Physics 2233 : Additional Examples

There are examples related to material covered after chapter 35.

Electromagnetic Waves (chapter 31; sections 5,6,9)

(NOTE: see the examples pdf for chapter 31.)

In the wave model of EM phenomena, the waves represents electric and magnetic fields varying at a usually very high frequency, travelling at the speed of light in the medium.

If the intensity of the waves is I (watts per square meter), the underlying E and B field strengths in a vacuum can be found to be: $< I >= \frac{1}{2}\epsilon_o c E_o^2 = \frac{1}{2}\frac{c}{\mu_o}B_o^2$ (Note that $E = cB$.)

where $\epsilon_o = 8.85 \times 10^{-12} C^2/N \cdot m^2$ and $\mu_o = 4\pi \times 10^{-7} T \cdot m/A$

(If travelling through a material, ϵ_o and μ_o are replaced with the ϵ and μ of that material.)

Photons (chapter 37; sections 2,3)

In the particle model for light and other electromagnetic phenomena, the energy is being carried by tiny massless particles called photons.

Each photon carries an energy of $E = hf = hc/\lambda$ where f is the usual frequency, λ is the wavelength, and $h = 6.63 \times 10^{-34}\ J \cdot s$ (called Planck's constant).

Each photon also carries a regular linear momentum of $p = E/c = hf/c = h/\lambda$

Since it carries momentum (even though it's massless), it can transfer this momentum to physical objects during 'collisions'. If I represents the intensity ($watts/m^2$) of the EM wave, then this transfer of momentum represents a pressure (called the **radiation pressure**) of $P = I/c$ (if the photons are completely absorbed) or $P = 2I/c$ is the photons are perfectly reflected back. Note: here P represents **pressure**, not power, so $P = F/A$ (force/area).

Matter Waves (chapter 37, section 7)

Borrowing from the relationship above, quantum mechanics shows that physical particles moving have an effective wavelength of $\lambda = h/p$ where $p = mv$ is the linear momentum of the object.

Particles can display wave-like phenomena (such as diffraction) as a result.

Nuclear Decay (chapter 41, section 8)

The nucleus of an atom usually consists of a number of protons and neutrons collected in a space on the order of $10^{15}\ m$ across. The electrical force repelling the protons is overcome by another force that attracts the neutrons and protons together called the 'strong nuclear force'.

Certain combinations of neutrons and protons can be energetically unstable though and eject parts of the nucleus to reach an overall lower energy state. The rate at which this occurs is controlled by a **decay constant** λ (nothing to do with wavelength; the symbol is just being re-used here), which represents the fraction of nuclei that will decay in a given time interval, so $\Delta N/N = -\lambda \Delta t$ from which $N = N_o e^{-\lambda t}$

The time it takes for half of the nuclei to decay is called the **half life** of the material: $t_{1/2} = 0.693/\lambda$.

Photons (1)

A 100 W lightbulb will emit light over a wide spectrum of frequencies and wavelengths, but assume it's emitting at a wavelength of exactly $\lambda = 550\ nm$.

(Note: see the chapter 31 examples also, which relate this intensity to the underlying E and B fields and the momentum represented by this intensity.)

(a) How much energy does each photon carry?

A single photon carries an energy of $E = hf = hc/\lambda$ and here $\lambda = 550\ nm = 550 \times 10^{-9}\ m$ so:

$E = (6.63 \times 10^{-34}\ J \cdot s)(3 \times 10^{8}\ m/s)/(550 \times 10^{-9}\ m) = 3.62 \times 10^{-19}\ J$.

Energies this small are often quotes in units of **electron-volts** (the energy involved in accelerating something with a charge of $1e$ across a potential of 1 $volt$, so $1\ eV = 1.609 \times 10^{-19}\ J$.

Here then, we have: $E = (3.62 \times 10^{-19}\ J) \times \frac{1\ eV}{1.609 \times 10^{-19}\ J} = 2.25\ eV$.

(This is enough energy to eject electrons from some atoms, creating a current in what is called the **photo-electric effect** which was seen in Lab 10.)

(b) How many photons/second is the lightbulb emitting?

The bulb has an intensity of 100 W or 100 J/s, so in one second the bulb emits 100 J of energy. If each photon is only carrying an energy of $3.62 \times 10^{-19}\ J$, how many (N) do we need to represent that much energy?

$(N)(3.62 \times 10^{-19}\ J) = 100\ J$ so $N = 2.76 \times 10^{20}$.

The light bulb is emitting that many photons every second.

Photons (2)

Extending the previous example, if we're standing 1 m away from the light bulb and hold out our hand, how much force should we feel? (Assume the photons are entirely reflected by your hand. That isn't true, of course, since some are reflected and make their way to our eyes so we can actually see our hand, and some are absorbed and warm up our hands, but here let's assume they're entirely reflected.)

There are two approaches we can take here: one we covered, and one we did not.

Method we covered : radiation pressure (P) for fully reflected EM waves is $P = 2I/c$, where I is the intensity at the given location. Here, we have a 100 W bulb with it's light spread around a sphere of radius 1 m, representing an area of $S = 4\pi r^2 = (4)(\pi)(1)^2 = 12.6\ m^2$.

The intensity here then is $I = (100\ W)/(12.6\ m^2) = 7.94\ W/m^2$.

The radiation pressure for fully reflected light should be $P = 2I/c = (2)(7.94)/(3 \times 10^8) = 5.29 \times 10^{-8}\ N/m^2$.

This is force per area though, so we'll need to multiply by the area of our palm. Assume it's roughly a square 7 cm (0.07 m) on each side, so the area would be about $5 \times 10^{-3}\ m^2$.

$P = F/A$ so $F = PA = (5.29 \times 10^{-8}\ N/m^2)(5 \times 10^{-3}\ m^2) = 2.64 \times 10^{-10}\ N$.

Method we did not cover : Each second, the bulb is emitting a huge number of photons, each carrying some tiny amount of momentum which is being transferred to our hand. This thus represents a change in momentum of our hand occurring in a given time interval (one second, say), which is a force (recall $F_{avg} = \Delta p/\Delta t$ from Physics I). This gives us another path to compute the force that our hand should feel, so let's try this approach:

From the first example we found a huge number of photons are emitted per second by the bulb. Each carries some momentum: $p = hf/c = h/\lambda = (6.63 \times 10^{-34}\ J \cdot s)/(550 \times 10^{-9}\ m)$ or $p = 1.205 \times 10^{-27} J\ s/m$ but joules are a measure of energy so their fundamental units are $kg\ m^2/s^2$ so working through the units we end up with just $kg\ m/s$ which is the usual metric units for momentum. $p = 1.205 \times 10^{-27}\ kg\ m/s$. That's the momentum that each photon has when it 'collides' with our hand. They're being completely reflected so the change in momentum of our hand is twice that in the opposite direction, so **each photon** is introducing a Δp of $2.4 \times 10^{-27}\ kg\ m/s$ to our hand.

How many are landing on our hand in one second?

Well, in one second, we found that the bulb emits 2.76×10^{20} photons but they're spread around in all directions, so at a distance of 1 m from the bulb, they're spread around over an area of $S = 4\pi r^2 = 12.6\ m^2$. We found above that the palm of our hand has an area of about $5 \times 10^{-3}\ m^2$ so the **fraction** of these photons that fall on our hand would be $\frac{5 \times 10^{-3}\ m^2}{12.6\ m^2}$ or about 4×10^{-4}.

Of the 2.76×10^{20} photons the bulb is emitting, only $(2.76 \times 10^{20}) \times (4 \times 10^{-4}) = 1.1 \times 10^{17}$ of them land on our hand in one second.

Each is creating a Δp on our hand of $2.4 \times 10^{-27}\ kg\ m/s$ so the total Δp on our hand (in one second) would be: $(2.4 \times 10^{-27}\ kg\ m/s) \times (1.1 \times 10^{17}) = 2.64 \times 10^{-10}\ kg\ m/s$.

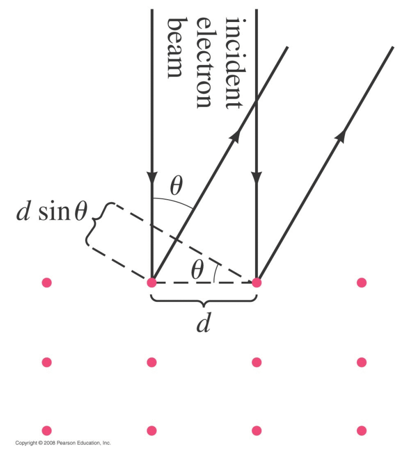
That much momentum change is happening each second and $F_{avg} = \Delta p/\Delta t$ so our hand is feeling a force of $F = 2.6 \times 10^{-10}\ kg\ m/s^2 = 2.64 \times 10^{-10}\ N$.

(Same result, and in either case way too weak to feel.)

Matter Waves

In 'electron diffraction', beams of electrons falling on a thin film of material will 'reflect' off the atoms (separated by about 0.2 nm). If this were light, reflecting off tiny reflective surfaces (like a CDROM), we'd treat this as a diffraction-grating type problem, and would find that constructive and destructive interference would be happening with the light at various different wavelengths, creating the rainbow patterns we see.

If the electrons just behaved like normal matter, they should bounce off the atoms in random directions and create a blur of electrons reflecting back in all directions. That is not what we see, though.
With tiny particles of matter, like electrons, we actually see the same diffraction phenomenon occurring, with the electrons behaving like waves with a wavelength of $\lambda = h/p$ where $p = mv$ is the momentum of the electron.
If we accelerate electrons across a 100 V potential, at what angles will we see strong reflections?



First, we'll need the momentum of the electrons. They'll have an energy of 100 eV after crossing this potential difference, representing an energy in joules of: $E = (100\ eV) \times \frac{1.609\times10^{-19}\ J}{1\ eV} = 1.609 \times 10^{-17}\ J$.

That represents a kinetic energy of $K = \frac{1}{2}mv^2 = 1.609 \times 10^{-17}\ J$ and using the electron mass of $m = 9.11 \times 10^{-31}\ kg$ we find $v = 5.943 \times 10^6\ m/s$ (which is far enough below the speed of light we can ignore relativity...).

The wavelength of this electron then will be $\lambda = h/p = h/(mv) = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(5.943\times10^6)} = 1.22 \times 10^{-10}\ m$ or about 0.122 nm.

The wavelength is just a bit smaller than the spacing between the reflection points and we'll have constructive interference where $d \sin\theta = m\lambda$ so here $\sin\theta = m\lambda/d = (m)(0.122\ nm)/(0.2\ nm) = (m)(0.61)$ so it looks like there is only one solution, at $m = 1$ (well, and $m = -1$) so we'll get a strong signal of electrons reflecting at $\theta = \pm 37.6^o$, instead of electrons being scattered back at random angles.

Radioactive Decay (1)

(a) What is the decay constant of ${}^{238}_{92}U$, whose half-life is 4.5×10^9 years?

Converting the half-life to standard units: $t_{1/2} = (4.5\times10^9\ year)\times\frac{3.156\times10^7\ sec}{1\ year} = 1.42\times10^{17}\ sec$

The decay constant and half-life are related via: $t_{1/2} = 0.693/\lambda$, so here:
$\lambda = 0.638/(1.42 \times 10^{17}\ s) = 4.49 \times 10^{-18}\ s^{-1}$.

That would be the **fraction** of ${}^{238}_{92}U$ atoms in a sample that would decay in each second.

(b) Suppose we have a small 10 *gram* sample of rock that contains some ${}^{238}_{92}U$ atoms. If we hold a Geiger counter 10 *cm* away from the rock, it reads about 1 decay each second. How many atoms of ${}^{238}_{92}U$ are in the sample? (Assume the surface area of the end of the Geiger counter is 8 cm^2.)

The radioactive atoms will be emitting α particles in random directions, and we're only detecting the ones that happen to fall on the area given. The area of a sphere 10 *cm* in radius would be $S = 4\pi r^2 = (4)(\pi)(10\ cm)^2 = 1257\ cm^2$ and we're only seeing the ones that fall in the 8 cm^2 area of the end of the Geiger counter. If we scale this up to the entire sphere, the count would be $(1\ per\ second) \times \frac{1257}{8} = 157\ s^{-1}$. 157 nuclei are decaying each second, and we're only seeing the one that enters the detector each second.

The decay constant came from the relationship that: $\frac{\Delta N}{N} = -\lambda\Delta t$. Here, the number of ${}^{238}_{92}U$ nuclei is decreasing by 157 each second, so we have:

$(-157)/N = -(4.49 \times 10^{-18}\ s^{-1})(1\ s)$ from which $N = (157)/(4.49 \times 10^{-18}) = 3.5 \times 10^{18}$.

That's how many ${}^{238}_{92}U$ nuclei must be in our sample. That sounds like a lot, but let's see.

(Note how rare this decay is though. We have 3.5×10^{18} of this type of Uranium atom in our sample, but in one second, only 157 of them decay. It takes a long time - 4.5 **billion** years for half the sample to decay.)

How much would that weigh? If we look it up, the mass of a ${}^{238}_{92}U$ atom is 238.05 u where u means 'atomic mass units' and $1\ u = 1.66 \times 10^{-27}\ kg$ so each atom has a mass of $(238.05\ u) \times \frac{1.66\times10^{-27}\ kg}{1\ u} = 3.95 \times 10^{-25}\ kg$.

We apparently have 3.5×10^{18} such atoms in our sample, so the total mass of the ${}^{238}_{92}U$ contained in the sample is $(3.95 \times 10^{-25}\ kg/atom) \times (3.5 \times 10^{18}\ atoms) = 1.38 \times 10^{-6}\ kg$ or $1.38 \times 10^{-3}\ gram$, which is very tiny.

(c) What fraction of the rock is this isotope of Uranium?

We found that 1.38×10^{-3} grams of our 10 *gram* sample consists of ${}^{238}_{92}U$, so that means that $(1.38 \times 10^{-3})/(10) = 1.38 \times 10^{-4} = 0.000138$ would be the fraction of the rock that is this isotope of Uranium. The other 0.999862 fraction of the rock is something else.

In nature, Uranium comes in various isotopes, but over 99 percent of it occurs in this particularly stable form. Less than 1 percent is in the form of the ${}^{235}_{92}U$ isotope used in nuclear weapons.

Radioactive Decay (2)

The activity of a radioactive source decreases by 2.5% in 31 hours. What is the half-life of this source?

<u>Version 1 : the picky, long version</u>

When we have some number of radioactive atoms, the number remaining after some time t has passed is $N(t) = N_o e^{-\lambda t}$ but we need to be careful here. The **activity** represents how many nuclei are decaying: it's how many clicks our Geiger counter reads (per time), for example. It's basically dN/dt and not N itself.

If we differentiate our N equation then, $dN/dt = -\lambda N_o e^{-\lambda t}$. This is the rate that the number of nuclei is dropping each second, so the number of counts we receive each second would be the opposite. Basically: $activity = (\lambda N_o)e^{-\lambda t}$

The quantity in parentheses is essentially the initial activity, which then drops off exponentially with time. Let's use R to represent our count rate (activity) so we can write this as: $R(t) = R_o e^{-\lambda t}$.

We're told here that at $t = 31\ hours = 111,600\ sec$, the rate has dropped to $R(t) = 0.975 R_o$ (it's dropped 2.5 percent from it's initial value, meaning it's now just 97.5 percent or 0.975 the initial rate).

Finally we can put this together: $R(t) = R_o e^{-111600\lambda}$ so $0.975 R_o = R_o e^{-111600\lambda}$ or $0.975 = e^{-111600\lambda}$.

Taking the natural log of both sides: $ln(0.975) = -111600\lambda$ so $-0.0253 = -111600\lambda$ or finally $\lambda = 0.0253/(111600\ s) = 2.27 \times 10^{-7}\ s^{-1}$.

Now that we have λ we can find the half life: $t_{1/2} = 0.693/\lambda = 3.05 \times 10^6\ sec$, or about 848 hours. Comparing that to the original information, 31 hours is only a small fraction of this half-life, so the activity shouldn't have had time to drop very much, so dropping only a couple of percent seems plausible.

<u>Version 2 (much shorter)</u>

Radioactive decay is a probabilistic phenomenon: some fraction of what we have will decay in a given period of time. If we have twice as much to start with, twice as many will decay (per second), so our count rate would double. Basically, the count rate is proportional to the amount of the stuff we have, so if the count rate has dropped by 2.5%, then the amount of stuff we have has also dropped by that same amount, and we can use our $N(t)$ equation directly.

$N(31\ days) = 0.975 N_o = N_o e^{-\lambda t}$ which immediately leads to the same equation (and result) we had above.

Radioactive Decay (3)

A $385-g$ sample of pure carbon contains 1.3 parts in 10^{12} (atoms) of ${}^{14}_{6}C$. How many disintegrations occur per second? (Note: this is similar to the first Radioactive Decay example, but we're starting from the other end this time.)

If we have N atoms, the decay rate λ gives the fraction of those atoms that will decay per second, so $(N)(\lambda)$ will be the actual number of decays per second.

We can find λ from the half-life. Appendix F in the book gives the half-life of ${}^{14}_{6}C$ to be 5730 $years$, or $(5730\ year) \times \frac{3.156\times10^{7}\ sec}{1\ year} = 1.808 \times 10^{11}\ sec$.

$t_{1/2} = 0.693/\lambda$ so $\lambda = 0.693/t_{1/2} = 3.83 \times 10^{-12}\ s^{-1}$. That's the tiny fraction of these atoms which will decay each second.

How many atoms of ${}^{14}_{6}C$ are in the sample though? We know that 1.3 parts in a trillion are this isotope, so how many carbon atoms in all are there in the sample?

If we look at a periodic table, the atomic mass of Carbon is given as 12.011 which means that Avogadro's number of these atoms weigh 12.011 $grams$. We have 385 $grams$ of the stuff, so we must have $(385)/(12.011) = 32.05$ times Avogadro's number of atoms present. $N_a = 6.022 \times 10^{23}$ so we have $(32.05)(6.022 \times 10^{23}) = 1.93 \times 10^{25}$ atoms of carbon in total.

The fraction of them that are the isotope we want is $\frac{1.3}{1\times10^{12}}$ so we have $(1.93 \times 10^{25}) \times \frac{1.3}{1\times10^{12}} = 2.51 \times 10^{13}$ atoms of ${}^{14}_{6}C$.

We found above that the fraction that decay each second is $\lambda = 3.83 \times 10^{-12}\ s^{-1}$ so the number of decays we should see (per second) would be: $(2.51 \times 10^{13}\ atoms) \times (3.83 \times 10^{-12}\ s^{-1}) = 96$ decays/sec.