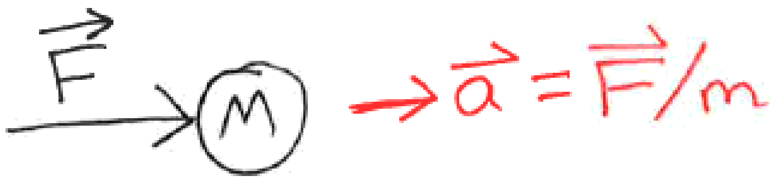
PH2213 Fox : Lecture 07
Chapter 4 : Dynamics : Newton's Laws of Motion

Force vs Acceleration

The first few chapters dealt with motion (1-D and beyond), focusing in particular on situations where the acceleration is a constant.

Forces are what cause an object to accelerate. A **vector force** $\vec{F}$ applied to an object of mass m will cause that object to accelerate in the same direction of $\vec{F}$ and with a magnitude of $a = |\vec{F}|/m$, so we can write that as $\vec{a} = \vec{F}/m$.



Force Units : MKS Metric System

- Rearranging $\vec{a} = \vec{F}/m$ into the more common form: $\vec{F} = m\vec{a}$, let's replace each term with its units to determine what units forces must have:
- $[force\ units] = [kg][m/s^2] = kg\ m/s^2 =$ **Newtons (N)**

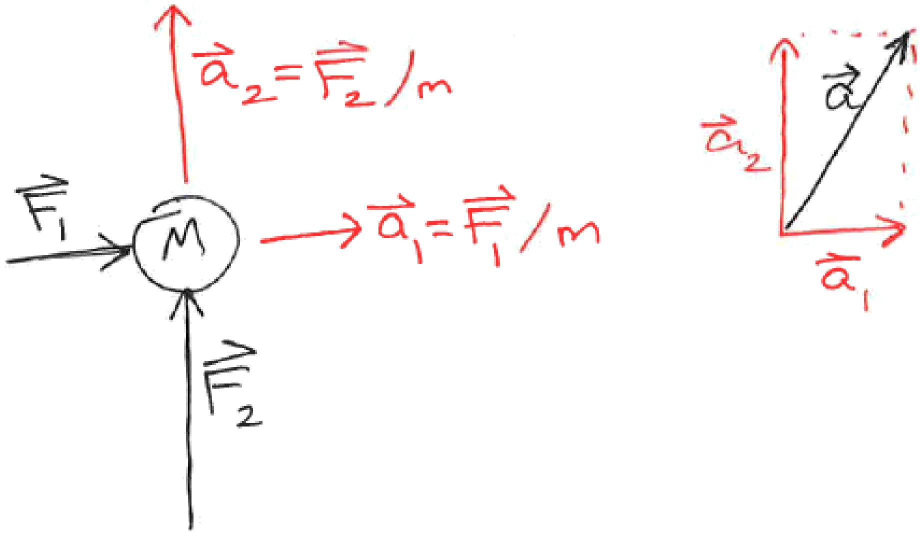
Force Units : English System

- One common English unit of force is the **pound** :
- $1\ lb = 4.44822\ N$ or $1\ N = 0.2248\ lb$

Force Units : CGS Metric System

- In chemistry and biology, the CGS version of the metric system is common where masses are measured in **grams** and lengths in **centimeters**.
- In this system, one $\boxed{\text{dyne}}$ of force will cause a $\boxed{1\ gram}$ mass to accelerate at $\boxed{1\ cm/s^2}$.
- $1\ dyne = 1 \times 10^{-5}\ N$

Multiple Forces



- $\vec{F}_1$ applied to an object of mass m produces an acceleration of $\vec{a}_1 = \vec{F}_1/m$
- $\vec{F}_2$ applied to an object of mass m produces an acceleration of $\vec{a}_2 = \vec{F}_2/m$
- The two forces applied at the same time produces **both accelerations simultaneously**: $\vec{a} = \vec{a}_1 + \vec{a}_2$
- $\vec{a} = \vec{a}_1 + \vec{a}_2 = \vec{F}_1/m + \vec{F}_2/m = (\vec{F}_1 + \vec{F}_2)/m$ or rearranging: $\vec{F}_1 + \vec{F}_2 = m\vec{a}$.
- When **multiple forces** are present, the object's acceleration is the same as if a **single** force equal to the sum of the individual forces (the 'net Force') had been applied: $\boxed{\vec{F}_{net} = \sum \vec{F}_i}$
- This leads to the most common form: $\boxed{\sum \vec{F}_i = m\vec{a}}$

That final point is important to understanding and analyzing situations where forces are involved. Many forces may be acting on some object, but we can combine them (as vectors) into a single force to determine how the object will react.

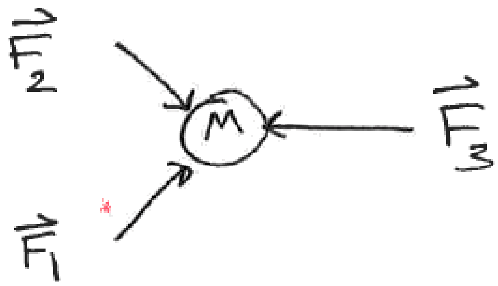
(NOTE: technically that's true only for the 'point-mass' model of objects. Later on, we'll see how applying forces to different locations on an object can induce rotations as well.)

The reverse is also true. Even if only a single force is present, it can be convenient to replace it with two forces (aligned with the coordinate system chosen) which add up to the original force vector.

We'll see how this all plays out as we work through several examples.

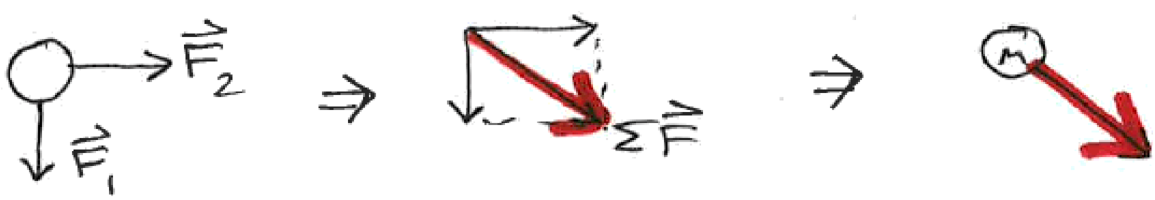
Newton's Laws

- **First Law** : if there is no net force on an object, it will continue to move with a **constant velocity** (which could be zero but doesn't have to be).
  $\boxed{\sum \vec{F}_i = 0 \text{ implies } \vec{a} = 0}$ and equivalently: $\boxed{\vec{a} = 0 \text{ implies } \sum \vec{F}_i = 0}$

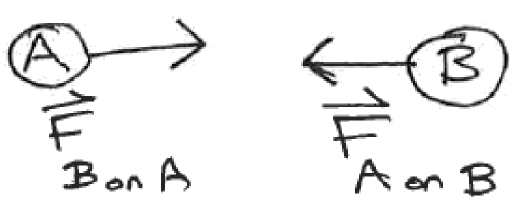
  

  NOTE: this, plus a corresponding equation for rotational forces (torques) is the basis for the **statics** course many (most?) of you will end up taking.

- **Second Law** : if there is a net force on an object, then it will cause that object to accelerate according to $\boxed{\sum \vec{F}_i = m\vec{a}}$
  Note the picky **wording** there: The forces acting **on an object** are what produce the acceleration of **that** object. Other forces may be present, and may indirectly work their way onto the object, but ultimately it's the forces acting directly **on** the object that are all it 'knows' about.

- **Third Law** : all forces are **interactions** between objects. If Ⓐ exerts a force on Ⓑ, then Ⓑ exerts an **equal** and **opposite** force on Ⓐ. Same magnitude, exactly opposite direction. (Think magnets, charged objects, masses attracting each other via gravity, etc.)

There are many different types of forces, but in this class we'll focus mostly on gravity, externally applied forces, normal forces, elastic (spring) forces, and friction. Electric and magnetic forces are left to PH2223, and other forces like what occurs inside the nucleus of an atom get briefly touched upon at the end of PH2233.

Gravity: $\vec{F}_g$

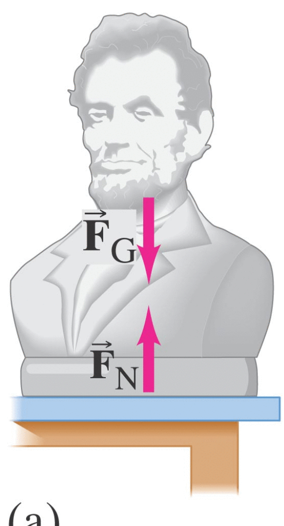
Objects with mass will attract one another. We'll cover this in more detail in chapter 6, but for now if we restrict ourselves to situations on or very near the surface of a large body (planet or moon) a good approximation for the force of gravity is $\vec{F}_g = m\vec{g}$.

Example: suppose we have an object of mass m that is released from rest above the floor. Determine it's acceleration.

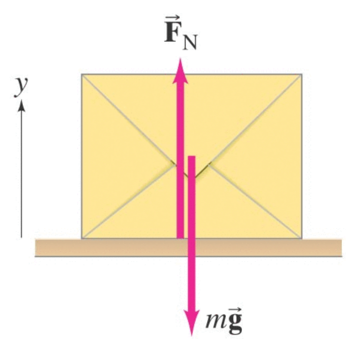
Once the object is released, the only force acting on it is gravity so Newton's 2nd law states $\sum \vec{F}_i = m\vec{a}$ and in this case our 'sum' only has one term, so: $\vec{F}_g = m\vec{a}$ but $\vec{F}_g = m\vec{g}$ so replacing the left side we have: $m\vec{g} = m\vec{a}$ or simply $\vec{a} = \vec{g}$. If an object only has gravity acting on it, it's acceleration will be $\vec{g}$, which is a constant, so all our equations-of-motion machinery from chapters 2 and 3 become available to analyze it's motion.

Normal Force : F_N

Suppose our object is sitting on a table with a horizontal, flat surface. Gravity is still acting on the object, but it doesn't appear to be accelerating, so there must be another force present. That force is called the **normal force**. Basically, the atoms that make up the object can't pass through the atoms that make up the table.



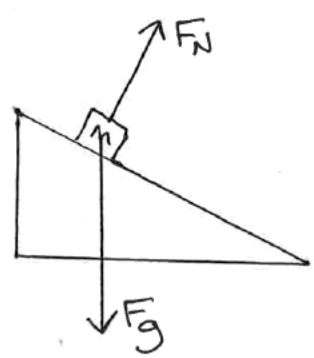
(a)



(a) $\Sigma F_y = F_N - mg = 0$

The name **normal force** refers to the fact that it's direction is normal (i.e. perpendicular) to the contact surface between the two objects. It just prevents things from moving through each other.

Suppose the object is sitting on a sloping surface. It can slide along the surface fine, it just can't move **through** it. The normal force is perpendicular to the surface:



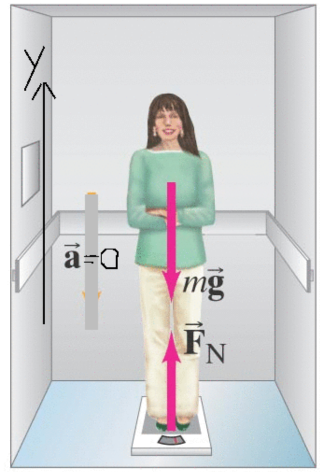
Example: Object on scale, in elevator AT REST

Suppose a 60 kg person is standing on a scale, in an elevator. If the elevator is at rest, what will the scale read?

Newton's Laws apply separately to each 'object', so if we apply them to the person, they have a mass so gravity is exerting a force of $\vec{F} = m\vec{g}$ downward. They're not passing through the scale, so it must be exerting a force upward on them of $\vec{F}_N$.

Applying Newton's 2nd law to the person: $\sum \vec{F}_i = m\vec{a}$ or:

$\vec{F}_g + \vec{F}_N = 0$. Let the Y direction be pointing vertically upward. Then picking off the Y (that is, the $\hat{j}$) components in that equation, we have $-mg + F_N = 0$ or $F_N = mg = (60\ kg)(9.8\ m/s^2) = 588\ N \times \frac{1\ lb}{4.44842\ N} = 132.2\ lb$.



Short Detour : Focusing on the scale now, we see that (from Newton's third law) that if the scale is exerting a force upward of mg on the person, the person is also exerting a force downward on the scale of mg. Basically that's what the scale is designed to provide: how much force is something applying to it. In the old days, there would be a spring inside the scale, and the weight of the object placed on it would compress the spring, causing (via an arrangement of mechanical levers) a pointer on a dial to respond. More recently, this force-on-the-scale causes a crystal inside the scale to be compressed which directly produces a small current or voltage that a circuit converts into a number displayed on a screen.

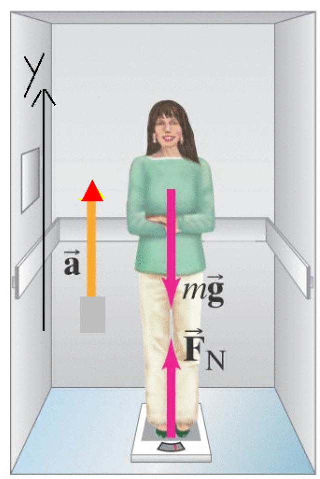
Key Point : Note that our analysis would be identical if the elevator were moving up or down, as long as it's moving at a constant velocity. Remember that $\vec{a}$ on the RHS of Newton's Laws is the acceleration, not the velocity. A constant velocity (whether zero or not), means that $\vec{a} = d\vec{v}/dt$ is zero.

Example: Object on scale, in upward accelerating elevator

Suppose the same person is in an elevator that is accelerating **upward** at 2 m/s^2. What will the scale read? (See Example 4-8 in the textbook also, where the elevator is accelerating downward.)

Focusing on the person again, they're accelerating upward along with the elevator, so $\Sigma\vec{F} = m\vec{a}$ and the RHS is no longer zero.

Picking off the vertical (Y) component of that equation: $-mg + F_N = ma$ or $F_N = mg + ma = (60\ kg)(9.8\ m/s^2) + (60\ kg)(2\ m/s^2) = 588 + 120 = 708\ N$ (or 159.2 lb). The scale must be exerting that much force upward on the person, so the person is exerting the same amount of force downward on the scale, producing a reading that is higher than before.

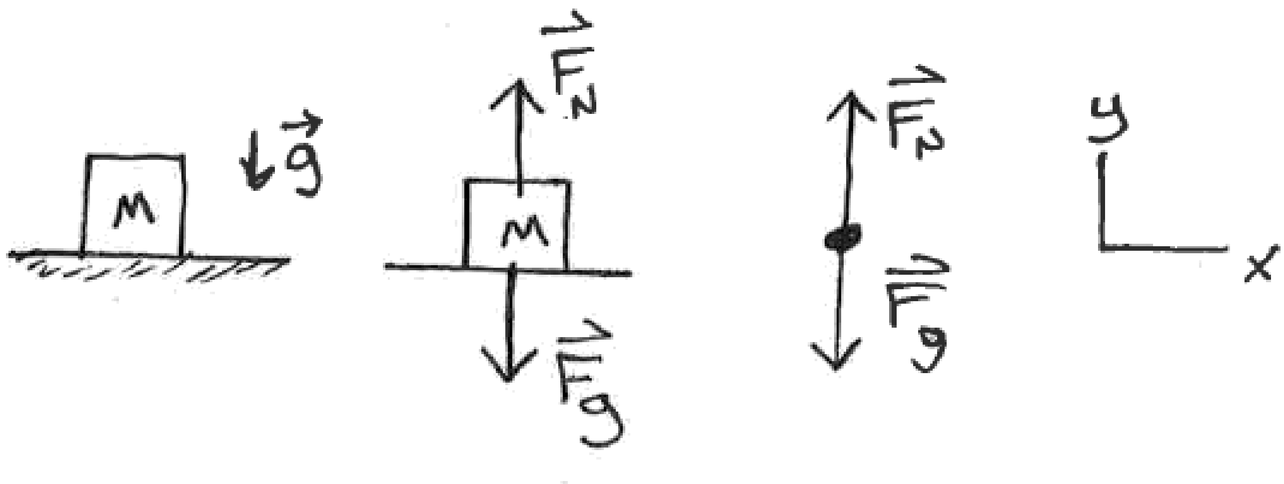


This scenario might represent:

- * elevator starting at rest, then rising
- * elevator moving downward but then coming to a stop

In both cases the **acceleration** vector is upward. Don't confuse acceleration with the direction of the velocity. Remember that acceleration is the **derivative** of velocity: how the velocity is **changing**.

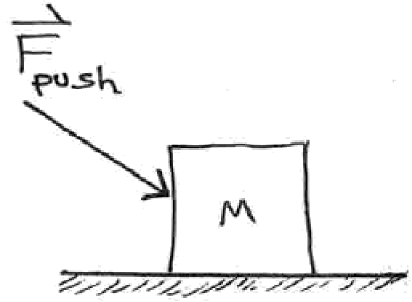
Steps Involved in Applying Newton's Laws



Scenario | Forces Present | Free-body Diagram | Appropriate Coordinates

- Problem statement : visualize (sketch) what's going on
- Select the object to which we'll apply Newton's Laws
- Note **all** forces acting on **that** object
  - if the object has mass, then we have gravity: $\vec{F}_g$
  - if the object is touching another object, we have a normal force $\vec{F}_N$
  - any other forces?
- Optional: free-body diagram version
  - replace object with point mass
  - add force vectors
- Select an appropriate coordinate system
  - forces here are entirely in the vertical direction, so that should be one of the coordinate axes
- Apply Newton's Laws: $\sum \vec{F}_i = m\vec{a}$
  $\sum F_x = ma_x$ and separately $\sum F_y = ma_y$ and so on

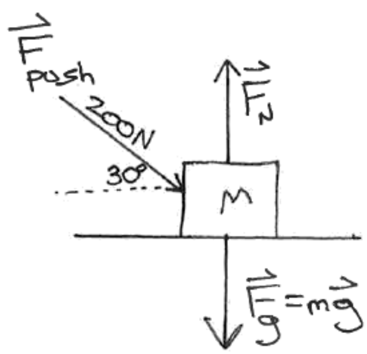
Example: $M = 100\ kg$ crate sitting on floor; being pushed with $|F| = 200\ N$ directed 30^o below the horizontal. Determine the acceleration of the crate across the floor.

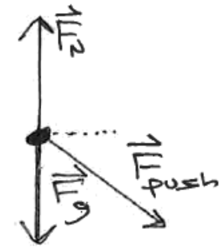


Object : the crate

Forces Present :

- It has a mass, so $\vec{F}_g$ vertically downward
- The $200\ N$ pushing force at the given angle
- A normal force $\vec{F}_N$ keeping the crate from moving through the floor

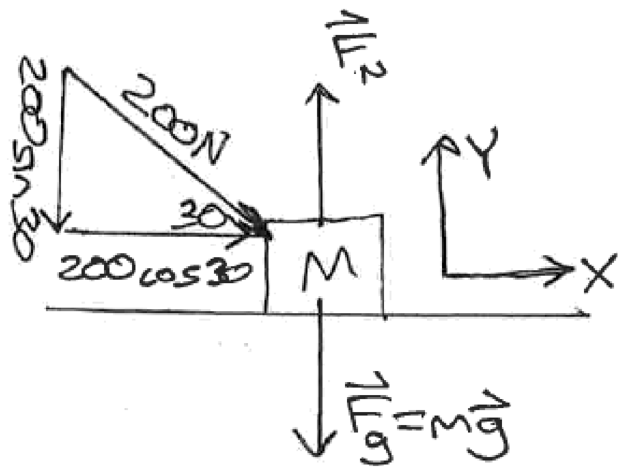




Free-body version

Choose a coordinate system : the crate will slide across the floor to the right, so we'll elect to use $+X$ to the right, and $+Y$ vertically upward.

Resolve vectors into components : any force vectors that aren't already entirely in X or Y will need to be converted into their X and Y components. Here, the pushing force is to the right and down, so it's replaced with a Y component pushing down, and an X component pushing to the right.



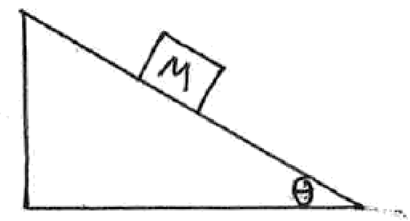
Apply Newton's Laws

X direction : $\sum F_x = ma_x$ so pick off the X components of all the force vectors acting on the crate. With our coordinate system choice, $\vec{F}_N$ and $\vec{F}_g$ are entirely in the Y direction, so their X components are zero, leaving only the X component of the pushing force:

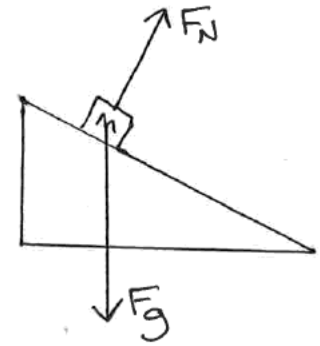
$\sum F_x = ma_x$ so here: $200 \cos 30 = (100)(a_x)$ or $a_x = +1.732\ m/s^2$.

Y direction : $\sum F_y = ma_y$ so pick off the Y components of all the force vectors acting on the crate. $\vec{F}_g$ has a magnitude of $F_g = mg = (100\ kg)(9.8\ m/s^2) = 980\ N$ and is directed straight down, so it has a Y component of $-980\ N$. The pushing force has a Y component of $200 \sin 30 = 100\ N$ and is also in the negative Y direction, so $F_{push,y} = -100\ N$. Collecting them: $-980 - 100 + F_N = 0$ or $F_N = 1080\ N$.

Example: Object of unknown mass M is placed (at rest) on a 30^o incline. It slides down the incline until it reaches the floor, travelling a distance along the incline of 2 m. How fast is the object moving when it reaches the bottom of the ramp?



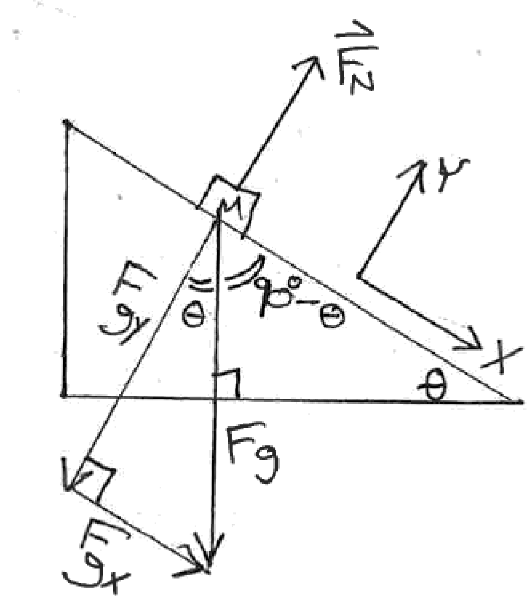
Forces acting on the object :



- It has a mass, so $\vec{F}_g$ vertically downward
- A normal force $\vec{F}_N$ keeping the object from moving through the ramp.

Choose a coordinate system : the object will slide down the ramp so let's use a **rotated coordinate system** with $+X$ parallel to the ramp, and $+Y$ perpendicular to the ramp and aimed up away from the ramp.

Resolve force vectors into components : F_N is entirely in the Y direction, so nothing to do there, but we'll need to convert $\vec{F}_g$ into components in our rotated coordinate system. Propagating the angle of the incline around a bit we end up with an angle inside the F_g 'triangle'. $\vec{F}_g$ has a magnitude of $F_g = mg$ straight down, so we'll have $|F_{gx}| = mg\sin\theta$ in the rotated +X direction, and $|F_{gy}| = mg\cos\theta$ in the rotated negative Y direction, so $F_{gy} = -mg\cos\theta$.

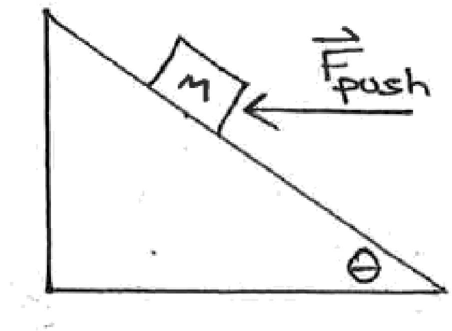


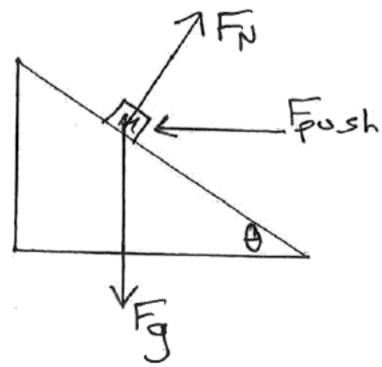
Apply Newton's Laws

X direction : $\sum F_x = ma_x$ so pick off the X components of all the force vectors acting on the crate. With our coordinate system choice, we only have F_{gx} in the X direction, so $mg\sin\theta = ma_x$ or $a_x = g\sin\theta = (9.8\ m/s^2)\sin(30^o) = 4.9\ m/s^2$. The object starts at rest so we can find it's speed at the bottom of the ramp: $v_x^2 = v_{ox}^2 + 2a_x\Delta x$ or $v_x^2 = (0)^2 + (2)(4.9\ m/s^2)(2\ m)$ from which $|v_x| = 4.427\ m/s$ (and it's sliding along the ramp, so $v_y = 0$).

Y direction : $\sum F_y = ma_y = 0$ since the object isn't changing it's Y coordinate at all as it slides down the ramp. Picking off Y components of all the force vectors acting on the crate: $F_N - mg\cos\theta = ma_y = 0$ so $F_N = mg\cos\theta$. (We don't know the mass here, so can't determine a numerical value for F_N but we do see that it'll be less than the full weight (mg) of the object.

Example: An object of $M = 100\ kg$ is sitting on a 40^o ramp. If we want to hold the object in place by pushing on it horizontally as shown in the figure, how much force do we need to apply?



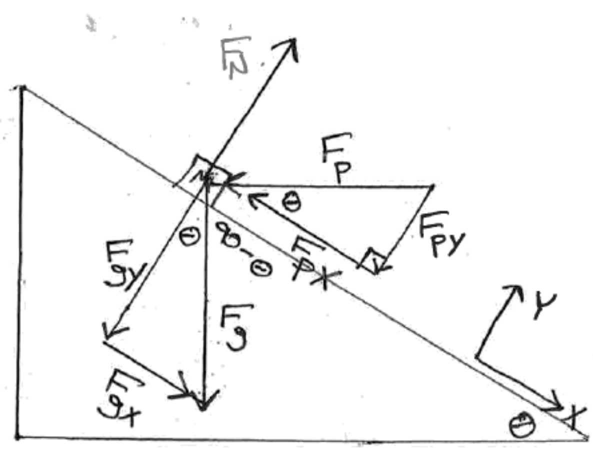


Forces acting on the object :

- It has a mass, so $\vec{F}_g$ vertically downward
- A normal force $\vec{F}_N$ keeping the object from moving through the ramp.
- The horizontal pushing force $\vec{F}_{push}$

Choose a coordinate system : without the pushing force, the object would slide down the ramp so let's use a rotated coordinate system with $+X$ parallel to the ramp, and $+Y$ perpendicular to the ramp and aimed up away from the ramp.

Resolve vectors into components : F_N is entirely in the Y direction, so nothing to do there, but we'll need to convert the others. Propagating the angle of the incline around a bit we see that $|F_{gx}| = mg \sin\theta$ aimed in the **+X** direction, $|F_{gy}| = mg \cos\theta$ aimed in the **-Y** direction, $|F_{push,x}| = F_{push} \cos\theta$ aimed in the **-X** direction, and $|F_{push,y}| = F_{push} \sin\theta$ aimed in the **-Y** direction.
See the Resolving Vectors information on Canvas, which is a link to: `https://newton.ph.msstate.edu/~fox/ph2213/vectors/index.html`



Apply Newton's Laws

X direction : $\sum F_x = ma_x = 0$ (zero since we want the object to just sit there), so pick off the X components of all the force vectors acting on the crate: $F_{gx} + F_{push,x} = 0$ or $mg \sin\theta - F_{push} \cos\theta = 0$ from which $F_{push} = mg \sin(\theta)/\cos(\theta) = mg \tan\theta$. Substituting in the information provided: $F_{push} = (100\ kg)(9.8\ m/s^2) \tan(40^o) = 822.3\ N$.

(We could do the Y direction to determine the amount of normal force present, but the X direction gave us what we were looking for so we'll just stop there.)