PH2213 : Examples from Chapter 5 : Applying Newton's Laws

Key Concepts

Newton's Laws (basically $\sum \vec{F} = m\vec{a}$) allow us to relate the forces acting on an object (left-hand side) to the motion of the object, through its acceleration (right-hand side).

Statics : if an object is not moving, or is moving at a constant speed, then $\vec{a} = 0$ which means that all the forces must cancel out, as vectors. That means that (separately) $\sum F_x = 0$ and $\sum F_y = 0$ (and $\sum F_z = 0$). This, coupled with the geometry of the situation (the various lengths, angles, etc) allows us to determine information about the forces (such as tension in cables).

Dynamics : if an object is accelerating, then the forces are not balanced (and vice versa). If we have an acceleration, then all our machinery from chapters 2 and 3 on equations of motion can be brought in to help analyze the situation.

Kinetic Friction : If the object is **moving**, the friction between it and the surface is it sliding on is given by $\boxed{f_k = \mu_k F_N}$. μ_k is the coefficient of kinetic friction and depends on the two materials involved.

Static Friction : If the object is **not yet moving**, then a different kind of friction exists that changes its magnitude as needed to keep the object from moving. It can only do this up to some maximum value though, so we write this type of friction as:
$\boxed{f_s \leq f_{s,max} \text{ where } f_{s,max} = \mu_s F_N}$

Circular Motion : In uniform circular motion, the acceleration vector is directed toward the center of the circle and has a magnitude of $\boxed{a_c = v^2/r}$. Since there is an acceleration, there must be some force causing the object to move in this circle (perhaps the tension in a string, or static friction between the tires of a car and the road).

Common Errors

- SEE the list given for chapter 04 where Newton's laws were introduced.
- Only one type of friction is present at a time. If the object is not yet moving, only static friction is present. Once it starts moving, only kinetic friction is present.

Notation Convention : Giancoli uses F_N as the symbol to represent the **normal force**, but you'll often see the letter **n** (or even N) used instead. I've tried to update most of the notes to reflect this, but you'll occasionally still see n used in figures and the text, and will likely encounter it in that form in some of your other courses.

Example 1 : Friction : Box Sliding to a Stop : Suppose we have a box of mass 10 kg sliding across a horizontal floor with an initial speed of 5 m/s. The coefficient of kinetic friction between the box and the floor is $\mu_k = 0.6$. How long does it take the box to come to a stop? How far did it travel?

Coordinates : Let +Y be vertically upward, and +X be in the direction the box is moving.

What are all the forces acting on the block? We have its own weight mg downward, some normal force F_N upward keeping the box from going through the floor, and some force of friction, acting to oppose the motion, so friction will be pointing in the direction opposite the motion.

$\sum F_x = ma_x$: we have f_k as the only force in the X direction. The **magnitude** of the force of kinetic friction is $f_k = \mu_k F_N$ and it is in the direction opposite motion, so $\sum F_x = ma_x$ becomes $-\mu_k F_N = ma_x$. That's as far as we can go at this point since we don't know the normal force yet.

$\sum F_y = 0$ here since the box isn't accelerating (or moving at all) in the Y direction. Taking all the Y components of the forces present, we have: $F_N - mg = 0$, so in this simple case we have $F_N = mg = 98.0N$.

Now that we know the normal force, we can find the force of kinetic friction: $f_k = (0.6)(98.0) = 58.8\ N$. $-f_k = ma_x$ leads to $-58.8 = (10)(a)$ or $a = -5.88\ m/s^2$.

Now that we know the acceleration of the box, we can apply all the machinery from the earlier chapters related to equations of motion to determine how far the box slides before coming to a stop and how long, in seconds, that took:

$v^2 = v_o^2 + 2a\Delta x$ becomes: $0^2 = (5\ m/s)^2 + (2)(-5.88)\Delta x$ or $\Delta x = (-25)/(-11.76) = +2.13\ m$.

$v = v_o + at$ so $(0) = (5\ m/s) + (-5.88\ m/s^2)(t)$ or $t = 0.85\ s$.

Generic Version : Let's look at this problem generically where we have a box of mass m sliding across the floor with an initial speed of v_o, and a coefficient of kinetic friction of μ_k. Then the magnitude of the force of kinetic friction will be $f_k = \mu_k F_N$.

$\sum F_x = ma_x$ becomes: $-f_k = ma_x$ or $-\mu_k F_N = ma_x$.

$\sum F_y = 0$ becomes: $F_N - mg = 0$ or $F_N = mg$ so replacing F_N in the $\sum F_x$ equation:

$-\mu_k(mg) = ma_x$ and dividing the equation by m yields: $\boxed{a_x = -u_k g}$.

Substituting this value for a_x into our v^2 equation: $v^2 = v_o^2 + 2a_x\Delta x$ so when the box comes to a stop: $(0)^2 + v_o^2 + (2)(-\mu_k g)(\Delta x)$ which we can rearrange (careful with signs) into: $\boxed{\mu_k = \frac{v_o^2}{2g\Delta x}}$ which gives us a simple way to determine the value of μ_k between two objects. If we can get the object moving with some known initial speed and then measure how far it slides before coming to a stop, we can easily compute the value of μ_k (for that pair of materials).

We can also rearrange the equation to solve for Δx and find that: $\boxed{\Delta x = \frac{v_o^2}{2\mu_k g}}$ which gives us the 'stopping distance' for an object with an initial speed and coefficient of friction.

Note that the stopping distance is **inversely** proportional to the coefficient, which means that the object will slide longer if the interface is more slippery. The stopping distance is also proportional to the **square** of the initial speed. If you are in a car and slam on the brakes so that the tires slide against the road, driving twice as fast means the stopping distance is 2^2 or 4 times longer.

Example 2 : Pushing Object Initially at Rest : Suppose we have the same 10 kg box as before but it is now initially at rest.

(a) How much (horizontal) force do we need to apply to start the box moving?

In this case, the box is not initially moving, so static friction is present between the box and the floor. The harder I push on the box, the stronger static friction holds the box back, but static friction can only provide up to $f_{s,max} = \mu_s F_N$ before giving up.

The geometry of this problem is the same as the previous, so looking in the vertical direction we have $\sum F_y = ma_y = 0$ which implies that $+F_N - mg = 0$ or $F_N = mg$. In the horizontal direction, we have the pushing force F being opposed by the frictional force. I can keep increasing the pushing force right up until it reaches $f_{s,max}$ before the box will start moving, so this will occur when $F = f_{s,max} = \mu_s F_N = \mu_s mg = (0.8)(10\ kg)(9.8\ m/s^2) = 78.4\ N$.

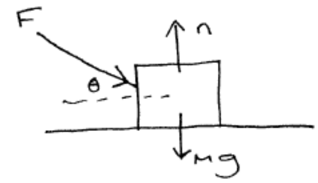
(b) If we continue to push with that same force, what acceleration will the box have?

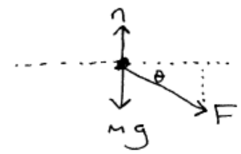
The box now breaks free of the floor and starts sliding in the +X direction. Now we have me pushing with 78.4 N and being opposed by kinetic friction of magnitude $f_k = \mu_k F_N = (0.6)(10\ kg)(9.8\ m/s^2) = 58.8\ N$. The net force in the X direction on the box is now $\sum F_x = +78.4 - 58.5 = 19.6\ N$ and the crate will start accelerating: $\sum F_x = ma_x$ so $19.6 = 10a_x$ or $a_x = 1.96\ m/s^2$.

10 kg box on an interface with $\mu_s = 0.8$ and $\mu_k = 0.6$. Then $F_N = mg = 98.1\ N$ and $f_{s,max} = \mu_s F_N = (0.8)(98.1\ N) = 78.48\ N$.

This is a 'real world' problem we often encounter: it takes more force to start something moving that it does to keep it moving. This is particularly a problem with certain combinations of materials. A sheet of glass laying on top of another sheet of glass has a very high coefficient of static friction, but a very low coefficient of kinetic friction, so it takes a lot of force to start one of the sheets moving, but once it starts moving it accelerates dangerously rapidly.

Example 3 : Force an an Angle (1) : Suppose we try to move our 10 kg box by **pushing** on it at an angle. Let $m = 10\ kg$, the coefficients of static and kinetic friction between the box and floor are $\mu_s = 0.8$ and $\mu_k = 0.6$. We found earlier that it took about 78 N to get the crate moving. Suppose we push with $F = 100\ N$ but directed 30^o below the horizontal. Does the box move? If so, what is its acceleration?
Coordinates: let +Y be vertically upward, and +X be to the right.



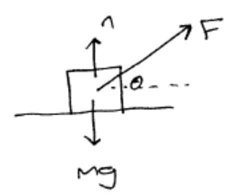


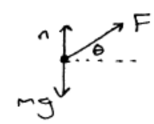
We have friction here, which means we're going to need to determine the normal force, so let's deal with that direction first:

$\sum F_y = ma_y = 0$ so $-mg + F_N - F\sin\theta = 0$ or: $-(10)(9.8) + F_N - (100)\sin 30 = 0$ or $-98 + F_N - 50 = 0$ or finally $F_N = 50 + 98 = 148$.

$\sum F_x = ma_x$ but is it moving or not? Is the force applied enough to cause it to move? Let's look at the X forces. We have $F\cos\theta = (100)(0.866) = 86.6N$ to the right. How much force is friction providing though? $f_{s,max} = \mu_s F_N = (0.8)(148.1) = 118.5N$. Static friction can provide **up to** that amount of force. Hm... that means that the $86.6N$ force I am applying is **not** enough to cause the crate to move. Even though I'm providing **more force** than before, some of it is going into the **normal force** and causing static friction to be higher, and the box will just sit there.

Example 4 : Force an an Angle (2) : Suppose we try **pulling** the crate along with a force angled upward this time. When we were pushing the crate with a perfectly horizontal force, it took about 78 N to get the box moving, so this time suppose we're pulling with just 70 N of force, but at the 30^o angle shown.
$\sum F_y = ma_y = 0$ so $-mg + F_N + F\sin\theta = 0$ so: $-(10)(9.8) + F_N + (70)\sin 30 = 0$ or $-98 + F_N + 35 = 0$ or finally $F_N = 98 - 35 = 63N$.
$\sum F_x = ma_x$ but is it moving or not? Is the force applied enough to cause it to move? Let's look at the X forces. We have $F\cos\theta = (70)(0.866) = 60.6N$ to the right. How much force is friction providing? $f_{s,max} = \mu_s F_N = (0.8)(63) = 50.4$, so this time around, friction is **not** providing enough force to keep the crate from moving.



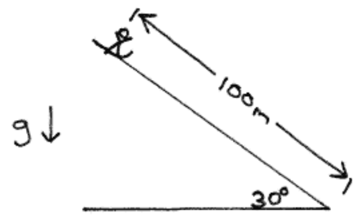


Now that we know that the crate will move, we need to redo the problem using kinetic friction instead of static friction. Nothing else has changed in the geometry of the problem - all the angles are the same, etc. The only thing that is different now is that we have kinetic friction f_k instead of static friction. Our computation for f_k is identical to what we did for f_s but now we have to use μ_k instead of μ_s.

$f_k = \mu_k F_N = (0.6)(63) = 37.8N$. Thus $\sum F_x = (F\cos 30) - f_k = (70)(0.866) - (37.8) = 22.82N$. Finally, $\sum F_x = ma_x$ becomes: $22.82 = 10a$ or $a = 2.282\ m/s^2$.

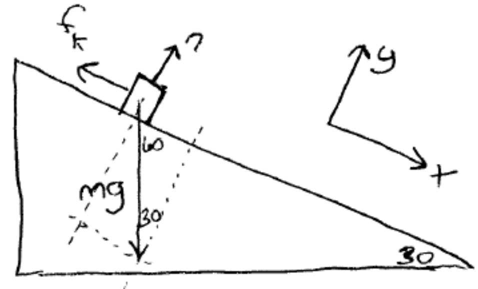
General Comment : Pushing downward on an object increases the normal force and thus increases the frictional force, making it harder to move the object. Pulling upward had the opposite effect, reducing the normal force and thereby reducing friction, making it easier to move the object.

Example 5 : Object on Incline with Friction : Skier on a Slo



Suppose we have an 80 kg skier at the top of a 100 $meter$ long, 30^o slope, initially at rest. Suppose $\mu_s = 0.20$ and $\mu_k = 0.10$. What happens? Is there enough static friction to keep the skier in place? If not, how fast will the skier accelerate down the slope?

What are all the forces acting on the skier? We have their weight mg directed downward towards the center of the earth. We have a normal force perpendicular to the slope, keeping the skier from accelerating **into** the ground, and we have friction acting to oppose motion (so friction, whether it's static or kinetic, would be directed as shown in the figure).



We'll **define a coordinate system** with +X aimed along the slope, starting at the skier's initial location, pointing downslope. +Y will be pointing up perpendicular away from the sloping surface.

Friction and the normal force are already aligned with one of the coordinate axes, but F_g is not, so we'll need to get its components. Moving the angles around, we find a right triangle we can use to determine the components of F_g. Here, we see that the weight has a component down-slope of $mg \sin 30$ and a component in the -Y direction of magnitude $mg \cos 30$.

(a) **Assume static** and see if the situation is possible.

Component of the skier's weight along slope: $mg \sin 30 = (80)(9.8)(0.5) = 392\ N$

Normal force: $F_N = mg \cos 30 = (80)(9.8)(0.866) = 679.0\ N$

Static friction: $f_{s,max} = \mu_s F_N = (0.2)(679.0) = 135.8\ N$

Note that the down-slope component of the skier's weight is **much** larger than the maximum amount of retaining force that static friction can provide, so apparently there is not enough friction to keep the skier in place. Result: the skier **will** start moving immediately.

(b) Now that we know the skier will accelerate down the slope, we need to **redo** the analysis, this time with **kinetic** friction. But really there's very little **new** work needed here, since nothing in our geometry has changed. The only difference is that now we have kinetic friction: i.e. we need to use μ_k instead of μ_s in computing the amount of friction present. So now: $f_k = \mu_k F_N = (0.1)(679.0) = 67.9$ so $\sum F_x = 392 - 67.9 = 324.1\ N$

$\sum F_x = ma_x$ so so $(324.1) = (80)(a)$ or $a = 4.05\ m/s^2$

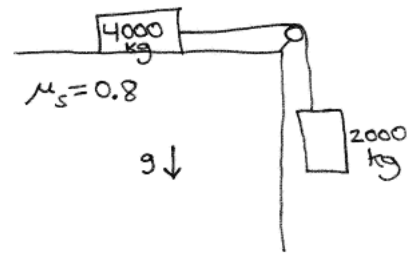
How fast is skier moving at bottom of slope?

$v^2 = v_o^2 + 2a_x \Delta x$ or $v^2 = 0^2 + (2)(4.05)(100) = 810$ from which $|v| = 28.5\ m/s$ (about 64 miles/hour).

We know the skier is moving downslope, which is our +X direction so $v = +28.5\ m/s$.

How long does it take the skier to reach the bottom of the hill? $v_x = v_{ox} + a_x t$ so $(28.5\ m/s) = (0.0\ m/s) + (4.05\ m/s^2)(t)$ from which $t = 7.04\ s$.

Example 6 : Jurassic Park Version 2 : at one point a dinosaur has pushed a trailer of mass 2000 kg over the edge of a cliff. It hangs there, connected to another trailer of mass 4000 kg which remains on the ground as shown in the figure. Let the coefficient of static friction here be $\mu_s = 0.8$



(a) Verify that the present situation is stable (i.e. static friction is providing enough force to keep the trailers from sliding over the edge).

For this part, we are assuming that everything is static - nothing is moving. Applying Newton's Laws to the hanging object, the sum of all the forces in the vertical direction must be zero, since the object is not accelerating. We have some tension in the cable, so looking at this object, we have T upward, and the weight of the object downward. $\sum F_y = ma_y = 0$ so $+T - mg = 0$ or $T = mg = (2000\ kg)(9.8\ m/s^2) = 19,600\ N$

Let's look at the part of the trailer that is sitting on the ground now. In the X direction, we have the tension pulling it to the right, and static friction acting to the left. That's all we have in the X direction, so will this static friction be enough to resist the tension trying to pull the object to the right?

The maximum amount of static friction will be $f_{s,max} = \mu_s F_N$ so we'll need to find the normal force here. Looking in the vertical direction and all the forces acting on **this** object, we have it's weight downward, and the normal force upward, so $+F_N - mg = 0$ or here $F_N = mg = (4000\ kg)(9.8\ m/s^2) = 39,200\ N$. The **frictional** force then can be up to $f_{s,max} = \mu_s F_N = (0.8)(39,200\ N) = 31,360\ N$.

Thus it looks like we have more than enough 'friction budget' available to keep the trailer in place.

(b) The dinosaur is not happy with that and decides to push on the trailer. How much force does the dinosaur need to provide to overcome static friction and cause the trailers to start to move?

The dinosaur needs to add some addition force in the +X direction: just enough force that it plus the tension that is already there will be enough to reach the maximum static friction, so $F + 19600 = 31360$ or $F = 11,760\ N$. Anything less and there's still enough left in the static friction budget to resist the motion. Once we add this much force though, static friction has reached its limit and the object can start to move.

Example 7 : Car and Unbanked Turn : Suppose we have a car taking an offramp or otherwise moving in a circular path. How fast can the car travel before it starts to slip? (Assume the roadway is flat and horizontal, not tilted up at an angle.)

Suppose the circular path the car is on has a radius of $r = 70\ m$ and that the coefficient of static friction between the car's tires and the road is 0.9, which would be typical for good tires and dry road.

The car is moving in a circle, so it has a centripetal acceleration of $a_c = v^2/r$. If there is an acceleration, there must be a force to provide it, so we need a force of $F = ma = mv^2/r$ to keep the car from slipping.

The force we have available is static friction between the tires and the road (since the tires **roll** along the road, rather than slide). The maximum amount of force that static friction can provide is $f_{s,max} = \mu_s n$

Looking in the vertical direction at the forces acting on the car, it's not accelerating in that direction so $\sum F_z = 0$ which implies that $F_N - mg = 0$ or $F_N = mg$. That means that static friction will provide up to $f_{s,max} = \mu_s F_N = \mu_s mg$.

If we want to travel as fast as possible, we'll be working right up against that limit, with static friction providing as much force as it can, so at this extreme limit: $mv^2/r = \mu_s mg$ and here we see that the mass of the car cancels out on both sides of this equation. The mass of the vehicle wasn't provided but apparently it doesn't matter. Dividing the equation by m, we are left with: $\boxed{v^2 = \mu_s gr}$

Doing this symbolically leads to some generally useful information about designing off-ramps. The better the coefficient of static friction between the tires and the road, the faster the car can go, and also the larger the arc the faster it can go. This relationship is **not** linear though. For a fixed value of μ_s, if we want to double the speed, we'll need to make the radius of curvature **four** times larger. That makes the length of the roadway four times longer. Since area is proportional to the radius squared, we'd have to buy 4^2 or 16 times as much land. Certainly cheaper to just post a lower speed limit on the offramp...

For our particular situation, with $r = 70\ m$ and $\mu_s = 0.9$, cars can take travel along this circular section with a speed of: $v^2 = (0.9)(9.8\ m/s^2)(70\ m) = 617.4\ m^2/s^2$ or $|v| = 24.85\ m/s$ (about $55.6\ miles/hr$)

If we have **wet pavement and bald tires**, the coefficient of static friction may be only $\mu_s = 0.4$ (and probably less) which reduces our maximum speed to:

$v^2 = (0.4)(9.8)(70) = 274.4$ or $|v| = 16.57\ m/s$ (about 37 $miles/hr$).

(In either event, we would want to post a speed limit that is smaller than these right-up-against-the-limit values...)

Example 8 : CDROM : What is the acceleration at the outer edge of a 40X CDROM?

A few years ago, it was claimed that CDROM drive speeds had gotten so fast that cheap CD's were flying apart in them.

History: the original 1X CDROM drives spun at a frequency that varied between 200 RPM and 500 RPM (the speed changes depending on what part of the disk the laser is reading so that the linear speed of the point under the laser is roughly the same at all times).

The radius of a typical CDROM is about 6 cm.

For now, let's just assume a 1X drive spins with a **constant frequency** of 360 $RPM's$. A 40X drive then would spin 40 times faster, and would have a frequency of $40 \times 360\ RPM = 14,400\ RPM$.

We're not in the right units yet though; we need frequency in revolutions per second so:

$\frac{14{,}400\ rev}{1\ min} \times \frac{1\ min}{60\ s} = 240 s^{-1}$.

How fast is a point on the outer edge moving?

$v = 2\pi r/T$ or $v = 2\pi r f = 2\pi(0.06\ m)(240\ s^{-1}) = 90.5\ m/s$ (about 202 miles/hr!).

What is the acceleration of a point on the edge?

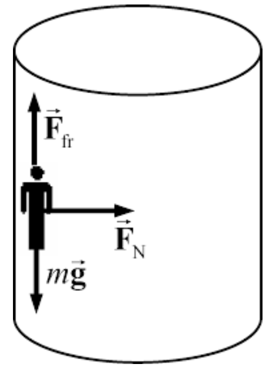
$a_c = v^2/r = (90.5)^2/0.06 = 136,440\ m/s^2$ or about 13,900 g's.

That's a lot. Recall from an earlier 1D motion example of the bullet stopping in water that we found acceleration of magnitude $45,000\ m/s^2$ which was enough to shatter the bullet into fragments, and the acceleration here is much larger. The scenarios are somewhat different though. With the bullet, the force of contact occurs suddenly and propagates as a shockwave through the material; with the CDROM, it spins up over a much longer period of time. In this Mythbusters episode they were eventually able to cause CDROM's to shatter by spinning them fast enough. They attached disks to a high speed router and had to spin them at over $20,000\ rev/min$ for them to break though.

Addendum: what will the acceleration be on the outer edge if the CD is spinning at $20,000\ rev/min$? We could go through all the same steps as above, but **we don't need to**. Note that $a_c = v^2/r$ so is proportional to v^2. Also, $v = 2\pi r f$ so v is proportional to the frequency. Thus a_c is proportional to f^2.

Here, we're multiplying the frequency by a factor of $20000/14400 = 1.42857$ which means that the acceleration will be higher by a factor of that value squared, or 2.0408. The acceleration at the outer edge then in this case will be $a_c = (2.0408)(136,440\ m/s^2) = 278,450\ m/s^2$ or about $28,400\ g's$.

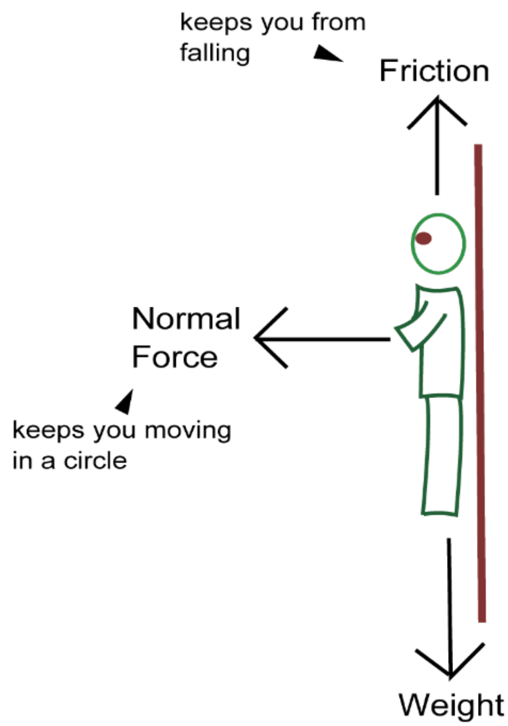
Example 9 : Gravitron : Version 1



A 'gravitron' is a carnival ride consisting of a large circular 'room' that can rotate. People stand up against the curved wall and the room is spun up to a high enough speed that the floor can actually drop away and friction will keep people from sliding down.

Let's use Newton's Laws to analyze this situation and determine how fast the ride needs to rotate for this to work.

In this figure, the rotation axis is over to the left of the person in this snapshot (so it's flipped around relative to the figure above...).
The person is rotating in a circle, which means they're undergoing a **radial acceleration** of $a_r = v^2/r$. Newton's Laws require $\Sigma\vec{F} = m\vec{a}$ so if we have an acceleration (vector) in the radial direction we must also have a force (vector) in that direction.
Fortunately we do: in this case it's the normal force. The force of gravity is straight downward, and with the floor gone gravity would be pulling the person downward, so static friction here must be upward to prevent that.
Let's use a coordinate system where one of the axes is in the direction of the acceleration vector (so radially inward) and then the other axis has to be perpendicular to that so let's use a Y axis with +Y vertically upward.



Applying Newton's laws then:

- Vertical direction: $\Sigma F_y = ma_y$ so $f_s - mg = 0$ or $\boxed{f_s = mg}$ meaning that we need the magnitude of the static friction to equal the weight of the person.
- Radial direction (with positive towards the rotation axis, which in this snapshot would be to the left): $\Sigma F_r = ma_r$ so $\boxed{F_N = mv^2/r}$.

The amount of static friction actually present is **up to** it's maximum value of $f_{s,max} = \mu_s F_N$, so looking at the second boxed equation, we see that the faster the ride rotates, the higher F_N will be, meaning a higher value for $f_{s,max}$. If the ride is rotating **very** fast, we'll easily have enough friction to overcome gravity.

Let's look at the edge case though: suppose we only have **just enough** static friction to hold the person in place, with nothing left over. So we're looking at the case where $f_s = f_{s,max} = \mu_s F_N$.

We found that we need $f_s = mg$ and also that $F_N = mv^2/r$ so making those substitutions:

$f_s = \mu_s F_N$ becomes: $mg = \mu_s mv^2/r$. m appears in every term of this equation, so we can cancel that out, leaving us with: $\boxed{g = \mu_s v^2/r}$ or $\boxed{v^2 = rg/\mu_s}$.

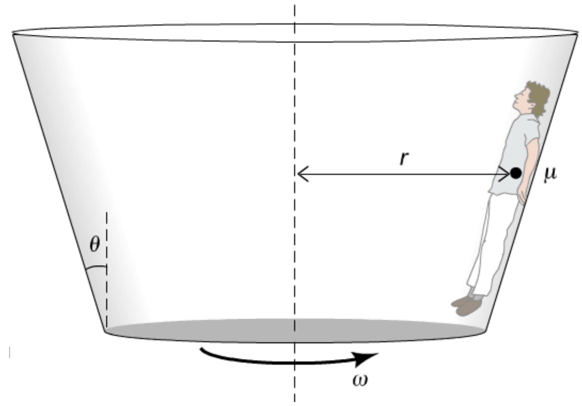
Let's use some realistic numbers here and compute the minimum safe speed and relate that to the g's of acceleration the person would feel, and also compute things like the period and frequency of the room's rotation.

The coefficient of static friction between clothes and the material the wall is made of can vary quite a bit. For safety reasons we'll use the lowest value we expect to encounter which is apparently around 0.25 and let's assume the radius of the room is $r = 3\ m$.

- **Speed** : Rearranging the equation: $v = \sqrt{rg/\mu_s} = \sqrt{(3.0\ m)(9.8\ m/s^2)/0.25} = 10.84\ m/s$ (a bit over 24 miles/hr).
- **Radial Acceleration** : What radial acceleration does this represent? $a_r = v^2/r = (10.84\,m/s)^2/(3\,m) = 39.2\ m/s^2$ which is 4 g's, so that's probably safe.
- **Period** : How long does it take the room to make one revolution? That is, what is the period for this circular motion? For circular motion, $v = 2\pi r/T$ so $T = 2\pi r/v = 2\pi(3.0\,m)/(10.84\,m/s) = 1.74\ sec$, so it would take just under 2 seconds to make one complete resolution.
- **Frequency** : What is the frequency of the rotation? $f = 1/T = \frac{1}{1.74\ s} = 0.5753\ rev/sec$ or $f = 0.5753\frac{rev}{sec} \times \frac{60\ sec}{1\ min} = 34.5\ rev/min = 34.5\ RPM$.

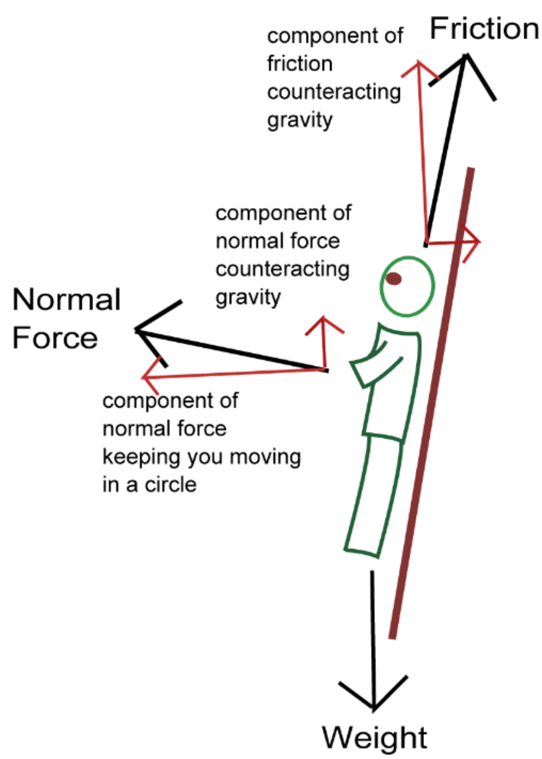
Example 10 : Gravitron : Version 2

Some of these rides have walls that are tilted out a little instead of being vertical. As we'll see, the net effect is that it's easier to produce the static friction we need to keep the person from sliding down the wall when the floor disappears.



Let's use Newton's Laws to analyze this situation and determine how fast this version of the ride needs to rotate.

In this figure, the rotation axis is over to the left of the person in this snapshot (matching the first figure above).

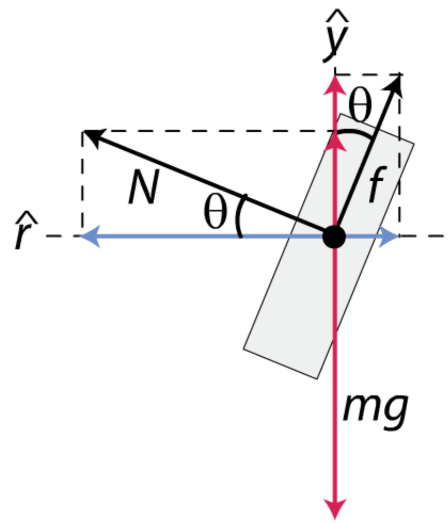


The person is rotating in a circle, which means they're undergoing a **radial acceleration** of $a_r = v^2/r$. Newton's Laws require $\Sigma\vec{F} = m\vec{a}$ so if we have an acceleration (vector) in the radial direction we must also have a force (vector) in that direction.

The force of gravity is straight downward, and with the floor gone gravity would be pulling the person downward, so static friction here must be upward (directed along the wall) to prevent that.

Unlike the simpler example, in this case we have **two** forces that will have **components** in the radial direction (i.e. in the direction of $\vec{a}$, as needed). Both $\vec{F}_N$ **and** $\vec{f}_s$ have radial components here, so this analysis will be a bit more complicated than the first example.

Let's use a coordinate system where one of the axes is in the direction of the acceleration vector (so radially inward) and then the other axis has to be perpendicular to that so let's use a Y axis with +Y vertically upward.
We've got an angle to deal with now, so here is a **free-body** version showing the force vectors acting on the person and the angle(s) involved.



Applying Newton's laws then:

- Vertical direction: $\Sigma F_y = ma_y$ so $F_N \sin\theta + f_s \cos\theta - mg = 0$ or $\boxed{F_N \sin\theta + f_s \cos\theta = mg}$
- Radial direction (with positive towards the rotation axis, which in this snapshot would be to the left): $\Sigma F_r = ma_r$ so $\boxed{F_N \cos\theta - f_s \sin\theta = mv^2/r}$.

As we did in the simpler example, let's look at the edge case where f_s is as high as it can be: we're using all the friction 'budget' we have to keep the person in place, so we're looking at the case where $f_s = f_{s,max} = \mu_s F_N$.

Replacing f_s with $\mu_s F_N$ in each of the two boxed equations yields:

- $F_N \sin\theta + \mu_s F_N \cos\theta = mg$ or $F_N(\sin\theta + \mu_s \cos\theta) = mg$
- $F_N \cos\theta - \mu_s F_N \sin\theta = mv^2/r$ or $F_N(\cos\theta - \mu_s \sin\theta) = mv^2/r$

Suppose we just **divide** the second equation by the first:

- $\frac{F_N(\cos\theta - \mu_s \sin\theta)}{F_N(\sin\theta + \mu_s \cos\theta)} = \frac{(mv^2/r)}{mg}$

F_N appears in both terms on the left side, so will cancel out. m appears in both terms on the right side so will cancel out, leaving us with:

- $\frac{\cos\theta - \mu_s \sin\theta}{\sin\theta + \mu_s \cos\theta} = \frac{v^2}{rg}$

Finally, rearranging this to solve for the speed:

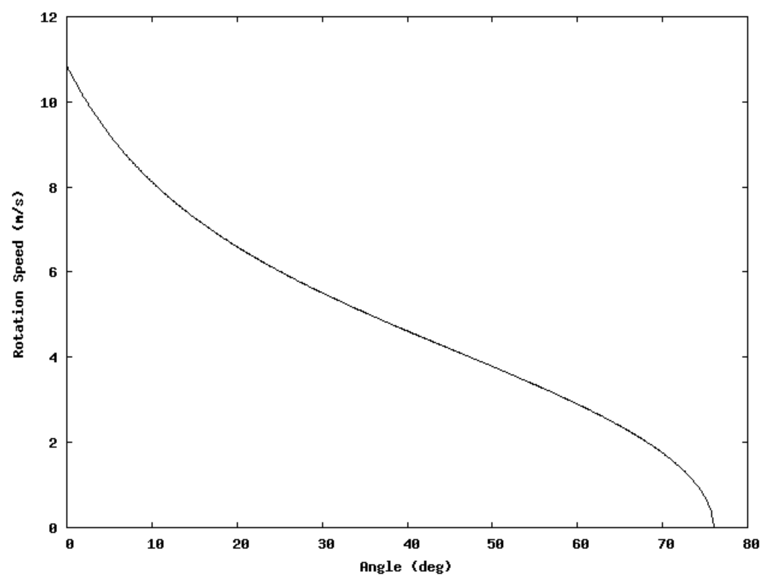
- $v^2 = rg(\frac{\cos\theta - \mu_s \sin\theta}{\sin\theta + \mu_s \cos\theta})$

First, let's check that this is consistent with the simple(r) version of the example. As the wall becomes vertical (i.e. when $\theta = 0^o$), this more generic version becomes the same situation as we had in the first example. Plugging in $\theta = 0^o$ here, $\cos 0 = 1$ and $\sin 0 = 0$ so we end up with $v^2 = rg(\frac{1-0}{0+\mu_s})$ or $v^2 = rg/\mu_s$ which is exactly what we had before.

In the previous example (vertical walls) we found we needed $v = 10.84\ m/s$, which yields a radial acceleration of $39.2\ m/s^2$ or $4g$ which is probably safe but could be uncomfortable.

If we have the wall tilted over at 20^o from the vertical and assume we have the same $\mu_s = 0.25$ and $r = 3$, plugging in these values yields $v = 6.598\ m/s$, which is considerably slower than before. The radial acceleration that the person would feel now is just $a_r = v^2/r = 14.5\ m/s^2$ or just under 1.5 g's (which will make the insurance company happier).

If we let θ be a variable and solve for $v(\theta)$ we find an interesting graph:



If the walls are vertical ($\theta = 0^o$) the rotational speed needed is about $10.84\ m/s$ and as the angle increases (the walls start tilting over), the speed needed gradually reduces but then something odd happens at high angles. The speed needed drops to **zero** at about $\theta = 76^o$. What's happening there?

Well, in that case, the person is just at rest on a tilted wall (ramp) and we've done that problem before. At this angle, apparently the upslope static friction present is just enough to cancel out the downslope component of gravity. We did an example in class (and in the homework) where we found that the maximum angle where static friction will be enough to hold something in place on a ramp was found to be $\tan\theta = \mu_s$ **BUT** we defined our angle differently when we derived that result: we used an angle that was measured up from the horizontal, not down from the vertical as we're using here. Using **this** definition of the angle, this critical angle is $\cot\theta = \mu_s$ or equivalently $\tan\theta = 1/\mu_s$. For the $\mu_s = 0.25$ we have here, we find that $\theta = 75.96^o$: right where we see the graph drop to $v = 0$.

What if we have no friction at all?

We did a problem like this is class, with a car going around a circular ramp that was tilted up at an angle, so what happens here if we have no friction?

$v^2 = rg(\frac{\cos\theta - \mu_s \sin\theta}{\sin\theta + \mu_s \cos\theta})$

and letting $\mu_s = 0$ yields $v^2 = rg\frac{\cos\theta}{\sin\theta}$ or $v^2 = rg/tan\theta$.

Again, we're defining θ differently than we did in class: here we're measuring θ relative to the vertical. With $r = 3\ m$ and $\theta = 20^o$, we get $v^2 = 80.78\ m/s$, which implies a radial acceleration of $a_c = v^2/r = 26.9\ m/s^2$ or around 2.7 $g's$, which will make the insurance company nervous, so we better make sure there's at least some friction present.