

PH2213 : Examples from Chapter 1 : Units and Significant Figures

Key Concepts

Metric system : this course will use the MKS flavor of metric units (meters for lengths, kilograms for masses, and seconds for time). This leads to various composite units such as Joules for energy, Newtons for force, and so on.

Advantage: if you convert everything to standard units up front before starting the problem, then all results will come out in standard metric units as well.

Units Conversions : frequently units are given in non-standard units and will need to be converted. See the front of the book for a table of common conversion factors.

Prefixes : when dealing with particularly large or small quantities, a prefix is often used. The letter k in 1 kg represents a factor of 1000 so $1\text{ kg} = 1 \times 10^3\text{ g} = 1000\text{ g}$. The letter n in 1 nm represents a factor of 10^{-9} so 1.4 nm is equivalent to $1.4 \times 10^{-9}\text{ m}$. Basically each prefix is equivalent to a particular power of 10.

Uncertainty : measurements are rarely exact, so each measurement represents a range of possible values, denoted as: $x = 1.5 \pm 0.1$ for example, which means that x is somewhere between $1.5 - 0.1 = 1.4$ and $1.5 + 0.1 = 1.6$. These uncertainties will propagate through the solution to a problem, usually increasing the overall uncertainty in the result.

Key Equations

See tables in the book on conversion factors between various systems of units and the powers of ten represented by various letters.

Common Errors

- applying conversion factors ‘upside down’
- 1 m is 100 cm (not $1000\ldots$)
- Significant figures (maintain at least 4 s.f. in intermediate results so that final answer will be accurate to 3 s.f. (Don’t round off too much.))

NOTE: the online homework system usually requires an answer to be accurate to 2 significant figures, which means that any intermediate work you do should be kept to at least 3 or 4 significant figures.

1. Units Conversion : Speed Limit Sign

States that border Canada sometimes post speed limit signs in both English and metric units. What is the metric equivalent of 60 *mph*, in units of *m/s* and also *km/hr*?

(a) Converting 60 *mph* to *m/s*

60 *mph* means 60 $\frac{\text{mile}}{\text{hour}}$. We need to replace the english units of **mile** with a metric version. From the table of conversion factors, we see that 1 *mile* = 1609 *m* or equivalently $\frac{1 \text{ mile}}{1609 \text{ m}} = 1$ and $\frac{1609 \text{ m}}{1 \text{ mile}} = 1$.

We also want to replace **hour** with **sec** but 1 *hour* = 3600 *sec* so we have conversion factors: $\frac{1 \text{ hour}}{3600 \text{ sec}} = 1$ and $\frac{3600 \text{ sec}}{1 \text{ hour}} = 1$

Converting units is equivalent to multiplying an expression by **1** in the form of the factors given above, chosen so that we basically cancel out the unit we want to get rid of, replacing it with the one we want.

$$60 \frac{\text{mile}}{\text{hour}} \times \frac{1 \text{ hour}}{3600 \text{ sec}} \times \frac{1609 \text{ m}}{1 \text{ mile}} = 26.82 \text{ m/s}$$

(b) Converting 60 *mph* to *km/hr*

Here, we already have the time measured in the desired units (hours, in both cases) so only need to deal with the length part. 1 *mile* is the same as 1609 *m* and 1 *km* is 1000 *m* so:

$$60 \frac{\text{mile}}{\text{hour}} \times \frac{1609 \text{ m}}{1 \text{ mile}} \times \frac{1 \text{ km}}{1000 \text{ m}} = 96.54 \text{ km/hr}$$

(Note: this is close to 100 *km/hr* so you'll occasionally see highway signs that give a speed limit of 60 *mph* and 100 *km/hr*, even though that metric version is slightly faster than 60 *mph*.)

2. Uncertainty in an area measurement

Suppose the width and length of a rectangular area is measured two ways:

- The length is measured roughly, giving a value of $L = 10 \pm 1$ meters which is a percentage uncertainty of 10 percent.
- The width is measured more accurately, giving a value of $W = 10 \pm 0.1$ meters, which is an uncertainty of only 1 percent.

The area then is $A = LW$ or $A = (10 \pm 1)(10 \pm 0.1)$.

The minimum area will be when both numbers are at the low end of their ranges:

$$A = (10 - 1)(10 - 0.1) = (9)(9.9) = 89.1$$

The maximum area when both numbers are at the high end of their ranges:

$$A = (10 + 1)(10 + 0.1) = (11)(10.1) = 111.1$$

We want to write this compactly as $A \pm \Delta A$.

The average area will be $(89.1 + 111.1)/2 = 100.1$

The overall range from the smallest to the largest will be $2\Delta A$ so $\Delta A = \frac{1}{2}(111.1 - 89.1) = 11$.

Our (almost) final answer then will be $A = 100.1 \pm 11$.

The uncertainty in the final area is pretty large here, so the extra 0.1 on the average part is overwhelmed by that uncertainty. We'd probably just write this as $A = 100 \pm 11$.

NOTE that at the end here, our resulting area has a percentage uncertainty of 11 percent. We started with a 10 percent uncertainty in the length and a 1 percent uncertainty in the width and ended up with an 11 percent error in the overall area.

This is an important result: when we combine measurements that have errors, those errors generally ACCUMULATE, resulting in an even worse (fractional or percentage) error in the final result.

As far as homework and test problems go, it means that if you want to end up with a result that is accurate to (say) 2 significant figures, any intermediate calculations that you do should be kept to at least 3 and maybe 4 significant figures.

3. Uncertainty and Calculus

When we have explicit equations relating variables, we can sometimes derive useful relationships between the uncertainty in the overall result as a function of the uncertainty in the input measurements.

Suppose we have a cube of material and want to see how an uncertainty in the length of the sides is related to the uncertainty in the overall volume of the cube.

Let x be the length along each side of the cube. Then the volume of the cube will be $V = x^3$. If our side length has some uncertainty: $x \pm \Delta x$, how does that translate into the resulting volume calculation: $V \pm \Delta V$?

If we differentiate this equation with respect to x : $dV/dx = 3x^2$ which we can write as $dV = 3x^2 dx$.

Derivatives were introduced by looking at small intervals Δx and then taking the limit as the interval shrinks to zero, essentially turning Δx into dx and so on.

Let's do the reverse here, and turn our little differentials into small intervals. Then $\Delta V = 3x^2 \Delta x$.

Dividing this equation by the equation $V = x^3$ we end up with: $\boxed{\frac{\Delta V}{V} = 3 \frac{\Delta x}{x}}$.

We can read this as saying that the fractional (or percentage) uncertainty in the volume of the cube is **three times** larger than the fractional (or percentage) uncertainty in the measurement of the side length.

If we can only measure the length of the sides to 1 percent, this turns into a 3 percent error in the volume.