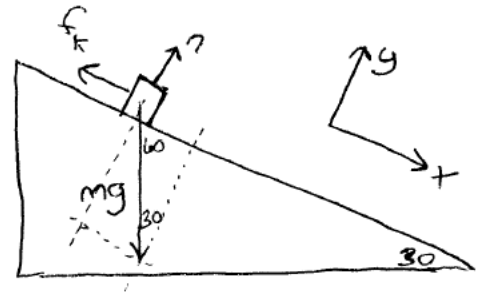
5. Object on Incline with Friction : Skier on a Slope

Suppose we have an 80 kg skier at the top of a 100 meter long, 30° slope, initially at rest. Suppose  $\mu_s = 0.20$  and  $\mu_k = 0.10$ . What happens? Is there enough static friction to keep the skier in place? If not, how fast will the skier accelerate down the slope?



**What are all the forces acting on the skier?**

We have their weight  $mg$  directed downward towards the center of the earth. We have a normal force perpendicular to the slope, keeping the skier from accelerating **into** the ground, and we have friction acting to oppose motion (so friction, whether it's static or kinetic, would be directed as shown in the figure).



We'll **define a coordinate system** with +X aimed along the slope, starting at the skier's initial location, pointing downslope. +Y will be pointing up perpendicular away from the sloping surface.

Friction and the normal force are already aligned with one of the coordinate axes, but  $F_g$  is not, so we'll need to get its components. Moving the angles around, we find a right triangle we can use to determine the components of  $F_g$ . Here, we see that the weight has a component down-slope of  $mg \sin 30$  and a component in the -Y direction of magnitude  $mg \cos 30$ .

(a) **Assume static** and see if the situation is possible.

Component of the skier's weight along slope:  $mg \sin 30 = (80)(9.8)(0.5) = 392 \text{ N}$

Normal force:  $n = mg \cos 30 = (80)(9.8)(0.866) = 679.0 \text{ N}$

Static friction:  $f_{s,max} = \mu_s n = (0.2)(679.0) = 135.8 \text{ N}$

Note that the down-slope component of the skier's weight is **much** larger than the maximum amount of retaining force that static friction can provide, so apparently there is not enough friction to keep the skier in place. Result: the skier **will** start moving immediately.

(b) Now that we know the skier will accelerate down the slope, we need to **redo** the analysis, this time with **kinetic** friction. But really there's very little **new** work needed here, since nothing in our geometry has changed. The only difference is that now we have kinetic friction: i.e. we need to use  $\mu_k$  instead of  $\mu_s$  in computing the amount of friction present. So now:  $f_k = (0.1)(679.0) = 67.9$  so  $\sum F_x = 392 - 67.9 = 324.1 \text{ N}$

$\sum F_x = ma_x$  so so  $(324.1) = (80)(a)$  or  $a = 4.05 \text{ m/s}^2$

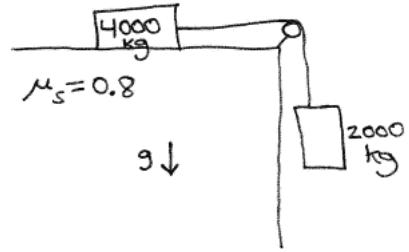
How fast is skier moving at bottom of slope?

$v^2 = v_o^2 + 2a_x \Delta x$  or  $v^2 = 0^2 + (2)(4.05)(100) = 810$  from which  $|v| = 28.5 \text{ m/s}$  (about 64 miles/hour).

We know the skier is moving downslope, which is our +X direction so  $v = +28.5 \text{ m/s}$ .

How long does it take the skier to reach the bottom of the hill?  $v_x = v_{ox} + a_x t$  so  $(28.5 \text{ m/s}) = (0.0 \text{ m/s}) + (4.05 \text{ m/s}^2)(t)$  from which  $t = 7.04 \text{ s}$ .

6. Example : Jurassic Park 2 : at one point a dinosaur has pushed a trailer of mass  $2000\text{ kg}$  over the edge of a cliff. It hangs there, connected to another trailer of mass  $4000\text{ kg}$  which remains on the ground as shown in the figure. Let the coefficient of static friction here be  $\mu_s = 0.8$



- (a) Verify that the present situation is stable (i.e. static friction is providing enough force to keep the trailers from sliding over the edge).

For this part, we are assuming that everything is static - nothing is moving. Applying Newton's Laws to the hanging object, the sum of all the forces in the vertical direction must be zero, since the object is not accelerating. We have some tension in the cable, so looking at this object, we have  $T$  upward, and the weight of the object downward.  $\sum F_y = ma_y = 0$  so  $+T - mg = 0$  or  $T = mg = (2000\text{ kg})(9.8\text{ m/s}^2) = 19,600\text{ N}$

Let's look at the part of the trailer that is sitting on the ground now. In the X direction, we have the tension pulling it to the right, and static friction acting to the left. That's all we have in the X direction, so will this static friction be enough to resist the tension trying to pull the object to the right?

The maximum amount of static friction will be  $f_{s,max} = \mu_s n$  so we'll need to find the normal force here. Looking in the vertical direction and all the forces acting on **this** object, we have its weight downward, and the normal force upward, so  $+n - mg = 0$  or here  $n = mg = (4000\text{ kg})(9.8\text{ m/s}^2) = 39,200\text{ N}$ . The **frictional** force then can be up to  $f_{s,max} = \mu_s n = (0.8)(39,200\text{ N}) = 31,360\text{ N}$ .

Thus it looks like we have more than enough 'friction budget' available to keep the trailer in place.

- (b) The dinosaur is not happy with that and decides to push on the trailer. How much force does the dinosaur need to provide to overcome static friction and cause the trailers to start to move?

The dinosaur needs to add some addition force in the  $+X$  direction: just enough force that it plus the tension that is already there will be enough to reach the maximum static friction, so  $F + 19600 = 31360$  or  $F = 11,760\text{ N}$ . Anything less and there's still enough left in the static friction budget to resist the motion. Once we add this much force though, static friction has reached its limit and the object can start to move.

7. Car and Unbanked Turn : Suppose we have a car taking an offramp or otherwise moving in a circular path. How fast can the car travel before it starts to slip?

Suppose the circular path the car is on has a radius of  $r = 70 \text{ m}$  and that the coefficient of static friction between the car's tires and the road is 0.9, which would be typical for good tires and dry road.

The car is moving in a circle, so it has a centripetal acceleration of  $a_c = v^2/r$ . If there is an acceleration, there must be a force to provide it, so we need a force of  $F = ma = mv^2/r$  to keep the car from slipping.

The force we have available is static friction between the tires and the road (since the tires **roll** along the road, rather than slide). The maximum amount of force that static friction can provide is  $f_{s,max} = \mu_s n$

Looking in the vertical direction at the forces acting on the car, it's not accelerating in that direction so  $\sum F_z = 0$  which implies that  $n - mg = 0$  or  $n = mg$ . That means that static friction will provide up to  $f_{s,max} = \mu_s n = \mu_s mg$ .

If we want to travel as fast as possible, we'll be working right up against that limit, with static friction providing as much force as it can, so at this extreme limit:  $mv^2/r = \mu_s mg$  and here we see that the mass of the car cancels out on both sides of this equation. The mass of the vehicle wasn't provided but apparently it doesn't matter. Dividing the equation by  $m$ , we are left with:  $v^2 = \mu_s gr$

Doing this symbolically leads to some generally useful information about designing off-ramps. The better the coefficient of static friction between the tires and the road, the faster the car can go, and also the larger the arc the faster it can go. This relationship is **not** linear though. For a fixed value of  $\mu_s$ , if we want to double the speed, we'll need to make the radius of curvature **four** times larger. That makes the length of the roadway four times longer. Since area is proportional to the radius squared, we'd have to buy  $4^2$  or 16 times as much land. Certainly cheaper to just post a lower speed limit on the offramp...

For our particular situation, with  $r = 70 \text{ m}$  and  $\mu_s = 0.9$ , cars can take travel along this circular section with a speed of:  $v^2 = (0.9)(9.8 \text{ m/s}^2)(70 \text{ m}) = 617.4 \text{ m}^2/\text{s}^2$  or  $|v| = 24.85 \text{ m/s}$  (about 55.6 *miles/hr*)

If we have **wet pavement and bald tires**, the coefficient of static friction may be only  $\mu_s = 0.4$  (and probably less) which reduces our maximum speed to:

$$v^2 = (0.4)(9.8)(70) = 274.4 \text{ or } |v| = 16.57 \text{ m/s (about 37 miles/hr).}$$

(In either event, we would want to post a speed limit that is smaller than these right-up-against-the-limit values...)

8. CDROM : What is the acceleration at the outer edge of a 40X CDROM?

A few years ago, it was claimed that CDROM drive speeds had gotten so fast that cheap CD's were flying apart in them.

History: the original 1X CDROM drives spun at a frequency that varied between 200 RPM and 500 RPM (the speed changes depending on what part of the disk the laser is reading so that the linear speed of the point under the laser is roughly the same at all times).

The radius of a typical CDROM is about 6 cm.

For now, let's just assume a 1X drive spins with a constant frequency of 360 *RPM's*. A 40X drive then would spin 40 times faster, and would have a frequency of  $40 \times 360 \text{ RPM} = 14,400 \text{ RPM}$ .

That's not in the right units; we need frequency in revolutions per second so:

$$\frac{14,400 \text{ rev}}{1 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 240 \text{ s}^{-1}.$$

How fast is a point on the outer edge moving?

$$v = 2\pi r/T \text{ or } v = 2\pi r f = 2\pi(0.06 \text{ m})(240 \text{ s}^{-1}) = 90.5 \text{ m/s (about 202 miles/hr)}.$$

What is the acceleration of a point on the edge?

$$a_c = v^2/r = (90.5)^2/0.06 = 136,440 \text{ m/s}^2 \text{ or about } 13,900 \text{ g's}.$$

That's a lot. Recall from our 1D motion examples for the bullet stopping in water that we found acceleration of magnitude  $45,000 \text{ m/s}^2$  which was enough to shatter the bullet into fragments, and the acceleration here is even larger. The scenarios are somewhat different though. With the bullet, the force of contact occurs suddenly and propagates as a shockwave through the material; with the CDROM, it spins up over a much longer period of time. In a Mythbusters episode they were able to cause CDROM's to shatter by spinning them fast enough. They attached disks to a high speed router and had to spin them at over  $20,000 \text{ rev/min}$  for them to break though.