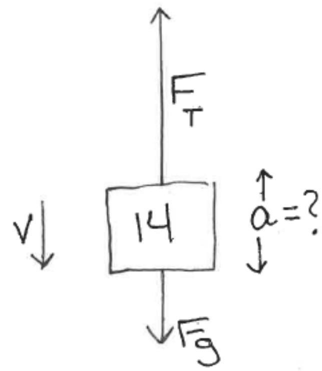
PH2213 Fox : Lecture 11
Chapter 5 : Using Newton's Laws

Test 2 Practice Problem HW04-15

A 14 kg bucket is being lowered vertically by a rope in which there is 132 N of tension.

(a) What is the acceleration of the bucket? (Up or down?)



Suppose the mass of the bucket is 12 kg but the tension remains the same 132 N.

(a) What is the acceleration of the bucket? (Up or down?)

Using a coordinate system with +Y pointing vertically upward, apply Newton's Laws:

$\sum F_y = ma_y$ so here $F_T - Mg = Ma$.

In the first case, we have $F_T = 132\ N$ and $M = 14\ kg$ so $F_g = Mg = 137.2\ N$ so: $(132\ N) + (-137.2\ N) = (14\ kg)(a_y)$ which leads to $a_y = -0.371\ m/s^2$.

That's a downward acceleration, and the velocity is already downward. $v = v_o + at$ so basically the object keeps moving downward and is picking up speed along the way, moving downward faster and faster.

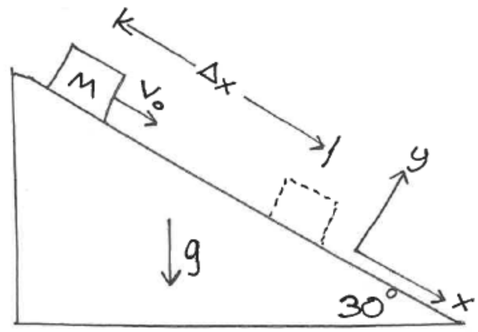
In the second case, we have $F_T = 132\ N$ but $M = 12\ kg$ so $F_g = Mg = 117.6\ N$ so $\sum F_y = ma_y$ becomes: $(132\ N) + (-117.6\ N) = (12\ kg)(a_y)$ which leads so $a_y = +1.2\ m/s^2$.

That's an **upward** acceleration. The original velocity was negative but the acceleration now is upward. The object is initially moving downward, but it slows down, comes to a stop, and then starts moving upward faster and faster.

The original velocity (up or down) of the bucket was **irrelevant** in our calculations.

Remember Newton's Laws relate forces to **accelerations**, not velocities. $a = dv/dt$ so Newton's Laws will tell us how the velocity will **change** over time

Test 2 Practice Problem HW05-YY



A 10 kg box is sliding down a rough 30^o incline as shown in the figure. In the upper position, the box is sliding down the incline at 50 cm/s but we observe that the box is slowing down and comes to a stop after sliding 50 cm along the ramp.

(a) What must the kinetic coefficientof friction be?

(First we'll convert everything to standard metric units so I don't have to keep writing them. Here, we convert the initial speed of 50 cm/s into 0.5 m/s, and the 50 cm distance into 0.5 m.)

Using the coordinate system in the figure, we have

$\sum F_x = ma_x$ and $\sum F_y = ma_y = 0$.

We have friction here and $f_k = \mu_k F_N$ so let's look in the Y direction first.

Resolving $F_g = mg = 98\ N$ into components, we have:

- $F_{gx} = (+)mg\sin\theta = +49\ N$.
- $F_{gy} = (-)mg\cos\theta = -84.87\ N$.

Applying Newton's Laws in the Y direction: $\sum F_y = ma_y$ so $F_N + (-84.87) = 0$ or $F_N = 84.87\ N$.

Applying Newton's Laws in the X direction: $\sum F_x = ma_x$ so $-f_k + F_{gx} = ma_x$ or $-(\mu_k)(84.87) + 49 = 10a_x$.

At this point we have two unknowns, **but** the problem gave us enough information to determine a_x. The object is moving in 1-D motion with a starting speed of 0.5 m/s, an ending speed of 0 m/s, and this occurs over a 0.5 m distance, so:

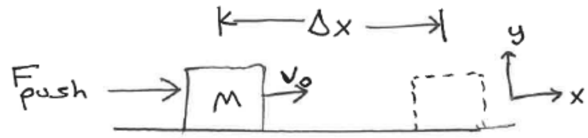
$v^2 = v_o^2 + 2a_x\Delta x$ becomes: $(0)^2 = (0.5)^2 + (2)(a_x)(0.5)$ from which $a_x = -0.25\ m/s^2$.

Now we can complete our previous equation, which ended with: $-(\mu_k)(84.87) + 49 = 10a_x$ but we know $a_x = -0.25\ m/s^2$ now so:

$-84.87\mu_k = 10(-0.25) - 49 = -51.5$ or $\boxed{\mu_k = +0.607}$.

Test 2 Practice Problem HW05-ZZ

A box of unknown mass M is sliding across the floor to the right at 1.5 m/s and is being pushed with a constant force of $F_{push} = 200\ N$.
The (flat, horizontal) floor is rough, with a coefficient of kinetic friction of $\mu_k = 0.6$ and we observe that the box is slowing down, coming to a stop after travelling 50 cm.



(a) What must the mass of the box be?

(First we'll convert everything to standard metric units so I don't have to keep writing them. Here, the only item not already in proper units is the distance, so convert 50 cm to 0.5 m.)

Using the coordinate system shown:

- $\sum F_x = ma_x$ so $-f_k + F_{push} = ma_x$
- $\sum F_y = ma_x = 0$ so $F_N - F_g = 0$ or $F_N = Mg$

Now $f_k = \mu_k F_N$ so we can write that as $f_k = \mu_k Mg = (0.6)(M)(9.8) = 5.88M$.

Substituting that into the F_x equation, we have: $\boxed{-5.88M + 200 = Ma_x}$.

Two unknowns still, but the problem notes that the object goes from 1.5 m/s to rest over a distance of 0.5 m so:

$v^2 = v_o^2 + 2a_x\Delta x$ becomes: $(0)^2 = (1.5)^2 + (2)(a_x)(0.5)$ from which $a_x = -2.25\ m/s^2$.

Substituting that into the boxed equation:

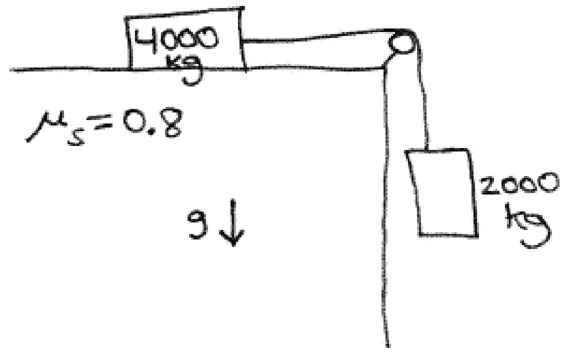
$-5.88M+200 = (M)(-2.25)$ so $200 = (-2.25)(M)+(5.88)(M) = 3.63M$ so finally $\boxed{M = 200/3.63 = 55.1\ kg}$.

Applying Newton's Laws is a **process**:

- What forces are involved?
- What coordinate system is appropriate?
- Convert vector forces into components.
- Finally apply Newton's Laws in X and Y as needed.

Ultimately this generates the equation(s) that apply to the given scenario and the rest is algebra.

Example : Jurassic Park 2 : at one point a dinosaur has pushed a trailer of mass **2000 kg** over the edge of a cliff. It hangs there, connected by a cable to another trailer of mass **4000 kg** which remains on the ground as shown in the figure. Let the coefficients of friction here be $\mu_\mathbf{s} = \mathbf{0.8}$ and $\mu_\mathbf{k} = \mathbf{0.6}$.



(a) Verify that the present situation is **stable** (i.e. static friction is providing enough force to keep the trailers from sliding over the edge).

(b) The dinosaur is not happy with that and decides to **push horizontally** on the trailer. How much force does the dinosaur need to provide to overcome static friction and cause the trailers to start to move?

(c) If the dinosaur **continues** to push with the same amount of force, determine the **acceleration** present, and the **tension** in the connecting cable.

(a) Verify the present situation is stable.

For this part, we are assuming that everything is static - nothing is moving. Applying Newton's Laws to the hanging object, the sum of all the forces in the vertical direction must be zero, since the object is not accelerating. We have some tension in the cable, so looking at this object, we have T upward, and the weight of the object downward. $\sum F_y = ma_y = 0$ so $+T - mg = 0$ or $T = mg = (2000\ kg)(9.8\ m/s^2) = 19,600\ N$

Let's look at the part of the trailer that is sitting on the ground now. In the X direction, we have the tension pulling it to the right, and static friction acting to the left. That's all we have in the X direction, so will this static friction be enough to resist the tension trying to pull the object to the right?

The **maximum** amount of static friction will be $f_{s,max} = \mu_s F_N$ so we'll need to find the normal force here. Looking in the vertical direction and all the forces acting on **this** object, we have it's weight downward, and the normal force upward, so $+F_N - mg = 0$ or here $F_N = mg = (4000\ kg)(9.8\ m/s^2) = 39,200\ N$. The **frictional** force then can be **up to** $f_{s,max} = \mu_s F_N = (0.8)(39,200\ N) = 31,360\ N$.

Thus it looks like we have more than enough 'friction budget' available to keep the trailer in place.

(b) Additional Horizontal Force Needed to Start Moving

The dinosaur needs to add some addition force in the +X direction: just enough force that it plus the tension that is already there will be enough to reach the maximum static friction, so $F_{push} + 19600 = 31360$ or $F_{push} = 11,760\ N$. Anything less and there's still enough left in the static friction budget to resist the motion. Once we add this much force though, static friction has reached its limit and the object can start to move.

$\boxed{\text{(c) Acceleration and Tension in Moving Scenario}}$

If we continue to push the (now moving) 4000 kg object to the right with $F_{push} = 11,760\ N$, what will be the acceleration of the objects?

- The 4000 kg object is moving to the right, so we'll let +X be to the right for that object.
- The 2000 kg object is moving downward, so we'll let +X be downward on that object.

Applying Newton's Laws to the hanging object

$\sum F_y = ma_y$ so $-F_T + mg = ma$ or here $-F_T + (2000)(9.8) = (2000)(a)$ or $\boxed{-F_T + 19600 = 2000a}$.

Applying Newton's Laws to the object on the ground

$\sum F_x = ma_x$ so $F_{push} + F_T - f_k = ma$

How much kinetic friction is present? (Remember: the objects are MOVING now, so we no longer have static friction - we have kinetic friction instead.) $f_k = \mu_k F_N$ and we already found F_N earlier, so $f_k = (0.6)(39200\ N) = 23520\ N$.

$\sum F_x = ma_x$ so $F_{push} + F_T - f_k = ma$ or $11760 + F_T - 23520 = 4000a$ or $\boxed{-11760 + F_T = 4000a}$

We now have two equations and two unknowns. F_T appears in each with opposite sign, so let's just add these two equations together, leaving us with:

$19600 - 11760 = 6000a$ or $\boxed{a = 1.307\ m/s^2}$.

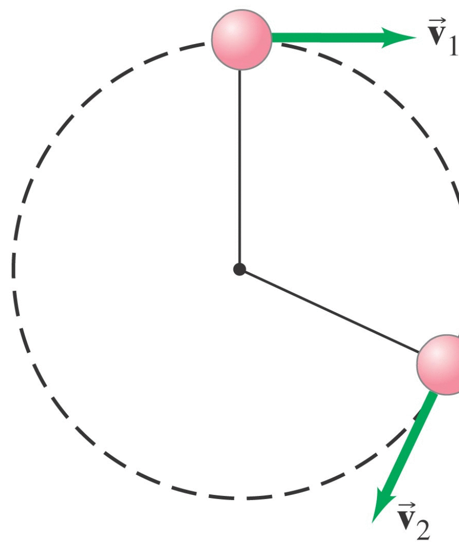
Plugging that value of a back into either of the boxed equations leads to $\boxed{F_T = 16987\ N}$. (Less than the 19600 N of tension we had back before the objects started moving, but remember we need that to be the case so that the unbalanced forces can create this acceleration.)

Uniform Circular Motion

Suppose we have an object moving along a circular path such that it's **speed** remains the same value.

Velocity is a vector and the velocity vector keeps changing direction as the object moves around the circle, so the only constraint we've added is that it's speed (i.e. the magnitude of the velocity $v = |\vec{v}|$ is constant).

This is called **uniform** circular motion.
(Note: we'll look at non-uniform circular motion in a later chapter, where the speed of the object can change as it moves around the circle.)



Circular Motion : Common Symbols and Definitions

- **T** : period : time per rotation
- **f** : frequency : rotations per time interval : $f = 1/T$
  Units: $s^{-1} = Hz$ (Hertz)
  Common: RPM (revolutions per minute)
- Object travels a distance of one circumference in one period:
  $v = (2\pi r)/(T) = 2\pi r f \qquad T = 2\pi r/v$

$\vec{a} = d\vec{v}/dt$ so even though the SPEED isn't changing here, the DIRECTION of the velocity vector IS changing, meaning we have an acceleration here, even though the object's speed isn't changing!

Let's determine what the acceleration is in such a scenario.

Acceleration

$\vec{a} = \frac{d\vec{v}}{dt}$

Consider a tiny time interval Δt

The velocity vector changes <u>direction</u> but not magnitude (same speed).

Object travels along the circular path a distance of $\Delta l = r\Delta\theta$ (see below).

Similar triangles: $\frac{\Delta v}{v} \approx \frac{\Delta l}{r}$

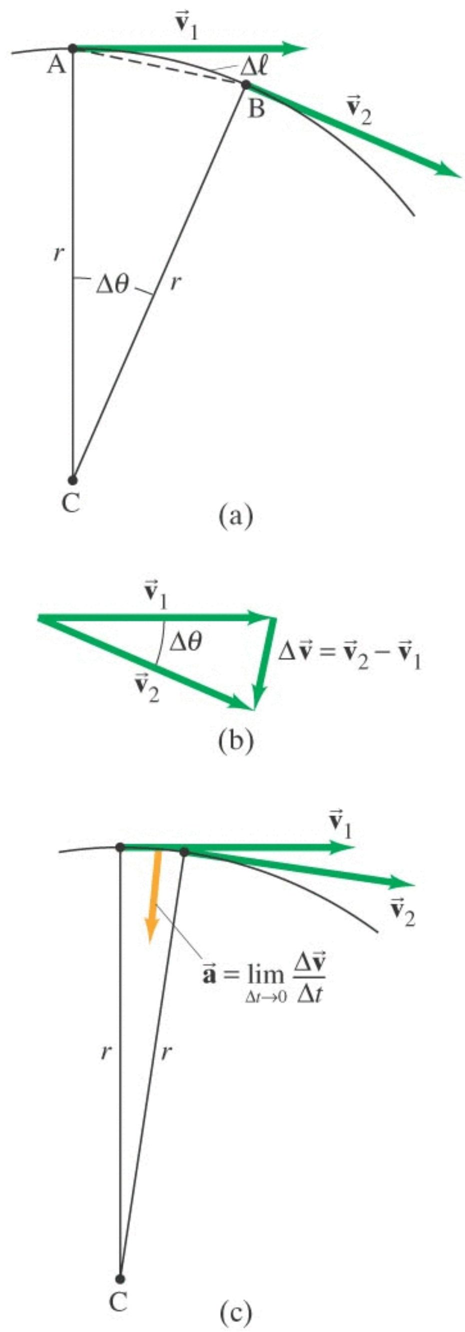
Rearrange: $\Delta v \approx \frac{v}{r}\Delta l$

$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{v}{r}\frac{\Delta l}{\Delta t}$

but $\lim_{\Delta t \to 0} \Delta l/\Delta t = v$ so: $a = v^2/r$

Direction: <u>radially inward</u> **Why?**

$\boxed{\text{Radial Acceleration: } a_r = v^2/r}$



Addendum: I claimed that the distance from A to B can (in the calculus limit) be approximated by the arc-length from A to B:

Angle (deg)	Arclength	Distance	Difference	%error
10.0000	1.745329350	1.743114889	0.002214462	0.126879294
1.0000	0.174532924	0.174530707	0.000002217	0.001269988
0.1000	0.017453294	0.017453291	0.000000002	0.000013340
0.0100	0.001745329	0.001745329	0.000000000	0.000000000

EXAMPLE : CD-ROM : Estimate the radial acceleration at the outer edge of a CDROM that is spinning in a 40X drive. (Note: original 1X drives spun at approximately 360 RPM, so 40X means it spins the disk at 40 times that rate or 14,400 revolutions/minute.) The radius of the disk is 6 cm.

The $14,400$ *revolutions/minute* value represents the frequency of this rotation (frequency being how many complete rotations or revolutions an object undergoes in a given period of time).

In order for everything to work out in standard units, we should convert that to rotations/second though:

$f = 14,400 \frac{rev}{min} \times \frac{1\ min}{60\ s} = 240\ rev/s$ (that is, the CD makes 240 complete rotations every second).

NOTE: 'revolutions' or 'rotations' or 'cycles' is just a counter, not a real physical unit. The actual physical units for frequency is just inverse seconds: $\boxed{f = 240\ s^{-1}}$.

A point on the very edge of the CD will be moving around in a circle of radius $r = 6\ cm = 0.06\ m$. The speed at that point is $v = 2\pi r/T = 2\pi r f$ so here $v = (2)(\pi)(0.06\ m)(240\ s^{-1}) = 90.48\ m/s$ (about 200 miles/hour).

The acceleration of a point on that outer edge will be $a_r = v^2/r = (90.48\ m/s)^2/(0.06\ m) = 136,440\ m/s^2$ or about $13,900\ g's$ of acceleration.

That is a pretty phenomenal acceleration. In an old Mythbusters episode, bullets travelling at up to 300 m/s were fired into water and they came to a stop in about a meter, which represents an acceleration of: $v^2 = v_o^2 + 2a\Delta x$ so $(0)^2 = (300\ m/s)^2 + (2)(a)(1\ m)$ from which $|a| = 45000\ m/s^2$ (about 4600 g's) and in that episode the bullets mostly shattered into fragments. Here we have about three times that acceleration and yet the plastic material making up the CD can withstand it.

In the video link below, the 'Slow Mo Guys' used a mechanical router to spin a CD at $23,000\ RPM's$ and filmed the results at a very high frame rate to see the results in extremely slow motion. What angular acceleration does that frequency represent?

We could repeat the same steps we did above, but **we don't need to**. Note that $a_r = v^2/r$ so is proportional to v^2. Also, $v = 2\pi r f$ so v is proportional to the frequency. $\boxed{\text{Thus } a_r \text{ is proportional to } f^2}$.

Here, we're multiplying the frequency by a factor of $23000/14400 = 1.59722$ which means that the acceleration will be higher by a factor of that value squared, or 2.5511. The acceleration at the outer edge then in this case will be $a_r = (2.5511)(136,440\ m/s^2) = 348075\ m/s^2$ or about $35,520\ g's$.

I recommend looking at **The Slow Mo Guys** channel on Youtube, or just type in the search term 'CD shattering' on youtube and it will directly link to many good videos. The CD's ends up shattering in a very interesting geometric pattern.

Here's a good one: `https://www.youtube.com/watch?v=zs7x1Hu29Wc` (see especially just after the 6 minute mark in the video).