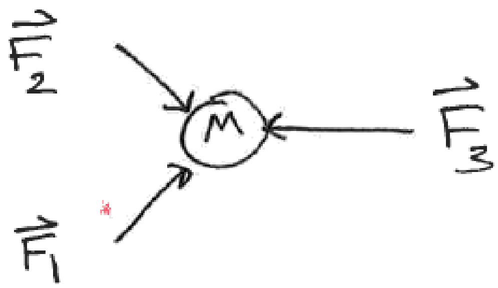
PH2213 Fox : Lecture 08
Chapter 4 : Dynamics : Newton's Laws of Motion

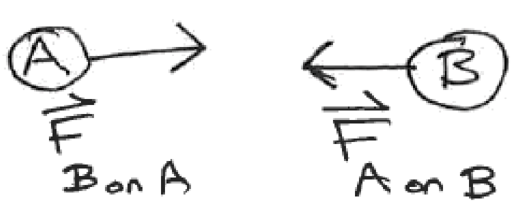
Newton's Laws

- **First Law** : if there is no net force on an object, it will continue to move with a **constant velocity** (which could be zero but doesn't have to be).
  $\boxed{\sum \vec{F}_i = 0 \text{ implies } \vec{a} = 0}$ and equivalently: $\boxed{\vec{a} = 0 \text{ implies } \sum \vec{F}_i = 0}$

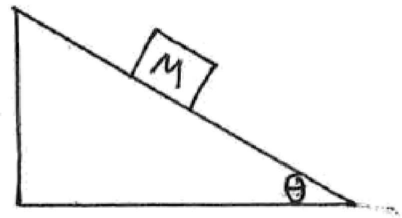


- **Second Law** : if there is a net force on an object, then it will cause that object to acceleration according to $\boxed{\sum \vec{F}_i = m\vec{a}}$
  Note the picky **wording** there: The forces acting **on an object** are what produce the acceleration of **that** object. Other forces may be present, and may indirectly work their way onto the object, but ultimately it's the forces acting directly **on** the object that are all it 'knows' about.

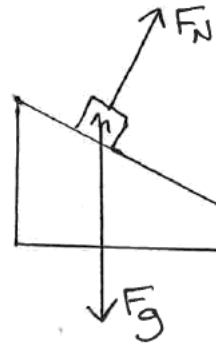
- **Third Law** : all forces are **interactions** between objects. If Ⓐ exerts a force on Ⓑ, then Ⓑ exerts an **equal** and **opposite** force on Ⓐ. Same magnitude, exactly opposite direction. (Think magnets, charged objects, masses attracting each other via gravity, etc.)



Example: Object of unknown mass M is placed (at rest) on a 30^o incline. It slides down the incline until it reaches the floor, travelling a distance along the incline of 2 m. How fast is the object moving when it reaches the bottom of the ramp?

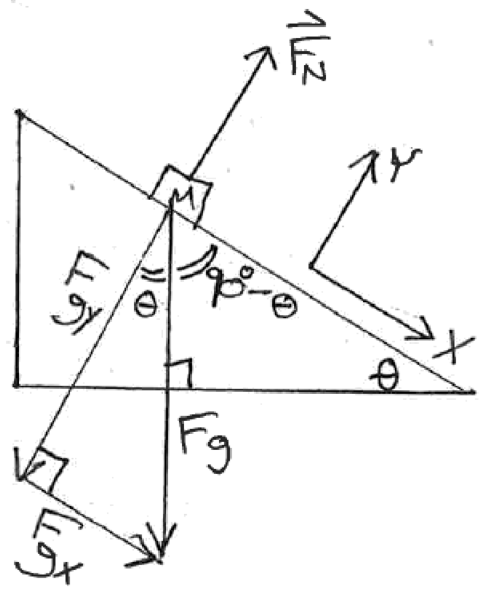


<u>Forces acting on the object</u> :



- It has a mass, so $\vec{F}_g$ vertically downward
- A normal force $\vec{F}_N$ keeping the object from moving through the ramp.

<u>Choose a coordinate system</u> : the object will slide down the ramp so let's use a **rotated coordinate system** with $+X$ parallel to the ramp, and $+Y$ perpendicular to the ramp and aimed up away from the ramp.
<u>Resolve force vectors into components</u> : F_N is entirely in the Y direction, so nothing to do there, but we'll need to convert $\vec{F}_g$ into components in our rotated coordinate system. Propagating the angle of the incline around a bit we end up with an angle inside the F_g 'triangle'. $\vec{F}_g$ has a magnitude of $F_g = mg$ straight down, so we'll have $|F_{gx}| = mg \sin\theta$ in the rotated +X direction, and $|F_{gy}| = mg\cos\theta$ in the rotated negative Y direction, so $F_{gy} = -mg\cos\theta$.

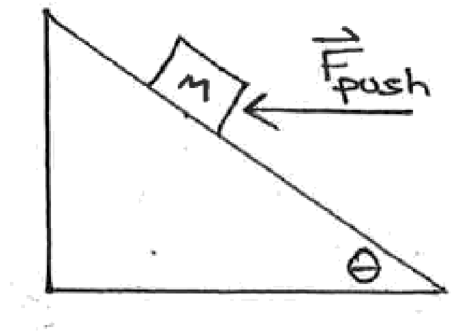


<u>Apply Newton's Laws</u>

X direction : $\sum F_x = ma_x$ so pick off the X components of <u>all</u> the force vectors acting on the crate. With our coordinate system choice, we only have F_{gx} in the X direction, so $mg\sin\theta = ma_x$ or $a_x = g\sin\theta = (9.8\ m/s^2)\sin(30^o) = 4.9\ m/s^2$. The object starts at rest so we can find it's speed at the bottom of the ramp: $v_x^2 = v_{ox}^2 + 2a_x\Delta x$ or $v_x^2 = (0)^2 + (2)(4.9\ m/s^2)(2\ m)$ from which $|v_x| = 4.427\ m/s$ (and it's sliding along the ramp, so $v_y = 0$).

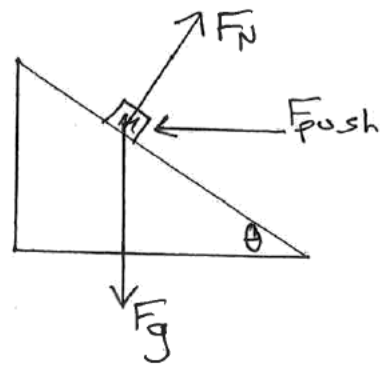
Y direction : $\sum F_y = ma_y = 0$ since the object isn't changing it's Y coordinate at all as it slides down the ramp. Picking off Y components of <u>all</u> the force vectors acting on the crate: $F_N - mg\cos\theta = ma_y = 0$ so $F_N = mg\cos\theta$. (We don't know the mass here, so can't determine a numerical value for F_N but we do see that it'll be less than the full weight (mg) of the object.

Example: An object of $M = 100\ kg$ is sitting on a 40^o ramp. If we want to hold the object in place by pushing on it horizontally as shown in the figure, how much force do we need to apply?



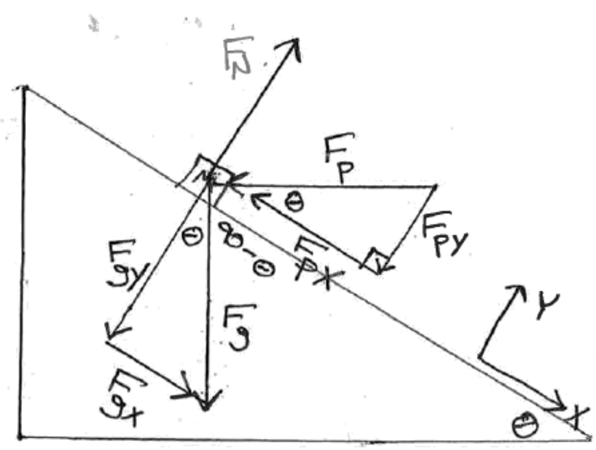
Forces acting on the object :

- It has a mass, so $\vec{F}_g$ vertically downward
- A normal force $\vec{F}_N$ keeping the object from moving through the ramp.
- The horizontal pushing force $\vec{F}_{push}$



Choose a coordinate system : without the pushing force, the object would slide down the ramp so let's use a rotated coordinate system with $+X$ parallel to the ramp, and $+Y$ perpendicular to the ramp and aimed up away from the ramp.

Resolve vectors into components : F_N is entirely in the Y direction, so nothing to do there, but we'll need to convert the others. Propagating the angle of the incline around a bit we see that $|F_{gx}| = mg \sin\theta$ aimed in the **+X** direction, $|F_{gy}| = mg\cos\theta$ aimed in the **-Y** direction, $|F_{push,x}| = F_{push}\cos\theta$ aimed in the **-X** direction, and $|F_{push,y}| = F_{push}\sin\theta$ aimed in the **-Y** direction.
See the Resolving Vectors information on Canvas, which is a link to: `https://newton.ph.msstate.edu/~fox/ph2213/vectors/index.html`



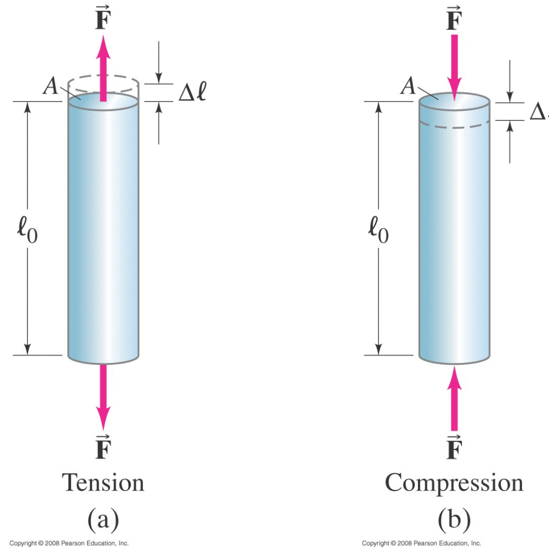
Apply Newton's Laws

X direction : $\sum F_x = ma_x = 0$ (zero since we want the object to just sit there), so pick off the X components of all the force vectors acting on the crate: $F_{gx} + F_{push,x} = 0$ or $mg\sin\theta - F_{push}\cos\theta = 0$ from which $F_{push} = mg\sin(\theta)/\cos(\theta) = mg\tan\theta$. Substituting in the information provided: $F_{push} = (100\ kg)(9.8\ m/s^2)\tan(40^o) = 822.3\ N$.

(We could do the Y direction to determine the amount of normal force present, but the X direction gave us what we were looking for so we'll just stop there.)

Tension Force : F_T

- Rubber band : stretched
- Molecular bonds being stretched
- Restoring Force along the stretched direction
- Same occurs with solid materials (wire, cable, rods, beams, ...)
- $\Delta l = \frac{1}{E}\frac{F}{A}l$
  (Young's modulus; PH2233)
- F_T itself is basically a scalar and is present along the entire length of the medium that has been stretched.



(Note: if the material can support it, similar effect if we compress the material instead of stretch itwe can also. Now called a **compression** force with the material pushing out instead of pulling in.)

Notational Convention

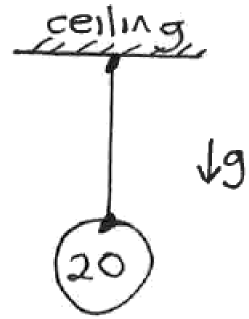
When multiple tensions are involved, this book sticks with the general symbol of $\mathbf{F_T}$ for tension force but then adds another subscript to denote the different wires/cables/etc that are involved.

This leads to variables like $\mathbf{F_{T_1}}$ or $\mathbf{F_{T_A}}$ and so on.

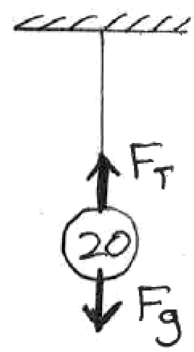
I will generally use the notation our previous textbook used, so the tensions in two strings might be called $\mathbf{T_1}$ and $\mathbf{T_2}$.

Until we get near the end of the course, any wires/cables/strings that appear in a scenario will be under tension (instead of compression), so it can be useful to think of them as rubber bands that have been stretched and are now pulling back inward towards themselves. The tension F_T (or T_1 or whatever we call it) will be some **positive** value as a variable, but then as a force it will always be pulling in on both ends.
It's useful to set up the problem that way so that if you end up with a negative value for the tension, that's a flag that something went wrong.
(Again, this all changes at the end of the course when we might have solid connecting elements that can support both compression and tension, but for now it's a good habit to get into.)

Example : Hanging Lamp (1) Suppose a 20 kg lamp is hanging from the ceiling by a wire. (a) Analyze the situation to determine the amount of tension in the wire. (b) What impact will this have on the ceiling?



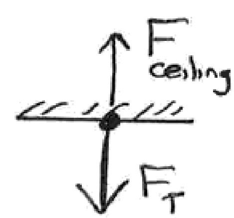
Focusing on the lamp, what forces are acting on it?
The lamp has mass, so $\vec{F}_g$ is acting downward on it with a magnitude of $F_g = mg = (20\ kg)(9.8\ m/s^2) = 196\ N$.
It's not accelerating (or even moving), so according to Newton's Laws, there must be some other force present (and acting upward on the lamp) that is counteracting the downward force of gravity.
This force is the tension in the wire: F_T.



If you pull on a rubber-band and stretch it out, the material pulls back on you to try and restore it's original length. The same occurs with strings, wires, cables, even heavy beams and girders in a building, the stretching just isn't (remotely) as obvious.
Here, the lamp is pulling on the wire, stretching it slightly, creating this tension in the wire which then pulls back with the same magnitude.

Applying Newton's Laws to this problem then: $\sum \vec{F} = m\vec{a} = 0$ so pulling out the Y (vertical) components of the two forces present:

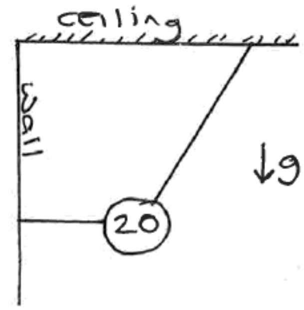
$F_T - F_g = 0$ or $F_T = F_g = mg = 196\ N$. The tension in the wire is 196 N.



We're not quite done yet though. This 196 N of tension (pulling inward) exists throughout the wire. If we focus on the point where the wire is connected to the ceiling, that point isn't accelerating either. Applying Newton's Laws at that point, we have the wire pulling downward with 196 N and that point isn't accelerating (or even moving), there must be some other force vertically upward at that point.

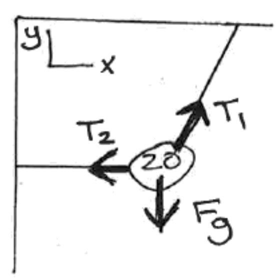
I've labelled that generically as $F_{ceiling}$. Whatever we've attached the wire to up there has to be able to provide that much force. Duct-taping the wire to a ceiling tile probably isn't enough, so maybe it's a beam or some other part of the ceiling structure.

Example : hanging lamp (2) Suppose the person using this room decides that the lamp is in the wrong place, so they attach another wire (horizontally) that is pulling the lamp over the side so that the original wire is now making a 30^o angle relative to the vertical. What tensions are present now?

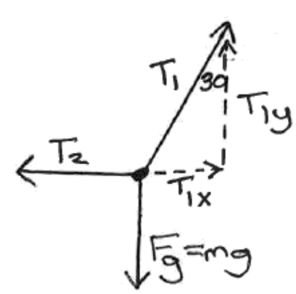


> NOTE: When multiple tensions are present, the F_T notation gets inconvenient quickly. We'll have two tensions here, so F_{T1} and F_{T2} maybe, but then we'll also need to deal with components of vectors, so do we add another subscript? F_{T1x}? That's a little unwieldy, so when multiple tensions are present I'm going to use a notation that a previous book used and label the tensions forces as just T_1, T_2, etc, with components denoted with T_{1x} and so on.

Newton's Laws apply to each object or point in the situation, so let's apply them to the lamp here. First, what are all the forces acting on the lamp? It has a mass, so we have $\vec{F}_g$ acting straight downward. There will be some tension T_1 in the upper wire, and where that wire touches the lamp it's exerting a force of magnitude T_1 in the direction shown. And there will be some tension T_2 in the horizontal wire, which will be pulling on the object to the left.



Using the coordinate system shown above (X positive to the right, and Y positive vertically upward) we have F_g entirely in the (negative) Y direction, T_2 entirely in the (negative) X direction, but T_1 will be inline with the wire, so it's pulling the object to the left and up, so we'll need to resolve that vector into X and Y components. Here:



- $T_{1x} = T_1 \sin(30^o)$ (to the right, so in our +X direction)
- $T_{1y} = T_1 \cos(30^o)$ (upward, so in our +Y direction)

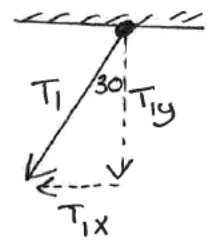
Now we can apply Newton's Laws to the lamp:

X direction : $\sum F_x = ma_x = 0$ so $-T_2 + T_1 \sin(30^o) = 0$ or $T_2 = 0.5T_1$. We don't know either, so that's as far as we can go at the moment.

Y direction : $\sum F_y = ma_y = 0$ so $T_1 \cos(30^o) - mg = 0$ or $T_1 = (20\ kg)(9.8\ m/s^2)/\cos(30^o) = 226.3\ N$.

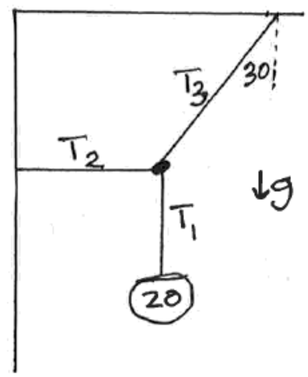
And now we can go back and find $T_2 = 0.5T_1 = 113.2\ N$.

To complete the analysis, let's look at the force the wire is exerting on the ceiling again.
This time, that wire is pulling at an angle and has a larger tension than before.
The wire is pulling on the ceiling with a force that has a downward component of magnitude $T_{1y} = T_1 \cos(30^o) = (226.3\ N)(0.866...) = 196\ N$, which is the same as we had in the earlier version, so no change yet BUT it's also pulling on that point to the LEFT with a force of $T_{1x} = T_1 \sin(30^o) = (226.3\ N)(0.5) = 113.2\ N$



Transverse forces like the one the wire is partly exerting on the ceiling in this version of the problem are called shear forces, and many (most?) materials are far more likely to fail in that direction first. Try pulling a toothpick apart by pulling outward on each end (i.e. your pulling force is 'inline' with the toothpick). It's unlikely you'll be able to. If instead you exert forces perpendicular to the toothpick, it will break much more easily. Even stronger materials like metals can have very different properties inline and perpendicular to their lengths. We may touch briefly on this near the end of the course, but most of you will have a whole class on the strength of materials (EM3213, and optionally ME4123) that will cover this in great detail.

Example : Hanging Lamp (3)



Suppose the person using this room decides that the lamp is in the wrong place, so they pull it over to the left by solidly attaching (welding?) a horizontal wire to the (originally) vertical wire, instead of to the lamp itself. Again, suppose the original wire is now making a 30^o angle relative to the vertical where it's connected to the ceiling, but becomes vertical below the 'weld' point. What tensions are present now? **NOTE: despite the similarity to the previous picture, this situation is quite different, as we'll see.**

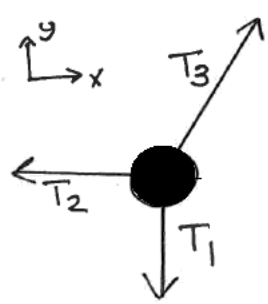
The first thing we need to determine is how many variables we actually have here. The original wire is still present, running from the ceiling down to the lamp, but it's now divided in two by that other wire running horizontally. That wire was said to be 'solidly' connected at that point, so it's not just a hook, it's a solid connection like a weld, in effect.

I'm going to argue that making a connection like this essentially splits the original wire into two elements, each of which can have a different amount of tension in it. This is easier to visualize with a string or rubberband. Imagine an object hanging vertically on a long piece of string. That's the same scenario as our original vertically-hanging lamp, so we know the tension in the string will just be the weight of the object. Now grab hold of the middle of the string firmly. We could release the top part of the string, reducing it's tension to zero while the bottom part of the string still maintains the same tension as before (equal to the weight of the object). Alternately, you could pull up on the top of the string, increasing the tension in the top half, while still leaving the tension in the bottom half the same as it was before. A solid connection like this, in effect, creates two wires that can have different tensions in them.

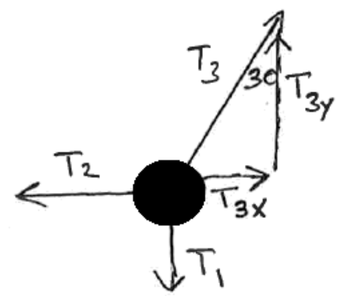
We also have to be careful applying Newton's Laws here. Remember that they apply to **an object**, so if we apply Newton's Laws to the lamp, there are only TWO forces acting on it: gravity downward, and the tension T_1 acting upward on it. None of the other forces present are acting on the lamp.

Apply Newton's Laws to the lamp : Here we have $F_g = mg = (20)(9.8) = 196\ N$ acting straight down (our negative Y direction), and T_1 acting straight up (our positive Y direction) so $\sum F_y = ma_y = 0$ leads to: $T_1 - 196 = 0$ or $T_1 = 196\ N$.
The tension in that piece of the wire is 196 N.

Apply Newton's Laws to the connection : We can now apply Newton's Laws to the point where the three wires come together. At that point, we have T_1 pulling downward, T_2 pulling to the left, and T_3 pulling off at the angle shown. What is **not** acting at this point is the force of gravity on the lamp. The connection point only 'sees' the forces acting on it. It's T_1 acting at that point, not F_g.



Using the coordinate system shown in the previous figure (X positive to the right, Y positive upward), we need to resolve T_3 into components as shown in this figure.
Here, we have:



- $T_{3x} = T_3 \sin(30^o)$ in the +X direction
- $T_{3y} = T_3 \cos(30^o)$ in the +Y direction

Finally, we can apply Newton's Laws at the connection point:

X direction : $\sum F_x = ma_x = 0$ so $-T_2 + T_3 \sin(30^o) = 0$ or $T_2 = 0.5T_3$.

Y direction : $\sum F_y = ma_y = 0$ so $-T_1 + T_3 \cos(30^o) = 0$ so $T_3 = T_1/cos(30^) = (196\ N)/(0.866...) = 226.32\ N$

And finally $T_2 = 0.5T_3 = 113.2\ N$.

- Wire 1 (the vertical wire the lamp is directly hanging from) has a tension in it of $T_1 = 196\ N$ (equal to the weight of the lamp)
- Wire 2 (the horizontal wire pulling the lamp toward the wall) has a tension of $T_2 = 113.2\ N$
- Wire 3 (connected to the ceiling) has a tension of $T_3 = 226.32\ N$ (somewhat higher than the weight of the lamp)

It is very important to understand this fundamental part of applying Newton's Laws to a system: they apply **separately** to each **object** or **point** in the system.

Focus on a single object (or a single point where forces are meeting) and apply Newton's Laws there using **only** forces that are acting **right there**.

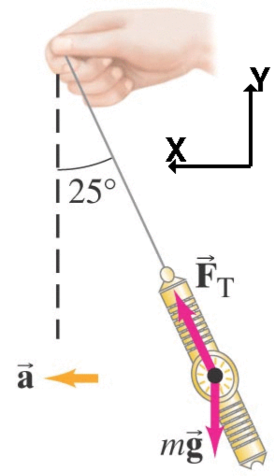
In the previous problem, it may seem picky to separate the tension T_1 from F_g as the force acting at the connection point but in more complex problems not doing so can easily get you off track.

You'll encounter systems in Statics and Dynamics where there are many unknowns, not just 3 like we had here, and being picky like this will be critical.

Reminder: the process of converting vectors into components is a skill that will be needed many times throughout the course. There is a section on my old course website that walks you through the process step by step, with separate pictures for each step:

`https://newton.ph.msstate.edu/~fox/ph2213/vectors/index.html`

Suppose we're sitting in an airplane that's about to take off, and are holding an object hanging on a piece of string. As the plane starts accelerating down the runway (assume the acceleration is constant), we notice that the object swings towards the back of the plane and hangs there with the string making an angle of 25^o relative to the vertical.
Find the connection between this angle and the acceleration of the plane. If it takes 19 seconds for the plane to take off (starting at rest), what's the take-off speed of the plane? How far did it travel on the ground before taking off?



(From the figure, the plane is accelerating to the left, so I defined my +X axis to be pointing to the left, with +Y vertically upward.)

Applying Newton's Laws (broken up into components):

X direction : The force of gravity is straight downward, so has no X component. The X component of the tension will be $F_T \sin\theta$ so $\Sigma F_x = ma_x$ becomes $\boxed{F_T \sin\theta = ma_x}$.

Y direction : We have the force of gravity straight down, and a component of F_T upward so $\Sigma F_y = ma_y$ becomes $F_T \cos\theta - mg = ma_y$ but we have no acceleration in the Y direciton, so $F_T \cos\theta - mg = 0$ or $\boxed{F_T \cos\theta = mg}$

Dividing the first boxed equation by the second: $\frac{F_T \sin\theta}{F_T \cos\theta} = \frac{ma_x}{mg}$ or $\boxed{a_x = g\tan\theta}$.

Apparently, if we can measure the angle the string makes with the vertical, we can determine the acceleration we're undergoing (basically we have a cheap accelerometer).

For this particular problem, $\theta = 25^o$ so here: $\boxed{a_x = (9.8\ m/s^2)\tan 25^o = 4.5698\ m/s^2}$. Having the acceleration means we can now use our equations of motion to determine various things.

The plane starts from rest and accelerates (at this now known a) for 19 seconds until it takes off, at which point $v = v_o + at = 0 + (4.5698\ m/s^2)(19\ s) = 86.83\ m/s$ (about 194 miles/hr, which is plausible).

How far did the plane travel on the ground before taking off? $x = x_o + v_{ox}t + \frac{1}{2}a_x t^2$. Let $x_o = 0$ be where the plane was initially located. It started at rest, so $v_{ox} = 0$ and we know the X acceleration, so: $x(t) = 0 + 0 + (0.5)(4.5698\ m/s^2)(19\ s)^2 = 825\ m$ (a little over 2700 feet).

We found that an acceleration of 'a' would cause the hanging object to swing back at an angle given by $a = g \tan\theta$.

If the plane is cruising at a constant speed with $a = 0$, the object will hang vertically ($\theta = 0$ using the angle the way it's defined in the figure).

If the plane is accelerating horizontally at $a = 1g = 9.8\ m/s^2$, this yields $\theta = 45^o$.

If we double the acceleration though, the angle doesn't double. $a = 2g$ yields $\theta = 63.43^o$

Unfortunately the scale is **not linear** but we can still create a scale that would let us translate the observed angle directly into the acceleration:

Horizonal Acceleration Chart		
Acceleration (in g's)	Acceleration (in m/s^2)	Angle (deg)
$0\ g$	$0.0\ m/s^2$	0.00^o
$1\ g$	$9.8\ m/s^2$	45.00^o
$2\ g$	$19.6\ m/s^2$	63.43^o
$3\ g$	$29.4\ m/s^2$	71.57^o
$4\ g$	$39.2\ m/s^2$	75.96^o
$10\ g$	$98.0\ m/s^2$	84.29^o
infinity	infinity	90.00^o