Chapter 27 Examples : Magnetism

Key concepts:

- Cross product: $\vec{A} \times \vec{B}$ produces a vector that is perpendicular to the plane formed by A and B, in a direction that can be determined via the right-hand rule (starting from A and curling towards the direction of B, your thumb is pointing in the direction of the cross product). The resulting vector has a magnitude of $AB \sin \phi$ where A and B are the magnitudes of the two input vectors and ϕ is the angle between them (the interior angle). (See Table 27-1 for examples.)

- Force on a charged particle in a uniform magnetic field: $\vec{F} = q\vec{v} \times \vec{B}$

- Force on a current-carrying wire in a uniform magnetic field: $\vec{F} = I\vec{l} \times \vec{B}$

- If a particle enters a uniform magnetic field that is perpendicular to its direction of motion, the magnetic force will cause it to move in a circular path of radius $r = \frac{mv}{qB}$. This circular motion has a period of $T = 2\pi m/(qB)$ or equivalently represents a frequency (called the cyclotron frequency) of $f = qB/(2\pi m)$

- In the presence of both electrical and magnetic forces, the forces simply add as vectors (along with any other vector forces present). $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

- Units: $1\ Tesla = 1N/(A \cdot m)$ (i.e. newtons per amp-meter). You may also see it denoted as webers per square meter).

  A full tesla field is very strong so a smaller unit of often used. One **gauss** is a ten-thousandth of a tesla: $1\ G\ =\ 1 \times 10^{-4}\ T$.

  The earth's magnetic field at its surface is around $0.5\ G$. (Note: the point on the Earth called the 'north pole' is actually a **south** magnetic pole, and vice versa.)

Using Magnetism to counter gravity

A particle of mass 0.195 g carries a charge of $-2.50 \times 10^{-8}\ C$. The particle is given an initial horizontal velocity that is due north and has magnitude $4.00 \times 10^4\ m/s$. What are the magnitude and direction of the minimum magnetic field that will keep the particle moving in the earth's gravitational field in the same horizontal northward direction?

Do we need to include the Earth's magnetic field here? Note that the particle is travelling due north, parallel with the earth's magnetic field. $\vec{F} = q\vec{v} \times \vec{B}$, which means that $\vec{v}$ and $\vec{B}$ are in the same direction, making the angle between them zero. $\vec{F} = qvB \sin\phi$ so the field of the **earth** has NO EFFECT on the motion of this particle, and we can ignore it and move on. (Actually the Earth's magnetic field is slightly tilted from the north-south longitude lines, so **may** be having an effect here, but let's ignore it for now - we'll find that the magnetic field we need in this problem to keep the particle floating will be hugely larger than the Earth's weak magnetic field.)

The force of gravity is downward and the particle is moving to the north and has a **negative** charge, so we have to do our right-hand rules carefully. If the magnetic field we are looking for is pointed to the East, then $\vec{v} \times \vec{B}$ will point down (right-hand rule) so the FORCE, being $\vec{F} = q\vec{v} \times \vec{B}$ will point UP (the negative charge on the particle caused that additional sign flip): exactly the direction we need to have a chance of cancelling out gravity.

Try magnetic fields in various other directions and determine the direction the force will be. A magnetic field in the N or S directions would result in zero force on the particle, since it's velocity would be parallel to the magnetic field, which makes $\vec{v} \times \vec{B}$ equal to zero. A magnetic field UP would result in $\vec{v} \times \vec{B}$ to the east, so $\vec{F} = q\vec{v} \times \vec{B}$ would be to the west (since q is negative here). And so on...

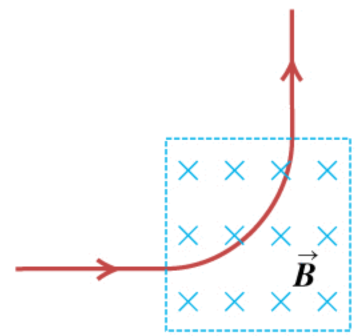
If the magnetic field is 'sort-of' eastward, the angle between $\vec{v}$ and $\vec{B}$ will be something other than 90 degrees, making $\sin\phi$ something less than 1. That means we'll need a stronger field to cancel gravity. We're looking for the smallest possible magnetic field that can cancel out gravity in this case, so $\vec{B}$ must be pointing exactly towards the east.

The magnitude of the force the magnetic field is exerting will be $F = qvB$ and we need that to equal the magnitude of the gravitational force (we've already taken care of the direction), so we need: $qvB = mg$ or $\boxed{B = \frac{mg}{qv}}$.

Substituting in the values we have here (and converting the mass from grams to kilograms): $B = \frac{(0.195 \times 10^{-3})(9.81)}{(2.50 \times 10^{-8})(4 \times 10^4)} = 1.91\ T$, which is an **extremely** strong magnetic field. The Earth's magnetic field has a strength of about 0.5 $gauss$ or 0.5×10^{-4} $tesla$ which is nearly 40,000 times weaker than what we found we needed here.

Charged Particle Moving in a Magnetic Field

A beam of protons traveling at 1.20 km/s enters a uniform magnetic field, and their motion is perpendicular to the field. The beam exits the field, leaving the field in a direction perpendicular to its original direction. (That is, the protons make a 90 degree turn as they pass through the magnetic field.) The beam travels a distance of 1.18 cm while in the field. What is the magnitude of the magnetic field?



Lots of verbiage there, but basically we have a charged particle moving in a uniform magnetic field, with the velocity perpendicular to the field. This is the situation which produces circular motion, and here the proton manages to trace out just a quarter-circle before it runs out of field and continues on as shown.

Up front, we'll convert the speed from 1.20 km/s to 1200 m/s, and the arclength $s = 1.18\ cm$ to $s = 1.18 \times 10^{-2}\ m$.

For a charged particle moving in such a field, we found (section 27-4) that the radius of the circle was related to the charge, speed and the strength of the magnetic field by: $R = \frac{mv}{qB}$. We're looking for the field B so we can rearrange this into the form: $B = \frac{mv}{qR}$. We can find the mass and charge of the proton from the tables at the very end of the book, and we know the velocity. What is the radius of this arc though? The path the proton makes in the field is exactly one quarter of a complete circle, so it represents a distance (along the arc) of one quarter of the circumference of a circle of radius R: $s = \frac{1}{4}(2\pi R)$ or $s = \frac{1}{2}\pi R$. We are given that $s = 1.18\ cm = 1.18 \times 10^{-2}\ m$ so $R = 2s/\pi = (2)(1.18 \times 10^{-2}\ m)/\pi = 7.512 \times 10^{-3}\ m$.

Finally we can put it all together: $B = \frac{mv}{qR} = \frac{(1.67\times10^{-27})(1200)}{1.6\times10^{-19})(7.51\times10^{-3})} = 1.67 \times 10^{-3}\ T$.

We can write this as $16.7 \times 10^{-4}\ T$ which is 16.7 $Gauss$. The earth's magnetic field is around 0.5 G so this field is about 33 times larger (making it a pretty weak field as magnets go...).

Magnetic Force on a current-carrying wire

A straight, vertical wire carries a current of 1.20 A downward in a region between the poles of a large superconducting electromagnet, where the magnetic field has a magnitude $B = 0.588\ T$ and is horizontal. What are the magnitude and direction of the magnetic force on a 1.00 cm section of the wire that is in this uniform magnetic field, if the magnetic field direction is: (a) east, (b) south, (c) 30.0^o south of west?

This is mostly to practice using the right-hand-rule.

XXXX ADD PICTURES HERE (top down view, with current going into the page). DESCRIBE the RHR process in each case

We found in section 27-3 that the force on a current-carrying wire is $\vec{F} = I\vec{l} \times \vec{B}$. The 'direction' of the length $\vec{l}$ is defined to be the direction the current is flowing in.

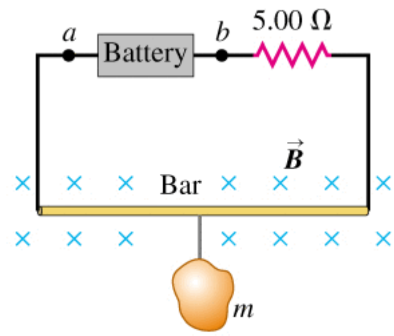
(a) Case of $\vec{B}$ to the east: here $\vec{l}$ points down and $\vec{B}$ points east, so (using the right-hand rule), $\vec{l} \times \vec{B}$ points directly south. The angle between $\vec{l}$ and $\vec{B}$ is 90^o in this case, so the magnitude of the force is $F = IlB \sin 90 = (1.20)(0.01)(0.588)(1.000) = 7.06 \times 10^{-3}\ N$.

(b) Case of $\vec{B}$ to the south: here $\vec{l}$ points down and $\vec{B}$ points south, so (using the right-hand rule), $\vec{l} \times \vec{B}$ points directly west. The angle between $\vec{l}$ and $\vec{B}$ is 90^o in this case, so the magnitude of the force is $F = IlB \sin 90 = (1.20)(0.01)(0.588)(1.000) = 7.06 \times 10^{-3}\ N$. (Same as before.)

(c) Case of $\vec{B}$ being 30 degrees south of west: From the sketch, the right hand rule tells us that $\vec{l} \times \vec{B}$ will point 30^o west of north (or equivalently 60^o north of west). The angle between $\vec{l}$ and $\vec{B}$ is still 90^o, so the magnitude of the force is still $F = IlB \sin 90 = (1.20)(0.01)(0.588)(1.000) = 7.06 \times 10^{-3}\ N$.

Magnetic Balance

The circuit shown in the figure is used to make a magnetic balance to weight objects. The mass m to be measured is hung from the center of the bar that is in a uniform magnetic field of 1.50 T, directed into the plane of the figure. The battery voltage can be adjusted to vary the current in the circuit. The horizontal bar is 60.0 cm long and is made of extremely light-weight material (i.e. ignore it's mass). It is connected to the battery by thin vertical wires that can support no appreciable tension: all the weight of the suspected mass m is supported by the magnetic force on the bar. A resistor with $R = 5.00\,\Omega$ is in series with the bar. The resistance of the rest of the circuit is negligible. (a) Which point a or b, should be the positive terminal of the battery? (b) If the maximum terminal voltage of the battery is 175 V, what is the largest mass that this instrument can measure?



The magnetic field will exert a force on the bar due to the current flowing through it of $F = I\vec{l} \times \vec{B}$. From the figure, the magnetic field $\vec{B}$ is pointing into the paper. Let's assume the current is flowing from the left to the right through the bar. That means that $\vec{l}$ is pointing to the right, which means that $\vec{l} \times \vec{B}$ will be, according to the right-hand rule, pointing UP: exactly the directly we need the magnetic force to be in, in order to have a chance of counteracting gravity. (If we assume the current is flowing from the right to the left, then $\vec{l}$ is to the left, and $\vec{l} \times \vec{B}$ will be pointing downward, in the same direction of gravity, so no chance of having those two balance each other out.)

OK: we know the current has to be flowing from the left to the right through the bar. That means that point a must be the positive terminal of the battery (answer to part (a)).

The current is exactly perpendicular to the magnetic field, so $F = I\vec{l} \times \vec{B}$ gives us a magnitude of $F = IlB \sin\phi = IlB \sin 90 = IlB$. We need this to equal the magnitude of the gravitational force, so $IlB = mg$ or $m = IlB/g$.

The current flowing depends on the voltage of the battery and the resistance of the circuit. Here we only have the one resistor, so $V = IR$ or $I = V/R$. The mass is related to the voltage then by: $m = IlB/g = \frac{VlB}{Rg}$. Here we see that the mass is directly proportional to the voltage. So if we can vary the voltage, we can adjust it to exactly mass the force of gravity.

For our particular device, $R = 5\ \Omega$, $l = 0.60\ m$, $B = 1.50\ T$ and $g = 9.81\ m/s^2$. The power supply can only put out a maximum of $V = 175\ volts$, so the largest mass we can measure will be $m = \frac{(175)(0.60)(1.5)}{(5)(9.81)} = 3.21\ kg$.

Balancing Electrical, Magnetic, and Gravitational Forces

You wish to hit a target from several meters away with a charged coin having a mass of 5.0 g and a charge of $+2500\ \mu C$. The coin is given an initial velocity of 12.8 m/s, and a downward uniform electric field with field strength 27.5 N/C exists throughout the region. If you **aim directly at the target** and fire the coin **horizontally**, what magnitude and direction of uniform magnetic field are needed in the region for the coin to hit the target?

Converting to standard metric units: $m = 5.0\ g = 5.0 \times 10^{-3}\ kg$ and $Q = 2500 \mu C = 2.5 \times 10^{-3}\ C$.

There are three forces acting on the coin: the coin has mass, so we have gravity. The coin has a charge and there is an electric field present, so we have an electrical force, and the coin is charged **and moving** through a magnetic field, so we also have a magnetic force. The verbiage of the problem basically says that we want the coin to travel in an exactly straight line, which means that these three forces need to cancel each other out.

The gravitational force is pointing down, obviously.

The electric force will be $\vec{F}_E = q\vec{E}$. Here $\vec{E}$ is pointing down and the charge is positive, so the electric force will be downward also.

We need the magnetic force to be exactly **upward** in order to cancel out these other two. The magnetic force will be $\vec{F}_B = q\vec{v} \times \vec{B}$. The coin has a positive charge, so we can find the direction of the force by just looking at $\vec{v} \times \vec{B}$ for various possible magnetic field directions until we find one that produces an upward force. Let's position ourselves at the initial location of the coin, with the coin flying away from us.

- If the magnetic field is pointing in the same direction as $\vec{v}$ (or in the exact opposite direction), then $\vec{v} \times \vec{B}$ will be zero, and that won't help.
- If the magnetic field is pointing UP, then $\vec{v} \times \vec{B}$ will be to our right, so that doesn't work.
- If the magnetic field is pointing DOWN, then $\vec{v} \times \vec{B}$ will be to our left, so that doesn't work.
- If the magnetic field is pointing RIGHT, then $\vec{v} \times \vec{B}$ will be downward, so that doesn't work.
- If the magnetic field is pointing LEFT, then $\vec{v} \times \vec{B}$ will be up, and we found it.

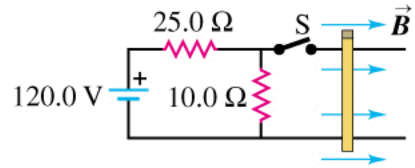
If the magnetic field is pointing 'mostly' LEFT but maybe a bit ahead or behind, the force will still be UP, but $\sin\phi$ will be something less than 1 and we'll need a stronger field to achieve our desired result. With the field pointing exactly to the left, we can get away with the smallest possible field.

With the magnetic field exactly to the left, the angle between the velocity and the field is 90^o so the magnitude of the force will just be $F_B = qvB$, and it will be directly upward. The electric force of qE and the gravitational force of mg are both directly downward. The sum of the forces then, taking UP to be the positive direction, will be $\Sigma F = -mg - qE + qbB$ and we want this sum to be zero so that the coin flies in an exactly straight line, so finally: $-mg - qE + qvB = 0$. Since we're interested in the strength of the magnetic field, we'll rearrange this to solve for B: $B = (mg + qE)/(qv)$.

With the values given here: $B = \frac{(5\times10^{-3})(9.81)+(2.5\times10^{-3})(27.5)}{(2.5\times10^{-3})(12.8)} = \frac{0.04905+0.06875}{0.032}$ or $B = 3.68\ T$ (a very large magnetic field).

Railgun - version 1 (pretty ineffective version)

A 3.00 N metal bar, 1.50 m long and having a resistance of 10.0 Ω rests horizontally on conducting wires connecting it to the circuit shown in the figure. The bar is in a uniform 1.60 T magnetic field and is not attached to the wires in the circuit. What is the acceleration of the bar just after the switch S is closed?



When we close the switch, some current will flow through the bar and this current being in a magnetic field will cause a force from $\vec{F}_B = I\vec{l} \times \vec{B}$. What direction will the current be flowing in?

When we close the switch, what does the circuit look like? We have current flowing from the positive battery terminal through the 25 Ω resistor, and then we encounter basically two resistors in parallel: the 10 Ω resistor already shown in the figure, plus the 10 Ω resistance of the movable bar. These two parallel resistors combine to produce an equivalent resistance of $\frac{1}{R} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$ so together they make a resistance of 5.0 Ω. Now this resistance is in series with the 25.0 Ω resistor, giving a total resistance in the circuit of $25+5$ or 30.0 Ω. The total current flowing from the battery then is $V = IR$ or $I = V/R = 120/30 = 4.0\ A$. When this current gets to the parallel-resistor-part of the circuit, there we have two identical 10 Ω resistors, so ultimately exactly 2.0 A of current will flow through the bar.

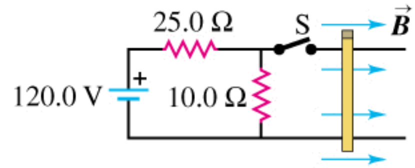
This current is flowing from the positive terminal of the battery towards the little network of resistors. From our previous work, we know that the current will be flowing 'down' (from the top of the figure towards the bottom) through the moving bar. This will cause a force from $\vec{F}_B = I\vec{l} \times \vec{B}$ in the UPWARD direction. The magnitude of this force will be $F_B = IlB = (2.0)(1.5)(1.6) = 4.8\ N$. We were given that the bar has a weight of 3.00 N though, so the two forces acting will be 4.8 N upward due to the external magnetic field, and 3.0 N downward due to gravity, giving us a net force of 1.8 N upward.

That's fine, but ultimately they asked for the actual acceleration on the bar, so we need to find the mass of the bar. It's weight is $W = mg$ so $m = W/g = (3.00\ N)/(9.81\ m/s^2) = 0.3058\ kg$. $F = ma$ so the acceleration felt by the bar will be $a = F/m = (1.8\ N)/(0.3058\ kg) = 5.88\ m/s^2$ (upward).

Note that when we close the switch, the bar will fly upward with the acceleration we just computed. That means it's flying up away from the wires it was resting on, and as soon as it detaches, the current will stop flowing, at which point there won't be any magnetic force on the bar anymore, and it will just fall back down. When it touches the wires, the current will flow again, causing it to be shot into the air again, and so on. So the bar will likely just bounce up and down on the wires. Not particularly useful...

Railgun - version 2 (more effective version)

Suppose we do the same problem but this time let the magnetic field direction is reoriented so that the field is pointing **down** into the 'page' here. (So take those $\vec{B}$ arrows and change them so they're pointing down into the figure.)
A 3.00 N metal bar, 1.50 m long and having a resistance of 10.0 Ω rests horizontally on conducting wires connecting it to the circuit shown in the figure. The bar is in a uniform 1.60 T magnetic field and is not attached to the wires in the circuit. What is the acceleration of the bar just after the switch S is closed?



(NOTE: Assume $\vec{B}$ is pointing down into the page, instead of the direction shown in the figure.)

The early discussion is the same as in the previous problem. We still end up with a 2 amp current flowing through the bar but let's look at the direction of the magnetic force now. $\vec{F} = I\vec{l} \times \vec{B}$ with $\vec{l}$ pointing from the top of the figure towards the bottom, and $\vec{B}$ pointing into the board, our cross product this time gives us the direction of the force being to the right in the figure. The bar is still feeling a force of gravity downward (keeping it in contact with the wires) but the magnetic force is causing the bar to accelerate to the right. The magnitude of the force is the same as we computed before: $F_B = IlB = (2.0)(1.5)(1.6) = 4.8\ N$. The mass of the bar was found to be 0.3058 kg so this will create an acceleration to the right of $m = F/m = (4.8)/(0.3058) = 15.7\ m/s^2$.

The downward force of gravity will keep the bar in contact with the wires, so the bar will continue to accelerate until it reaches the end of the 'track', leaving the mechanism with some reasonably high velocity.

(We'll see different and far more effective versions of this mechanism later.)