Physics 2233 : Chapter 31 Examples : Electromagnetic Waves

Maxwell's equations relate electric and magnetic fields in fairly complex ways, involving space and time derivatives of each other. Combining these equations reveals that wave-like solutions can exist, often modeled as:

$E = E_o \sin(kx - \omega t)$ and $B = B_o \sin(kx - \omega t)$ where $v = \omega/k = 1/\sqrt{\epsilon\mu}$, ϵ being the 'permittivity' constant that appears frequently in electric field equations and μ being the 'permeability' constant that appears frequently in magnetic field equations. These are physical properties of the materials involved but in the case of free space (vacuum), they take on the values ϵ_o and μ_o and the resulting EM wave velocity becomes $v = 1/\sqrt{\epsilon_o\mu_o} = c$ which is about $3.0 \times 10^8\ m/s$.

The E and B fields are pendiculular to one another and propagate in the direction given by the right-hand rule (direction of $\vec{E} \times \vec{B}$). (Figure 31-9)

Electromagnetic Spectrum

As with all waves, $v = \lambda/T = \lambda f$ so with EM waves, $\boxed{c = \lambda f}$. See figure 31-12 on page 823 for an overview of the names commonly given to various ranges of frequencies and wavelengths (radio, light, microwave, xrays, etc).

Energy in EM Waves

From PH2223 : **energy density** (J/m^3) in the presence of an electric or magnetic field:

$u_E = \frac{1}{2}\epsilon_o E^2$ and $u_B = \frac{1}{2}\frac{1}{\mu_o}B^2$

In the case of EM waves, both are present but $B = E/c$ and we can combine these. Doing so, we find that even though the magnetic field is immensely weaker than the electric field ($B = E/c$), the energy density of the magnetic field is exactly the same as that of the electric field in an EM wave. Overall, the total energy density in the EM wave field is: $u = \epsilon_o E^2$ which, with various substitutions, can also be written in several forms: $\boxed{u = \epsilon_o E^2 = \frac{1}{\mu_o}B^2 = \sqrt{\frac{\epsilon_o}{\mu_o}}EB}$

Note: these are **instantaneous** values. Since E and B are sinusoidal in nature, a more useful form for the **average** energy density relates it directly to the **amplitudes** of the waves E_o and B_o :

$$\boxed{< u >= \frac{1}{2}\epsilon_o E_o^2 = \frac{1}{2}\frac{1}{\mu_o}B_o^2 = \frac{1}{2}\sqrt{\frac{\epsilon_o}{\mu_o}}E_o B_o}$$

Intensity (I or S) : the rate at which energy is passing through a given area (energy per square meter per second, i.e. watts per square meter). Basically we have an energy density of u (above) moving at the speed of light c so can relate the intensity to the strength of the E (and/or B) fields that make up the EM wave:

$I = \epsilon_o c E^2 = \frac{c}{\mu_o}B^2 = \frac{1}{\mu_o}EB$

NOTE: this form is **not** typically used as-is since it represents **instantaneous** intensity, which is **varying in time** (since E and B are varying with time). See below for more useful forms.

NOTE: In this field, the intensity is usually given the symbol S instead of I and is the magnitude of the Poynting Vector, defined as $\vec{S} = \frac{1}{\mu_o}\vec{E} \times \vec{B}$. Note that the direction of the vector $\vec{S}$ gives the direction of propagation of the wave.

Since E and B vary (rapidly) with time, it is usually more convenient to work with the average intensity. Since E (and B) are sinusoidal, $< E^2 >$ essentially introduces a factor of $< sin^2() >$ which, from calculus is just 1/2 :

$$\boxed{< I >= \frac{1}{2}\epsilon_o c E_o^2 = \frac{1}{2}\frac{c}{\mu_o}B_o^2 = \frac{1}{2}\frac{1}{\mu_o}E_o B_o}$$

This relates the intensity (I or S) directly to the amplitudes E_o and/or B_o of the underlying EM waves.

Another common choice is to relate the average intensity to the RMS values of E and B instead of directly to their amplitudes, but since E and B are sinusoidal, this conversion is simple: $E_{rms} = \frac{1}{\sqrt{2}}E_o$ and $B_{rms} = \frac{1}{\sqrt{2}}B_o$.

Radiation Pressure : P

Note here that the symbol P is being used for **pressure** (and not, for example power...).

EM waves carry energy but also carry momentum (even though they have no mass). When an EM wave (light, radar, anything) strikes an object, it will transfer momentum to that object, resulting in a force. The larger the area, the more momentum will be transferred, so this is usually denoted as a pressure and depends on whether the EM waves are reflected or absorbed.

In the case of fully absorbed waves: $P =< I > /c$

In the case of fully reflected waves: $P = 2 < I > /c$

Example 1 : Game Lag

Electrical signals (such as information traveling through the internet) travel at nearly the speed of light. A game server is located 2000 km away. How much latency will this introduce into the game?

When I hit a key, this information must travel from my computer to the server and then from the server back to my computer. The signal is thus traveling $2 \times 2000\ km$ or $4000\ km$ or $4 \times 10^6\ m$.

$d = vt$ so this signal, traveling at the speed of light, represents a time delay of $t = d/v = d/c = (4 \times 10^6\ m)/(3 \times 10^8\ m/s) = 0.0133\ s$ or about $13.3\ ms$

If the signal has to travel via one of the geostationary satellites ('satellite internet'), the time delay will be much worse. These satellites orbit at roughly $35,800\ km$ above the surface of the earth so the signal must travel from your computer to the satellite, from the satellite to the server, server to satellite, and finally satellite to your computer. In doing so it's traversed four times the orbital height of the satellite, or $4 \times 35,800\ km = 143,200\ km$. In this case the time delay becomes $t = d/c = (1.43 \times 10^8\ m)/(3 \times 10^8\ m/s) = 0.477\ s$ or $477\ ms$ (making multiplayer games over satellite internet pretty unpleasant).

Example 2 : Solar Sail

At the orbital distance of the earth from the sun, the sun's power represents an intensity of about $1400\ W/m^2$. This yields a radiation pressure that is sufficient to affect satellites with large arrays of solar cells to power the satellites. This effect is usually undesirable since it gradually pushes the satellites out of their designated orbits, but can also be exploited to push interplanetary probes.

Suppose we have a large square sail, 1 km along each side, made of an extremely thin and light material. We orient the sail so that it points towards the sun, collecting as much momentum as possible. What acceleration would this provide if the overall mass of the satellite and sail is $1000\ kg$, and the satellite were presently located at the same distance from the sun that the earth is? How long would it take for this satellite to accelerate to $17\ km/s$ which is roughly the fastest any interplanetary probe travels in deep space, away from the gravitational effects of the planets?

Radiation pressure for fully absorbed energy is $P = < I > /c$, so here we have $< I > = 1400\ W/m^2$ giving us a pressure of $P = (1400)/(3 \times 10^8) = 4.67 \times 10^{-6}\ N/m^2$.

$P = F/A$ so the force on the object will be $F = PA = (4.67 \times 10^{-6})(1000\ m)^2 = 4.67\ N$.

$F = ma$ so $a = F/m = (4.67\ N)/(1000\ kg) = 0.00467\ m/s^2$.

How long will it take to reach $17.1\ km/s$ or $17100\ m/s$?

$v = at$ so $t = v/a = (17100)/(0.00467) = 3.67 \times 10^6\ s$ or only about $42\ days$.

(This idea has been tested on a small scale several times, and actually used in practice with the IKAROS probe to Venus in 2010.)

Example 2 : MSU Campus Radio Station

WMSV operates at a frequency of 91.1 MHz with a power output of $14,000$ $watt$.

(a) What is the wavelength of this EM wave?

$v = \lambda/T = \lambda f$ so $\lambda = v/f$ with v being the speed of light in air. This is close enough to c that we can use $\lambda = c/f = (3 \times 10^8\ m/s)/(91.1 \times 10^6\ s^{-1}) = 3.293\ m$.

(b) What is the intensity at a distance of 2 km from the antenna?

The intensity I (or S) is the power per area, in this case the area being the area of a sphere with a radius of 2000 m. Technically this is the average intensity so $< I >= power/(4\pi r^2) = (14000)/(4\pi 2000^2) = 0.000278\ W/m^2$.

(c) What is the amplitude of the electric and magnetic fields in the EM wave at this distance?

The average intensity is related to the amplitude of the electric field by: $< I >= \frac{1}{2}\epsilon_o c E_o^2$ so $E_o = \sqrt{2 < I > /(\epsilon_o c)}$ or:

$$E_o = \sqrt{\frac{(2)(0.000278)}{(8.85\times10^{-12})(3\times10^8)}} = 0.458\ V/m.$$

$B_o = E_o/c = 1.52 \times 10^{-9}\ T$

(d) What is the radiation pressure at this distance for waves that are absorbed?

In the case of fully absorbed waves, the pressure is related to the intensity by $P =< I > /c$ so here $P = (0.000278)/(3 \times 10^8) = 9.3 \times 10^{-13}\ N/m^2$.

(e) Suppose we have a 30 cm by 30 cm piece of aluminum foil (which completely reflects the waves) hanging vertically in the presence of these EM waves. Is the radiation pressure enough to cause the sheet of foil to deflect from the vertical to any observable degree?

Aluminum foil is about 0.016 mm thick and has a density of 2.7 gm/cm^3 so this little piece of foil has a mass of $m = (2.7\ gm/cm^3)(30\ cm)(30\ cm)(0.0016\ cm) = 3.9\ gm$ or $m = 3.9 \times 10^{-3}\ kg$. The force of gravity would be $F_g = mg = 3.8 \times 10^{-2}\ N$.

The radiation pressure will result in a force of $F = (pressure)(area) = (2)(9.3\times10^{-13}\ N/m^2)(0.3\ m)^2 = 1.68 \times 10^{-13}\ N$. (Note we've doubled the pressure since in this case the EM waves are reflected instead of absorbed.)

That's 11 orders of magnitude smaller, so any deflection would be infinitesimal.

(f) What is the average energy density at this distance?

The energy density of the EM wave is related to the strength of the wave. We have various forms for this, but we're looking for the time-averaged energy density (i.e. ignoring the megahertz fluctuations in u). For example, $< u >= \frac{1}{2}\epsilon_o E_o^2$

We computed above that $E_o = 0.458\ V/m$ so $< u >= (0.5)(8.85\times10^{-12})(0.458) = 2.02\times10^{-12}\ J/m^3$.

These sorts of energy densities exist for all EM sources surrounding us, so we are essentially surrounded by an energy density a few orders of magnitude larger than this, but still many orders of magnitude too small to power anything. The various gadgets that charge phones without wires use induction which requires the phone to be fairly close to the charging station.

Example 4 : Microwave Oven

A microwave oven has conducting metal panels on its sides (highly protected from being touched). A very high frequency EM source generates standing EM waves in this cavity. The frequencies are chosen to be highly absorbed by molecules in food, heating them up.

From PH2223, we know that at a conductor, the component of $\vec{E}$ parallel to the conductor has to be zero. We can rig this by creating standing EM waves such that $E = 0$ at these locations. This is similar to what we did with standing waves on a string and results in the same equation: some number of half-wavelenghts has to exactly fit between the two sides: $n(\frac{\lambda}{2}) = L$ where L say is the distance between the plates on the left and right sides of the oven. Thus these standing waves must have wavelenghts of $\lambda_n = (2L)/n$. We can relate this to frequency: $c = \lambda f$ for EM waves though, so the frequencies of these standing waves will be $f_n = n\frac{c}{2L}$.

A wavelength of 12.2 cm is highly absorbed by food. What frequency does this represent?

$f = c/\lambda = (3 \times 10^8\ m/s)/(0.122\ m) = 2.46\ GHz$

That is unfortunate since old cordless phones and things like DSL modems also operate near this frequency, and using a microwave oven would disrupt those.

These are standing waves, so the (say) width of the chamber has to be a multiple of half the wavelength, or some multiple of 12.2 $cm/2 = 6.1\ cm$. (If there are plates in the top and bottom, they would also be separated by some multiple of 6.1 cm.)

The peaks of these standing waves will be 6.1 cm apart. Since the intensity depends on the square of the amplitude of the wave, this results in uneven intensity, with hot and cold zones (hence the rotating platform in many microwave ovens).