Chapter 28 Examples : Sources of Magnetic Fields

Key concepts:

- $\mu_o = 4\pi \times 10^{-7}\ T\ m/A$ (called the permeability of free space); analogous to ϵ_o that appears in many electric field calculations
- Magnetic field around a straight wire: $B = \frac{\mu_o}{2\pi}\frac{I}{r}$ (Direction from RHR with thumb in direction of current.)
- Magnetic force between two parallel currents (wires) separated by a distance d: $F/L = \frac{\mu_o}{2\pi}\frac{I_1 I_2}{d}$ (Attractive if currents in same direction; repulsive if opposite direction)
- Ampere's law: $\oint \vec{B} \cdot d\vec{l} = \mu_o I_{encl}$
- Interior of a solenoid: $B = \mu_o n I$ where n is the number of loops per meter
- Interior of a toroid: $B = \frac{\mu_o N I}{2\pi r}$
- Biot-Savart : computing magnetic field from non-straight currents: $\vec{B} = \frac{\mu_o I}{4\pi}\int \frac{d\vec{l} \times \hat{r}}{r^2}$
- Magnetic field created by a moving point charge (not covered in this book): $\vec{B} = \frac{\mu_o}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$

Magnetic Field from a lightning strike

Lightning bolts can carry current up to approximately 20 kA. We can model such a current as the equivalent of a very long, straight wire. (a) If you were unfortunate enough to be 5.0 m away from such a lightning bolt, how large a magnetic field would you experience? (b) How does this field compare to one you would experience being 5.0 cm from a long, straight household current of 10 A?

(a) The magnitude of the magnetic field from a long current-carrying 'wire' is $B = \frac{\mu_o I}{2\pi r}$ so here, $B = \frac{(4\pi\times10^{-7})(20{,}000)}{2\pi\times5} = 8 \times 10^{-4}\ T$. (Note this is 8 gauss, which is about 16 times stronger than the magnetic field of the earth, so would at least briefly cause compasses to change direction.)

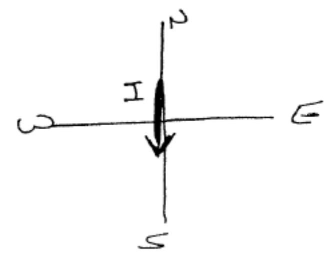
(b) For the household wiring, we have $I = 10A$ and $r = 0.05\ m$, so $B = \frac{\mu_o I}{2\pi r} = \frac{(4\pi\times10^{-7})(10)}{2\pi\times0.05} = 4.0 \times 10^{-5}\ T$.

The Earth's magnetic field is about $5 \times 10^{-5}\ T$, so close to the wire the magnetic field from the wire can easily be stronger than the Earth's field, and should cause significant deflections of a compass needle.

Fortunately, the current flowing in household wiring (and most wiring for that matter) is usually AC current, so it is changing direction back and forth at 60 times per second. At best, we should see the compass needle trying to swing back and forth that fast, and the inertia in the needle will make that difficult to see.

Compass Interference from Power Lines

Two hikers are reading a compass under an overhead transmission line that is 5.50 m above the ground and carries a current of 800 A in a horizontal direction from north to south. (a) Find the magnitude and direction of the magnetic field at a point on the ground directly under the conductor. (b) One hiker suggests they walk on another 50 m to avoid inaccurate compass readings caused by the current. Considering that the magnitude of the earth's field is of the order of 0.5×10^{-4} T, is the current really a problem?



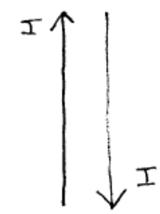
The figure shows a 'top down' view of the current. From the right hand rule, if you point your thumb in the direction of the current, your fingers will curl in the direction of the magnetic field. ABOVE the wire, the field will be pointing to the west. To the left of this current, the field will be pointing down. Directly underneath this wire, the field will be pointing to the EAST, and so on.

(a) The magnitude of the field from a long straight wire is $B = \frac{\mu_o I}{2\pi r}$ so here: $B = \frac{(4\pi\times10^{-7})(800)}{2\pi\times5.50} = 2.91 \times 10^{-5}$ T. The magnetic field of the earth is about 0.5 gauss, or 5×10^{-5} T so these are nearly the same, and being this close to the power line would definitely throw off the compass.

(b) If you walked 50 m directly away from the power line, you're now located 5.5 meters below and 50 meters off to the side, so the distance to the wire is now $r = \sqrt{50^2 + 5.5^2} = 50.3$ m, which makes the magnetic field magnitude equal to $B = \frac{\mu_o I}{2\pi r} = \frac{(4\pi\times10^{-7})(800)}{2\pi\times50.3} = 3.19 \times 10^{-6}$ T, which is now about 16 times weaker than the earth's field. (That's still enough to throw the compass off about 4 degrees though...)

BUT : This is all assuming the wire is carrying a DC current. Power transmissions lines use AC, so the current direction will flip back and forth 60 times a second - almost certainly too fast for the needle to deflect very far.)

Magnetic Forces in wiring



The wires in a household lamp cord are typically 3.0 mm apart center to center and carry equal currents in opposite directions. If the cord carries current to a 100 W light bulb connected across a 120 V potential difference, what force per meter does each wire of the cord exert on the other? Is the force attractive or repulsive? Is this force large enough so it should be considered in the design of the lamp cord? (Model the lamp cord as a very long straight wire.)

Let's look at the wire on the left:

From the right-hand rule, it will create a magnetic field circulating around that wire. With your thumb pointing in the direction of the current, your fingers curl in the direction of the field, so the left wire will create a magnetic field that is pointing downward into the page at the location of the wire on the right side. OK, so now we have a field pointing INTO the paper, and a current in that other wire flowing DOWN the page. The force a current-carrying conductor feels in a magnetic field is given by $\vec{F} = I\vec{l} \times \vec{B}$, which means that this force is pushing off to the right. So the left current is exerting a force repelling the right current. (A similar argument shows that the right-side current is repelling the left-side current.)

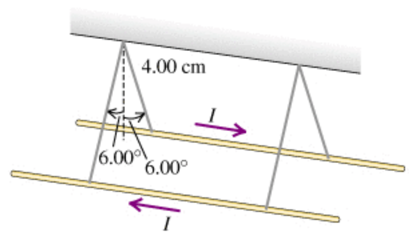
Section 28-2 shows that the force per unit length between two parallel conductors is: $\frac{F}{L} = \frac{\mu_o I I'}{2\pi r}$. Here, the current is the same in both wires, and we can find what it is from $P = VI$ or $I = P/V = (100)/(120) = 0.8333$ $amps$. The force (per meter) between the two wires in the lamp cord will be $\frac{F}{L} = \frac{(4\pi\times 10^{-7})(0.8333)(0.8333)}{2\pi\times(3\times 10^{-3})} = 4.6 \times 10^{-5}\ N/m$.

This is a pretty feeble force, probably smaller than the weight of the cords.

Since we've been mentioning DC vs AC currents in the prior problems, what effect would that have here? The actual current in the wiring will be AC. At one instant, the current in the 'left' wire will be in towards the bulb and that same current will be flowing out from the bulb in the 'right' wire, so the two currents are in opposite directions and will repel. A brief time later, the current reverses direction and the left wire has current out from the bulb and the right wire has current into the bulb. They're still in the opposite direction from one another, so will still be trying to repel each other. The magnitude of the force will fluctuate 60 times per second, but the direction will always be repulsive.

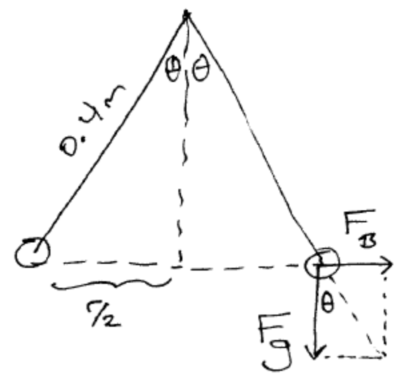
Magnetic Forces between wires

Two long, parallel wires hang by 4.0 cm cords from a common axis (see figure). The wires have a mass per unit length of 0.0125 kg/m and carry the same current in opposite directions. What is the current in each wire if the cords hang at an angle of 6.0^o with the vertical?



Section 28-2 shows that the force per unit length between two parallel conductors is: $\frac{F}{L} = \frac{\mu_o I I'}{2\pi r}$. Here, the current is the same in both wires, and when the current is flowing in opposite directions the force will be repulsive. (See the earlier problem 27 for this discussion.)

Looking along the wires then as shown in this figure, and focusing on, say, the wire on the right, we have the magnetic force pushing this wire outward, and gravity pulling it downward. Things are in equilibrium now, and we've done plenty of these types of problems in Physics 1. The balance of forces here requires that $\vec{F}_B$, $\vec{F}_g$ and the tension in the string add up to zero which ultimately implies that $\tan\theta = F_B/F_g$.



$F_B = \frac{\mu_o I^2 L}{2\pi r}$ and the distance between the two wires will be (from the trigonometry of the problem): $r = 2 \times (0.04 \sin\theta) = 0.08 \sin 6^o = 8.36 \times 10^{-3}$ m.

The gravitational force downward will be mg but we don't know the mass here. We DO know the mass per unit length, so we can write the mass as: $m = (mass\ per\ length)(L)$. The force of gravity is equal to $F_g = mg = \lambda L g$ where I've used λ to represent the mass per length.

We argued above that $\tan\theta = F_B/F_g$ so $F_B = F_g \tan\theta$ or $\frac{\mu_o I^2 L}{2\pi r} = \lambda L g \tan\theta$.

Note here we have L on both sides of this equation so we can cancel it out now:

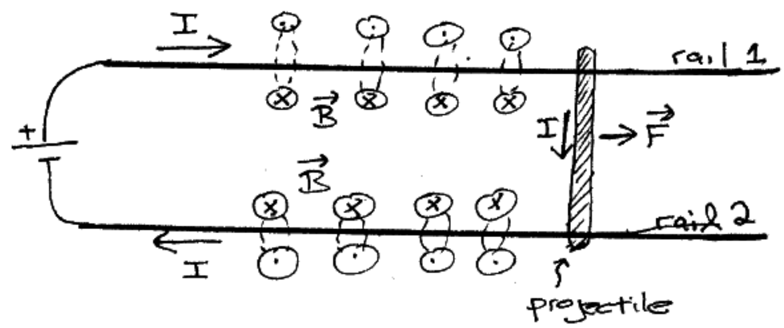
$\frac{\mu_o I^2}{2\pi r} = \lambda g \tan\theta$ or rearranging to solve for the current:

$I^2 = (2\pi r \lambda g \tan\theta)/(\mu_o) = (2\pi)(8.36 \times 10^{-3})(9.81)(0.0125)\tan 6^o/(4\pi \times 10^{-7}) = 538.2$ or finally $I = \sqrt{538.2} = 23.2$ $amps$.

Rail-gun updated

Using what we've learned so far, we can design another type of railgun that is self-contained and does not require an external source of magnetic field. (Note: this is a grossly simplified description that leaves out a lot of practical detail, but it's a start...)

The figure shows the geometry we'll be working with here. The two horizontal lines represent the 'rails' of the railgun, and we're looking down on the system from above. On the left, we have a source of voltage (I drew this as a battery, but we'll see later this will need to be a capacitor.) On the right, we've dropped a metal bar across the rails (so this metal bar is free to move).



Once the bar (which will become the projectile) is dropped across the rails, the circuit is complete and current will flow. Using the right-hand rule, the current flowing to the right in the bar at the top of the figure (rail 1) will create a magnetic field around that rail. With your thumb pointing in the direction of the current flow, that means that $\vec{B}$ will be pointing into the page in the space where the projectile exists. Looking at the other rail (rail 2), the current there is flowing to the left, and the RHR again implies that the $\vec{B}$ swirling around that rail will also be pointing into the page in the space where the projectile is located.

Looking at the projectile now, we have a current flowing towards the bottom of the page in the presence of a magnetic field pointing perpendicularly down into the page. The projectile will feel a force of $\vec{F} = I\vec{L} \times \vec{B}$ which is to the right, which will cause the projectile to accelerate to the right.

So that's the general idea here: the current flowing through this configuration generates a magnetic field which then causes the mobile part (the projectile) to accelerate down the rails until it leaves the system at a considerable velocity.

Let's try some arbitrary numerical values and see what happens.

Suppose $I = 10^6\ A$ and the tracks are separated by 5 cm.

(a) First, what will be the (approximate) force per unit length between these two tracks?

The equation we derived for the force between two current-carrying wires was for 'infinitely long' lines of current; here we have a meter or two of rails, separated by a few centimeters, so we won't be exactly correct, but it's a good first approximation:

$F/L = \frac{\mu_o}{2\pi}\frac{I_1 I_2}{r} = 2 \times 10^{-7}(10^6)^2/0.05 = 4 \times 10^6\ N/m.$

Hm. So even if our rails are only a few meters long, we have to deal with forces between them on the order of several **million** Newtons. For this reason, the 'rails' are usually just that: very solid metal rails instead of just wires, with the rails solidly connected to some base so they can't move, which is quite a feat considering the millions of Newtons of force that will exist between them.

(b) If the projectile has a mass of 1 kg, what will be its acceleration?

There is a current I flowing through the projectile, so there will be a force of $\vec{F} = I\vec{l} \times \vec{B}$ on it but again that was for a current in a constant magnetic field, which we don't quite have here. B will be much stronger near the rails and weakest at the point in the middle between the rails. Let's use the magnetic field at the midpoint to get an estimate at least of how much force we have.

From each wire: $B = \frac{\mu_o}{2\pi}\frac{I}{r}$ where here r will be halfway between the two wires. And we have the same magnitude (and direction) from the other wire, so overall we have twice that: $B = \frac{\mu_o I}{\pi r}$ is roughly the strength of the magnetic field in the midpoint.

The force on the payload will be $F \approx IrB$ so multiplying those out we end up with $\boxed{F \approx \frac{\mu_o}{\pi} I^2}$.

In this case, with $I = 10^6\ amp$, we get $F \approx 400,000\ N$.

If we use a 1 kg payload, $F = ma$ yields an acceleration of $400,000\ m/s^2$. From our 1-D equations of motion, $v^2 = v_o^2 + 2a\Delta x$ so if the rails are 2 m long, the payload will be ejected at the other end with a speed of $v \approx \sqrt{2a\Delta x} = 1300\ m/s$ or about 3000 $miles/hr$.

And that's for a railgun that's only 2 meters long.

(c) How much energy does the projectile have when it leaves the gun?

$K = \frac{1}{2}mv^2 = (0.5)(1)(1300)^2 = 850,000\ J$.

This is so much energy that the projectile tends to just become a lump of molten metal.

The largest railguns to date have achieved projectile energies about 10 or 20 times as large.

(d) How much POWER does this thing need to operate?

Well, we know it has to inject 850,000 joules in the brief time interval while the thing is operating. How long does it take the projectile to go from rest to being ejected? $v = v_o + at$ so $1300 = 0 + 400,000t$ so $t \approx 3 \times 10^{-3}\ s$.

$P = energy/time = (850,000\ J)/(3 \times 10^{-3}\ s) \approx 280MW$.

That's about the total power output of a large nuclear power plant...

So how do these things work in the real world? How can I get that much power, considering I only need it for a few milliseconds?

The only practical source is **capacitors**, or really a whole bank of lots of really big capacitors. Real-world rail-guns are physically large mostly because of the massive banks of large capacitors needed.

(Continued...)

(e) Hand-held railgun pistol?

Suppose we want a less extreme railgun - something that would launch a bullet-sized mass at a typical bullet velocity?

If we drop down to $m = 20\ gram = 0.02\ kg$ with a desired launch speed of $200\ m/s$, achieved over a distance of about $20\ cm$, this implies an acceleration of: $v^2 = v_o^2 + 2a_x\Delta x$ so $(200)^2 = (0)^2 + (2)(a)(0.2)$ so $a = 100,000\ m/s^2$ which means we need a force of $F = ma = (0.02)(100,000) = 2000\ N$.

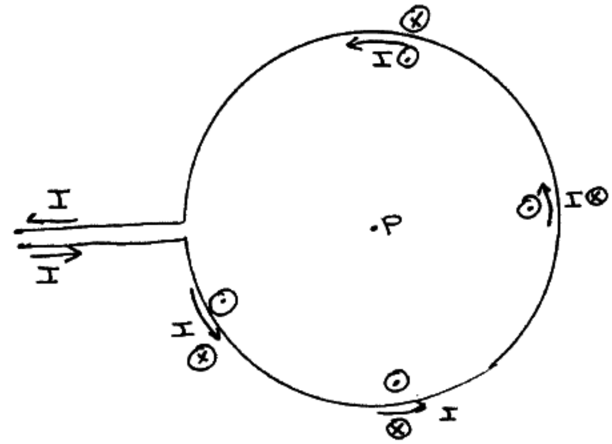
We found above that for this geometry, $F \approx \frac{\mu_o}{\pi} I^2$ so if we only need $2000\ N$ of force, we need a current of 'only' $71,000\ amp$. (Not sure I want to be holding anything in my hand that has that much current flowing through it, even for a few milliseconds...).

How much energy do we need to store in our capacitors? The bullet leaves the barrel with a kinetic energy of $K = \frac{1}{2}mv^2 = (0.5)(0.02)(200)^2 = 400\ J$. How large physically would the capacitor need to be to do that? High-tech supercapacitors have energy storage densities around $10,000\ J/kg$ and up so storing that much energy (or several times that, so we can fire multiple rounds) isn't out of the question.

You can find examples of home-built railguns on youtube, along with another approach using coils (see: coil guns) that are essentially solenoids on steroids...

Magnetic Field In a Loop of Wire

Suppose we bend a wire into a circle of radius R as shown in the figure. The wire carries a current of I amps. What is the strength and direction of the magnetic field at the center of the circle?

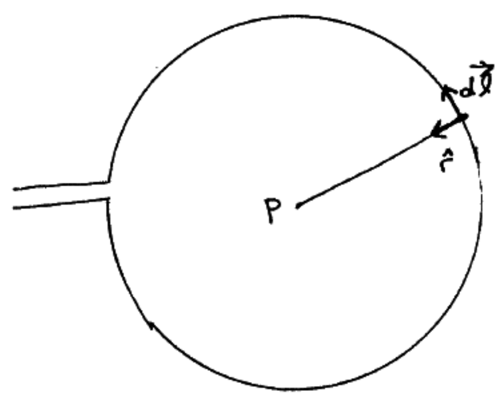


The Biot-Savart equation tells us that a little $d\vec{l}$ segment of wire carrying a current of I creates a magnetic field element $d\vec{B}$ at some point P equal to:

$$d\vec{B} = \frac{\mu_o I}{4\pi r^2} d\vec{l} \times \hat{r}$$

Before we pull in any equations, let's look at what sort of magnetic field this loop of wire is creating.

The two wires that are bringing the current into and out of the loop are nearly on top of one another, so their magnetic fields will cancel. More than that though - since those lines are basically radial towards or away from point P, the $d\vec{l}$ and $\hat{r}$ vectors are either parallel or anti-parallel, so their cross product is zero. Either way, we can ignore them.



Looking at a little element $d\vec{l}$ along the circumference of the circle, $d\vec{l}$ is perpendicular to $\hat{r}$ everywhere so the sine of the angle between them is 1. From the RHR, their cross product creates a vector coming straight up out of the page. Also, since each of those elements is located at $r = R$, they're all the same distance from the point P, so the magnetic field that each element contributes is identical.
The magnitude of each element will be: $dB = \frac{\mu_o I}{4\pi R^2} dl$ and now we can integrate over all of them: $B = \int dB = \int \frac{\mu_o I}{4\pi R^2} dl = \frac{\mu_o I}{4\pi R^2} \int dl$ where in the last step we pulled out everything that was a constant.

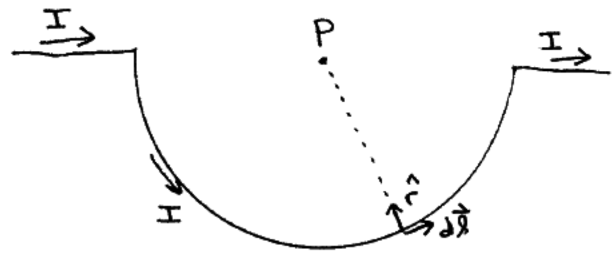
The integral of dl over a circle is just the circumference of the circle, or $2\pi R$ so $B = \frac{\mu_o I}{4\pi R^2}(2\pi R)$ which finally simplifies to $\boxed{B = \frac{\mu_o I}{2R}}$ and we previously determined that this field as a vector is pointing straight up out of the page.

Note how this behaves: the smaller the radius of the loop, the **larger** the field at the center of the loop will be. This can be a problem with integrated circuits where the currents may be small through some tiny piece of the circuit, but the sizes are so small that significant (albeit localized) magnetic fields can be generated which can affect currents flowing through other nearby parts of the circuit.

Also, we just encountered yet another right-hand rule. Here we have current flowing around a loop. If you curl your fingers in the direction of I, your thumb will point in the direction of the $\vec{B}$ created by that current loop. In this problem, the current was flowing counter-clockwise around the loop and produced a magnetic field coming up out of the loop's plane. If we turn the current around, the magnetic field will point down into the page.

Magnetic Field In a Partial Loop of Wire

Let's modify the previous problem so that the loop is just a semi-circle as shown in this figure. The radius of the semi-circle is still R, and still carrying a current of I. How strong (and in what direction) will the magnetic field be at point P located at the center of the circle?



What has changed from the previous problem? If we look at the two straight parts of the line, each little $d\vec{l}$ element along those lines will be pointing towards (or away from) point P, meaning they're parallel or anti-parallel to the $\hat{r}$ vectors that point from that element to the point of interest P. Those two segments of the wire won't contribute anything to the magnetic field at point P.

Along the semi-circle, each $d\vec{l}$ is still perpendicular to the $\hat{r}$ at that point, so $d\vec{l} \times \hat{r}$ is still just dl in magnitude, and represents a vector coming up out of the page.

The magnitude of each element is still: $dB = \frac{\mu_o I}{4\pi R^2} dl$ and now we can integrate over all of them: $B = \int dB = \int \frac{\mu_o I}{4\pi R^2} dl = \frac{\mu_o I}{4\pi R^2} \int dl$ where in the last step we pulled out everything that was a constant.

The only difference this time is we are not integrating over an entire circle - just half the circle. So $\int dl$ becomes half the circumference of the circle of just πR, leaving us with:

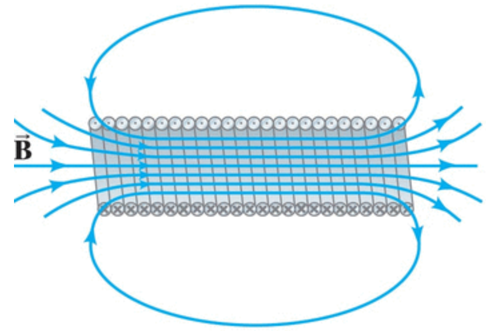
$B = \frac{\mu_o I}{4\pi R^2}(\pi R)$ which finally simplifies to $\boxed{B = \frac{\mu_o I}{4R}}$ and we previously determined that this field as a vector is pointing up out of the page.

This half-circle creates a magnetic field (at point P anyway) that is half the strength of the full loop.

NOTE: see also Example 28-13 in the book for a 'quarter-circle' loop where the wire is bent in a particular way that we can still ignore the field produced by wires coming into and going out from the arc.

Solenoids

A particular solenoid has a diameter of 1 cm and is 5 cm long. Wire is wrapped tightly around the cylinder from one end to the other, making 5000 $turns$ in total. How much current would we need to run through it to create a magnetic field of 1 T inside the solenoid?



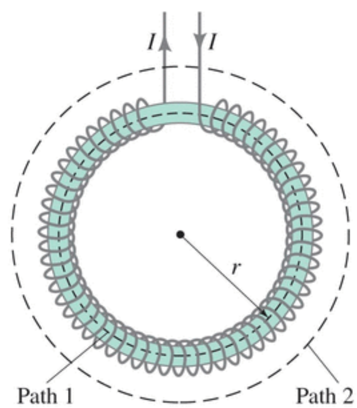
The radius or diameter of the solenoid is a red herring since it actually doesn't affect the strength of the magnetic field except near the edges.

The magnetic field within the interior of the solenoid has a magnitude of $B = \mu_o n I$ where n is the number of loops per meter the wire makes. We have 5000 loops of wire along a distance of 5 cm (0.05 m) so $n = (5000\ loops)/(0.05\ m) = 100,000\ loops/meter$.

We desire $B = 1\ T$ so: $B = \mu_o n I$ becomes $(1) = (4\pi \times 10^{-7}\ T\ m/A)(100,000\ m^{-1})(I)$ or $1 = 0.1256637..I$ so $I = 7.96\ amp$.

Toroids

Suppose we have a toroid that's about 2 meters across and we desire a magnetic field inside the toroid to be about 10 Tesla. What does that imply about the current and/or the number of turns of wire we need?



(a)

The magnetic field inside the toroid (i.e. inside the 'tube' the wire is wrapped around) is: $B = \frac{\mu_o N I}{2\pi r}$ where r is the distance from the center of the circle to the point inside the toroid, so the magnetic field is slightly non-uniform along the barrel of the tube.

With the given size and desired field strength, let's assume $r \approx 1\ m$ and $B \approx 10\ T$. Rearranging the equation a bit:

$$NI = 2\pi r B/\mu_o = 2\pi(1)(10)/(4\pi \times 10^{-7}) = 5 \times 10^7$$

We need a lot of loops, or a very high current, or some combination such that their product is quite large. We don't want to melt the wires, so are probably limited to a hundred amps or so. If $I = 100\ A$, then we'll need $N = 500,000$ loops of wire around the tube in order to create the desired field strength.

Toroidal shapes have been experimented with as part of developing the concept of fusion reactors where super-high temperature plasma is held in place by magnetic fields to keep it from touching the sides of the chamber.