PH2213 Fox : Lecture 21
Chapter 9 : Linear Momentum

Real Version of Newton's Laws

$$\boxed{\sum \vec{F} = \frac{d\vec{p}}{dt} \text{ where } \vec{p} = m\vec{v}}$$

$\vec{p} = m\vec{v}$ is called the **momentum** of an object.

If the mass is constant, $d(m\vec{v})/dt = m d\vec{v}/dt = m\vec{a}$ so this 'real' version reduces to what we've been using so far, as long as the mass is constant.

The 'real' version of Newton's Laws also handles cases where the mass of the object isn't constant. Examples are rockets, where most of the mass of the rocket is actually it's fuel, which is being consumed while the rocket engine is running. It also handles things like an open crate travelling along a conveyor as items are being added to it (thus increasing it's mass).

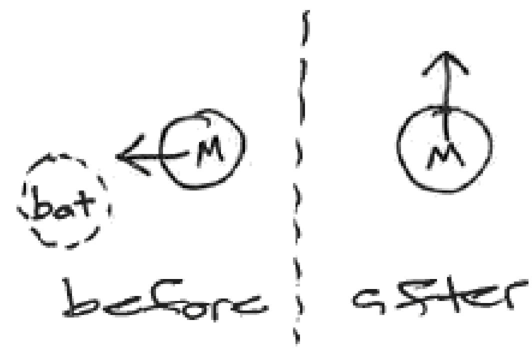
Implications

- $\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t}$
  If we see an object change its momentum over some interval Δt, we can determine the average force that must have been acting on it during that interval.
- Rearranging: $\Delta\vec{p} = \int \vec{F}(t)dt$ which we can write as
  $v(t) = v_o + \frac{1}{m}\int \vec{F}(t)dt$
  giving us a way to handle time-varying forces
- Conservation of momentum during interactions:
  $\sum\limits_{before} \vec{p} = \sum\limits_{after} \vec{p}$
  Breaking this vector equation into components:
  $\sum\limits_{before} (mv_x) = \sum\limits_{after} (mv_x) \qquad \sum\limits_{before} (mv_y) = \sum\limits_{after} (mv_y) \qquad$ etc.

We will find, as we work through the examples, that **mechanical energy**, our particular subset of energies in the form of $(K + U_g + U_s)$, is usually NOT conserved during collisions.

We will also find that the forces involved during the brief time intervals involved in most collisions are tiny compared to the force of the collision itself, so we can usually ignore any other forces acting on the object(s) during that tiny time interval.

Example : HW09-06 : Force on Baseball
A 0.145 kg baseball flying horizontally at 32 m/s strikes a bat and is popped straight up to a height of 36.5 m. If the contact time between the bat and the ball is 0.7 ms, calculate the average force the bat exerted on the ball during contact.



The force on the ball can be found by looking at how its momentum changes: $\vec{F}_{avg} = \frac{\Delta \vec{p}}{\Delta t}$ where $\Delta \vec{p} = \vec{p}_{after} - \vec{p}_{before}$ refers to the change in the momentum of the BALL alone.

Using a coordinate system with +X to the right, and +Y vertically upward, the ball is initially moving directly to the left, so we'd write its velocity as $\vec{v} = (32\ m/s)(-\hat{i})$ (32 meters/second in the negative X direction), so the ball's momentum before the collision would be $\vec{p}_{before} = m\vec{v}_{before} = -(0.145\ kg)(32\ m/s)\hat{i} = -4.64\hat{i}$ (with units of $kg\ m/s$ but if we put everything in standard metric units of meters, kilograms, seconds, etc, then the force will end up coming out in Newtons, so I'll drop writing all the units here).

After the collision, $\vec{p}_{after} = m\vec{v}_{after}$ but what velocity did the ball have right after the collision? We know it flew upward 36.5 m after the collision event was over, and the ball it flying vertically upward, so we can use work-energy or conservation of energy to determine what its velocity must have been (right after the collision).

Between the point just after the collision and the apogee point:

$(K + U_g + U_s)_{top} = (K + U_g + U_s)_{bottom} + W_{other}$

We have no springs here, and no 'other' work so:

$0 + mgh + 0 = \frac{1}{2}mv^2 + 0 + 0 + 0$ or just $mgh = \frac{1}{2}mv^2$ or finally $v = \sqrt{2gh} = \sqrt{(2)(9.8)(36.5)} = 26.75\ m/s$.

The ball is flying that fast, vertically upward, so $\vec{v}_{after} = +26.75\hat{j}$ (m/s) and $\vec{p}_{after} = m\vec{v}_{after} = (0.145)(26.75)\hat{j} = 3.88\hat{j}$

Finally we can compute the force on the ball:

$\vec{F}_{avg} = \frac{\Delta \vec{p}}{\Delta t} = \frac{\vec{p}_{after} - \vec{p}_{before}}{\Delta t} = \frac{(3.88\hat{j}) - (-4.64\hat{i})}{0.0007\ sec} = 5543\hat{j} + 6629\hat{i}$ (in Newtons).

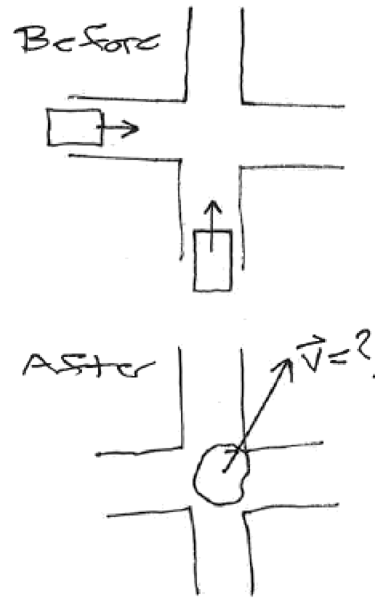
In the usual order (x,y,z), we should probably write that as $\vec{F}_{avg} = (6629\hat{i} + 5543\hat{j})\ N$. This force is to the right and up: the component to the right is stopping the leftward motion of the ball; the component up is giving it the vertical velocity it needed to reach the given apogee point.

From Newton's third law, the ball then will exert an equal but opposite force on the bat.

The magnitude of the force will be $|F| = \sqrt{F_x^2 + F_y^2} = 8641\ N$. What average acceleration did the ball undergo during its interaction with the bat? $F = ma$ so $a = F/m = (8641\ N)/(0.145\ kg) = 59591\ m/s^2$. Converting units, this is about 6080 $g's$ of acceleration. At the start of an earlier lecture, I showed a video of a baseball colliding with a bat and we saw how much the ball (and bat) deformed during the collision.

Example : 2-D Car Collision
A 1000 kg car travelling east at 20 m/s collides with a 2000 kg truck travelling north at 15 m/s. Assume this is a totally inelastic collision (objects 'stick together' as a result of the collision).

- At what angle does the combined object move off?
- If each driver has a mass of 100 kg, determine the force (as a vector, then as a magnitude) on each driver if the collision took $\Delta t = 0.1\ s$



Momentum is conserved as a vector, so let's break that (vector) equation into its components. The total X momentum before the collision must equal the total X momentum after the collision. Ditto for the Y (and Z) components. If momentum is conserved as a vector, its components must also be conserved.

We'll use a coordinate system with +X to the right (East, in the words of the problem), with +Y toward the top of the page (North).

Conservation of momentum in the X direction : the 1000 kg car is travelling entirely in the +X direction (so its velocity is entirely X), and the 2000 kg car is travelling entirely in the +Y direction (so it has a zero velocity component in the X direction):

After the collision, we have a combined 3000 kg object moving with some velocity vector $\vec{v}$, with components v_x and v_y

$\sum\limits_{before}(mv_x) = \sum\limits_{after}(mv_x)$ so $(1000\ kg)(20\ m/s) + (2000\ kg)(0\ m/s) = (3000\ kg)(v_x)$ from which $\boxed{v_x = 6.67\ m/s}$.

Conservation of momentum in the Y direction : the 1000 kg car is travelling entirely in the +X direction (so its Y velocity component is zero), and the 2000 kg car is travelling entirely in the +Y direction (so all its velocity is in its Y component):

After the collision, we have a combined 3000 kg object moving with some velocity vector $\vec{v}$, with components v_x and v_y

$\sum\limits_{before}(mv_y) = \sum\limits_{after}(mv_y)$ so $(1000\ kg)(0\ m/s) + (2000\ kg)(15\ m/s) = (3000\ kg)(v_y)$ from which $\boxed{v_y = 10.0\ m/s}$.

Speed and angle of wreck : Immediately after the collision, we have the X and Y components of the velocity of the combined object, so $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6.67)^2 + (10.00)^2} = 12.02\ m/s$ and $\tan\theta = v_y/v_x = 10/6.67 = 1.50$ so $\theta = 56.3^o$.

$\boxed{\text{Energy Lost}}$: How much (mechanical) energy was lost in this collision?

Before the collision, the car has $K = \frac{1}{2}mv^2 = (0.5)(1000)(20)^2 = 200,000\ J$ and the truck has $K = \frac{1}{2}mv^2 = (0.5)(2000)(15)^2 = 225,000\ J$ so overall there's $425,000\ J$ of mechanical energy just before the collision.

After the collision, we have a 3000 kg object moving at a velocity $\vec{v}$ but we found it's components to be $v_x = 6.67\ m/s$ and $v_y = 10\ m/s$. so it's speed is $v = \sqrt{(v_x)^2 + (v_y)^2} = 12.02\ m/s$. It thus has a kinetic energy of $\frac{1}{2}(3000\ kg)(12.02\ m/s)^2 = 217,000\ J$.

We apparently lost $425000 - 217000 = 208,000\ J$ of energy here: that much energy did 'other' work damaging the vehicles...

Impact on the Driver of the Car

Let's look at the force the driver of the car felt during the collision. They're being carried along with the car, so their momentum changes also.

Force on person in car: $\vec{F}_{avg} = \Delta\vec{p}/\Delta t = \frac{\vec{p}_{after} - \vec{p}_{before}}{\Delta t}$

$\vec{p}_{before} = m\vec{v}_{before} = (100)(20\hat{\imath} + 0\hat{\jmath}) = 2000\hat{\imath} + 0\hat{\jmath}$.

$\vec{p}_{after} = m\vec{v}_{after} = (100)(6.67\hat{\imath} + 10\hat{\jmath}) = 667\hat{\imath} + 1000\hat{\jmath}$.

$\vec{F}_{avg} = \frac{\vec{p}_{after} - \vec{p}_{before}}{\Delta t} = \frac{(667\hat{\imath} + 1000\hat{\jmath}) - (2000\hat{\imath})}{0.1} = -13333\hat{\imath} + 10000\hat{\jmath}$ (N). The magnitude of that force is $|F| = 16,700\ N$ and using $F = ma$ or $a = F/m$ the car's driver 'felt' an acceleration of about 167 m/s^2 or about 17 $g's$ during the 0.1 sec of the collision.

Impact on the Driver of the Truck

Going through the same process for the truck driver:

$\vec{p}_{before} = m\vec{v}_{before} = (100)(0\hat{\imath} + 15\hat{\jmath}) = 0\hat{\imath} + 1500\hat{\jmath}$.

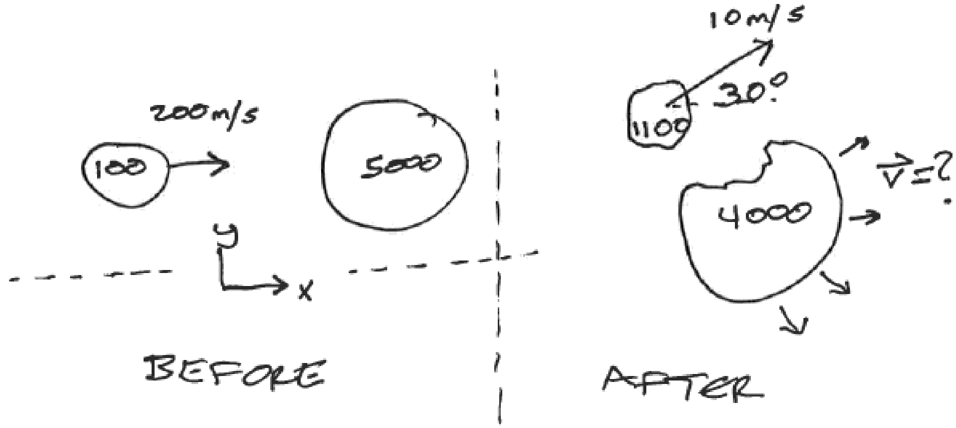
$\vec{p}_{after} = m\vec{v}_{after} = (100)(6.67\hat{\imath} + 10\hat{\jmath}) = 667\hat{\imath} + 1000\hat{\jmath}$.

$\vec{F}_{avg} = \frac{\vec{p}_{after} - \vec{p}_{before}}{\Delta t} = \frac{(667\hat{\imath} + 1000\hat{\jmath}) - (1500\hat{\jmath})}{0.1} = 6667\hat{\imath} - 5000\hat{\jmath}$ (N). The magnitude of that force is $|F| = 8300\ N$, which is about **half** the force the car driver underwent. Using the truck driver's 100 kg mass, this represents an acceleration of $a = F/m = 83\ m/s^2$ or roughly $8\frac{1}{2}\ g's$ of acceleration.

Reducing Impact of Crashes
Momentum conservation is unavoidable, so there's literally nothing we can do about the $\Delta\vec{p}$ term in our $\vec{F}_{avg}$ calculation. The only thing we might be able to control is the Δt. When old solid steel cars from the 50's collided, the interaction was very abrupt. The car, frame, etc didn't 'give' much. Modern cars are designed to absorb a lot of the energy though: the front and rear of the vehicles are designed to collapse, basically sacrificing the vehicle in order to spread out the Δt over which the collision occurred, thus greatly reducing the forces felt by the people inside.

Vector CoM : Spacecraft-Asteroid Collision

A 100 kg spacecraft travelling in the $+X$ direction at 200 m/s collides with a 5000 kg asteroid. (We're using a coordinate system attached to the asteroid at this instant, so it's velocity would be zero.) After the collision, the spacecraft dislodges a 1000 kg chunk of the asteroid and they move off together as shown.



- What must the velocity of the remaining 4000 kg part of the asteroid be?
- How much energy was lost in the collision?

Conservation of momentum in the X direction : $\sum_{before}(mv_x) = \sum_{after}(mv_x)$

$$(100)(200) + (5000)(0) = (1100)(10\cos 30^o) + (4000)(v_x) \text{ or } \boxed{v_x = 2.618\ m/s}$$

Conservation of momentum in the Y direction : $\sum_{before}(mv_y) = \sum_{after}(mv_y)$

$$(100)(0) + (5000)(0) = (1100)(10\sin 30^o) + (4000)(v_y) \text{ or } \boxed{v_y = -1.375\ m/s}$$

Kinetic Energy Before Collision : $\sum K = \frac{1}{2}(100)(200)^2 + 0 = 2,000,000\ J$

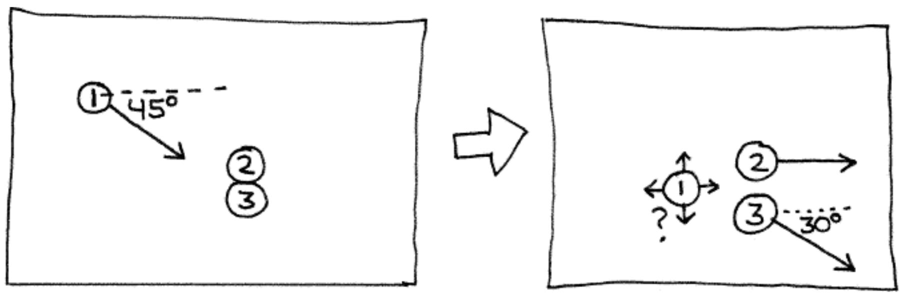
Kinetic Energy After Collision : The remaining big chunk of the asteroid has a speed of: $v = \sqrt{(v_x^2 + v_y^2)} = 2.9575\ m/s$ so after the collision: $\sum K = \frac{1}{2}(1100)(10)^2 + \frac{1}{2}(4000)(2.5882)^2 = 55000 + 17493 = 72494\ J$

Energy lost in collision: $\Delta E = E_{after} - E_{before} = -1,927,506\ J$ (about 96% of the original energy was lost here).

See the DART asteroid-redirection experiment from September 2022. In that test, the spacecraft had a mass of 610 kg and was travelling at about 6600 m/s. The asteroid (Dimorphos) had a mass between $1.3 \times 10^9\ kg$ and $4 \times 10^9\ kg$, and as a result of the collision the asteroid's velocity changed by a few mm/sec. (Seems insignificant, but even a change this small if done while an asteroid is still far enough away from Earth might be enough to cause it to miss us.)

$\boxed{\textbf{Example: Pool balls}}$: Two pool balls are touching one another on a flat, horizontal pool table as shown in the figure. We shoot the cue ball towards them with a speed of 6 m/s at the 45 degree angle shown. After the balls collide, we see that ball 2 heads straight in the $+x$ direction with a speed of 2 m/s and ball 3 heads off at the 30^o angle shown in the figure with a speed of 3 m/s.

What is the vector velocity of the cueball after this collision? (Assume all the balls have the identical 0.15 kg mass, and use coordinates such that $+x$ points off to the right and $+y$ points towards the top of the paper.)



Momentum is conserved as a vector, so $\sum\limits_{before} p_x = \sum\limits_{after} p_x$ and $\sum\limits_{before} p_y = \sum\limits_{after} p_y$.

Now, $p = mv$ and all the balls have identical masses here so let's apply CoM and just use the symbol M to represent that value.

$\boxed{\textbf{X direction}}$: Looking at just the x components first, initially we have one ball moving at a speed of 6 m/s at the angle shown. This velocity will have an x component of $(6\ m/s)(\cos 45) = 4.243\ m/s$ so the total momentum in the x direction initially is $(M)(4.243\ m/s)$. After the collision, we have ball 2 moving entirely in the x direction with a speed of 2 m/s, and ball 3 has an x component of velocity of $(3\ m/s)(\cos 30) = 2.598\ m/s$. We also have the 1 ball with some unknown x component of velocity of, say, v_x. The total momentum in the x direction after the collision then is: $(M)(v_x)+(M)(2\ m/s)+(M)(2.598\ m/s)$. But the momentum before and after must be the same, so: $(M)(4.243\ m/s) = (M)(v_x)+(M)(2\ m/s)+(M)(2.598\ m/s)$. We can divide this entire equation by M resulting in: $(4.243\ m/s) = (v_x)+(2\ m/s)+(2.598\ m/s)$ or $(4.243\ m/s) = (v_x)+(4.598\ m/s)$ or finally $\boxed{v_x = -0.355\ m/s}$.

$\boxed{\textbf{Y direction}}$: Looking at conservation of momentum in the y direction, the '1' ball has an initial y velocity of $(6\ m/s)(\sin 45) = 4.243\ m/s$ in the negative y direction, so the initial y momentum is $\sum p_y = -(M)(4.243\ m/s)$. After the collision, ball 2 has no momentum in the y direction. Ball 3 has a y velocity of $(3\ m/s)(\sin 30) = 1.5\ m/s$ in the negative y direction, and ball 1 has a y momentum of $(M)(v_y)$. So conservation of momentum in the y direction tells us that: $-(M)(4.243\ m/s) = (M)(v_y) + (0) + (M)(-1.5\ m/s)$ or $-4.243\ m/s = v_y - 1.5 m/s$ or $\boxed{v_y = -2.743\ m/s}$.

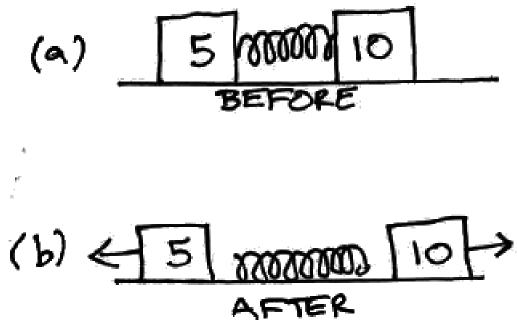
$\boxed{\textbf{Energy Loss}}$: Kinetic energy is usually not conserved in collisions. In this collision, the **initial** kinetic energy is $K = \sum K_i = \frac{1}{2}(0.15)(6\ m/s)^2 = 2.7\ J$

The speed of the 1 ball after the collision was $v_1 = \sqrt{(0.355)^2 + (2.743)^2} = 2.766\ m/s$ so the final kinetic energy after the collision was $K = \sum K_i = \frac{1}{2}(0.15)(2.766)^2 + \frac{1}{2}(0.15)(2)^2 + \frac{1}{2}(0.15)(3)^2 = 1.549\ J$, so about 43% of the initial energy was lost in the collision.

(Real pool-ball collisions usually represent a much smaller percentage loss.)

ADDENDUM : Spring and Two Objects (I)

Two boxes (on a frictionless surface) are pushed together with a spring between them and held in place (at rest). The spring has $k = 48000\ N/m$ and in figure (a) it's compressed by 5 cm.
When they're released, how fast will each box be moving when they separate from the spring?



Everything's happening in 1-D here (let's call the horizontal direction X with $+X$ pointing to the right on the page).

Conservation of momentum in the X direction : $\sum_{before}(mv_x) = \sum_{after}(mv_x)$, so: $(5)(0)+(5)(0) = (5)v_5 + (10)v_{10}$

(I'm pretty sure $\vec{v}_5$ will be off to the left, but let's just let the algebra determine that.)

Ultimately then, we have: $0 = 5v_5 + 10v_{10}$ or rearranging $\boxed{v_5 = -2v_{10}}$

Conservation of Energy : there's no 'collision' here, and no forces that would be adding or removing energy from this 'system', so the total energy after they separate will equal the total energy before, so:

$(K + U_g + U_s)_{after} = (K + U_g + U_s)_{before} + W_{other}$ (with $W_{other} = 0$ here).

Initially, we have nothing moving but we do have energy in the compressed spring of $U_s = \frac{1}{2}kd^2 = (0.5)(48000)(0.05)^2 = 60\ J$.

After they separate, we have no energy in the spring, but we have two moving objects with a total kinetic energy of: $\frac{1}{2}(5\ kg)(v_5)^2 + \frac{1}{2}(10\ kg)(v_{10})^2$ or $2.5v_5^2 + 5v_{10}^2$.

Energy being conserved, that means that $\boxed{60 = 2.5v_5^2 + 5v_{10}^2}$.

We have two equations and two unknowns now. We know from the first equation that $v_5 = 2v_{10}$ so making that substitutions: $60 = 2.5(2v_{10})^2 + 5v_{10}^2$ or $60 = 10v_{10}^2 + 5v_{10}^2$ or simply $60 = 15v_{10}^2$. Dividing each side by 15 we're left with $4 = v_{10}^2$ or $v_{10} = \pm 2$.

Looking at each solution:

- If $v_{10} = +2\ m/s$, then $v_5 = -2v_{10} = -4\ m/s$. So this solution has the 10 kg block moving to the **right** at 2 m/s and the 5 kg block moving to the **left** at 4 m/s. (Seems viable so far.)
- If $v_{10} = -2\ m/s$ then $v_5 = -2v_{10} = +4\ m/s$. This solution implies that after they move away from the spring, the 10 kg block is moving at 2 m/s to the **left** with the 5 kg block moving at 4 m/s to the right.

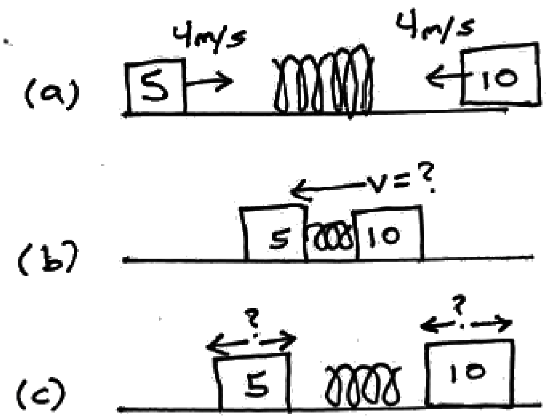
That second solution is definitely **not** possible since it means the two blocks had to pass through each other at some point, so the first option is the only possibility.

ADDENDUM : Spring and Two Objects (I)

Two boxes are sliding towards each other on a frictionless, horizontal surface as shown in the figure. A spring with $k = 48000\ N/m$ is placed between them.

The spring causes each object to slow down, reaching a point where they're moving together at the same velocity (which will also be the point of maximum compression for the spring). How fast are they moving together at this point?

The compressed spring will eventually push the boxes apart. How fast will each be moving when they separate?



Once they hit the spring from each side, it'll start compressing, eventually reaching some maximum amount of compression where the blocks are as close together as they're going to get. At that point they're no longer moving relative to one another, so they're moving at the same velocity v as shown in the middle figure.

Conservation of momentum in the X direction : $\sum\limits_{before}(mv_x) = \sum\limits_{after}(mv_x)$, so: $(5)(4) + (10)(-4) = (5)(v) + (10)(v)$ or $\boxed{-20 = 15v}$ from which $\boxed{v = -1.333..\ m/s}$. At their closest approach, with the spring maximally compressed (figure b), the blocks are moving to the **left** together at $1.333..\ m/s$.

How far did the spring compress?

There's nothing to add or remove energy to the system here, and the blocks never actually touch (no actually energy-losing collision occurs), so again we can use **both** CoM and CoE to find a solution.

In the upper figure, the total energy present is just the combined kinetic energy of the two blocks: $E_{before} = \frac{1}{2}(5\ kg)(4\ m/s)^2 + \frac{1}{1}(10\ kg)(4\ m/s)^2 = 40 + 80 = 120\ J$.

In the middle figure, we have the combined 15 ks object moving at $1.333..\ m/s$ for a total kinetic energy of $K = \frac{1}{2}(15\ kg)(1.333..\ m/s)^2 = 13.333..\ J$.

The missing $120 - 13.333.. = 106.666..\ J$ is the energy stored in the spring at that instant, which is $U_s = \frac{1}{2}kd^2 = 24000d^2$ so $24000d^2 = 106.666..$ which yields $d = 0.0667\ m$ or about $6.7\ cm$.

After separation

Momentum is conserved throughout this interaction, so the total momentum in figure (c) has to be the same as we started with: $-20\ kg\ m/s$.

In figure (c), we have $\sum mv_x = (5)(v_5) + (10)(v_{10}$ so apparently $5v_5 + 10v_{10} = -20$ or dividing by 5: $v_5 + 2v_{10} = -4$ which we could write as $\boxed{v_5 = -(4 + 2v_{10})}$.

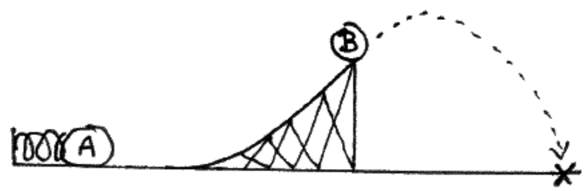
Once the blocks have completely separated from the spring, it's no longer storing any energy, so all the 120 J of energy we started with has moved back to the moving blocks now. The total kinetic energy present is: $\frac{1}{2}(5)(v_5)^2 + \frac{1}{2}(10)(v_{10})^2 = 120$ or $\boxed{2.5v_5^2 + 5v_{10}^2 = 120}$.

The two boxed equations can now be combined. Replacing v_5 in the later boxed equation with $v_5 = -(4 + 2v_{10})$ and combining some like terms, we end up with the quadratic equation: $15v_{10}^2 + 40v_{10} - 80 = 0$ which has two solutions:

- One solution is $v_{10} = -4\ m/s$, in which case $v_5 = -(4 + 2v_{10}) = +4\ m/s$. But those are exactly the original velocities of the two boxes, which means this is the solution where they just miss the spring entirely (or pass through it and each other), so it's certainly not the solution we're looking for.
- The other solution is $v_{10} = +1.333..\ m/s$ (meaning the 10 kg block bounced back to the right at a fairly slow speed). And in this case, we get $v_5 = -6.67\ m/s$, meaning the lighter block bounces back to the left. That seems viable!

ADDENDUM : 1-D collision example from a recent test.

A little toy car Ⓐ is being held in place against a compressed spring. When released, it shoots across the floor along an essentially frictionless track that curves up on the other end, where a second car Ⓑ is located (and is not moving). When car A undergoes a 1-D collision with car B, their little bumpers lock together.



This combined object now flies through the air, eventually crashing back down on the floor.

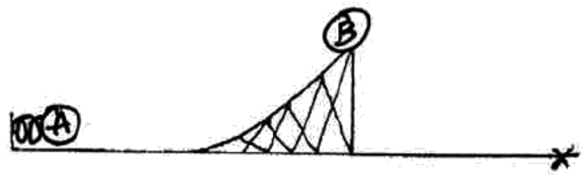
Car A has a mass of $m_A = 0.10\ kg$ and car B has a mass of $m_B = 0.30\ kg$. The spring here has a spring constant of **500 N/m** and was initially compressed by **0.1 m**. The end of the ramp where car B was sitting is located exactly **0.5 m** above the floor.

(a) How fast was car Ⓐ moving the instant just before colliding with car Ⓑ? ____________ m/s

(b) How fast was the 'combined object' moving when it hit the ground? ____________ m/s

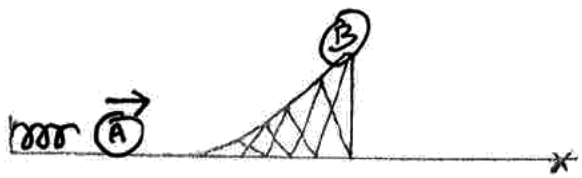
Don't overcomplicate this: no equations of motion are needed (or allowed anyway), so just focus on conservation of energy (where appropriate) and conservation of momentum (during the collision).

Position 1 : mass A is at rest touching the compressed spring. The mechanical energy of object A at this point is: $E_A = K + U_g + U_s = 0 + 0 + \frac{1}{2}kd^2$, so:
$E_A = (0.5)(500\ N/m)(0.1\ m)^2 = 2.5\ J$



(1) Object A and compressed spring

Position 2 : the spring has pushed the object off to the right. Basically all the U_s that had been stored in the spring has now been converted into kinetic energy of the object. The mechanical energy of object A at this point is: $E_A = K + U_g + U_s = \frac{1}{2}mv^2 + 0 + 0 = 2.5\ J$
We could use that to compute A's speed at this point: $(0.5)(0.1\ kg)(v)^2 = 2.5$ from which $|v| = 7.071\ m/s$.



(2) Object A pushed away by the spring

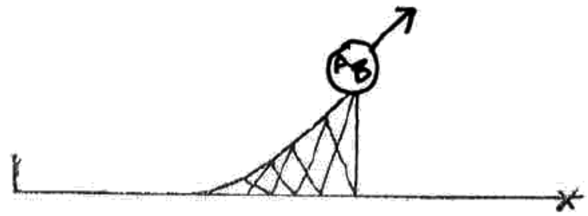
Position 3 : object A slides up the ramp (no friction here to remove energy) and is just about to collide with object B. Basically all the kinetic energy the object had in position 2 has been converted into two forms: gravitational potential energy, and some remaining kinetic energy. The mechanical energy of object A at this point is: $E_A = K + U_g + U_s = \frac{1}{2}mv^2 + mgh + 0 = 2.5\ J$

(3) Just before collision

(We still have the same 2.5 J of energy we started with, so technically we could have gone straight from position 1 to position 3 with our CoE equation.) $(0.5)(0.1\ kg)(v)^2 + (0.1\ kg)(9.8\ m/s^2)(0.5\ m) = 2.5$ from which $|v| = 6.340\ m/s$ which is the speed of object A the instant right before the collision.

Collision! Object A, moving at the speed we just found, collides and connects with object B, and that combined object (with a combined mass of 0.4 kg now) is now moving off with some new speed.

Momentum is conserved as a vector, so basically this is a 1-D equation. Car A is moving in some direction (parallel to the ramp, whatever direction that is), and after the collision that vector momentum has now been transferred to the combined object, so the direction remains the same. Let's call that direction our 'X' axis, then CoM in this rotated X direction will be:



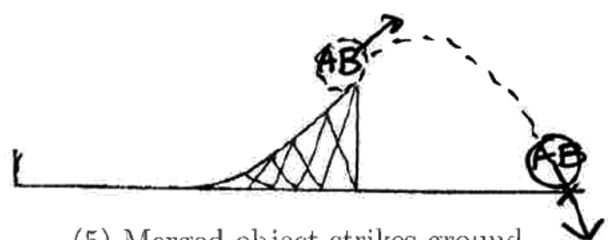
(4) Merged object just after collision (Momentum conserved, but energy lost)

$\sum(m_i v_i)_{before} = \sum(m_i v_i)_{after}$ so:
$(0.1\ kg)(6.340\ m/s) + (0.3\ kg)(0) = (0.4\ kg)(v)$ or $v = 1.585\ m/s$.

This is a totally inelastic collision, so we should be losing quite a bit of the mechanical energy during the collision. The instant just before the collision our 0.1 kg car moving at 6.340 m/s had a kinetic energy of $K = \frac{1}{2}(0.1\ kg)(6.340\ m/s)^2 = 2.01\ J$.

The instant after the collision, we have a 0.4 kg object moving at 1.585 m/s, representing a kinetic energy of $K = \frac{1}{2}(0.4\ kg)(1.585\ m/s)^2 = 0.502\ J$. Looks like we lost about 75% of our energy during the collision.

Position 5 : The combined object has left the top of the ramp with a now known speed of $v = 1.585\ m/s$ and we want to determine how fast it's moving when it hits the ground. From positions 4 to 5, we don't have any 'other' work (just gravity's involved and we can deal with that via U_g), so applying CoE between those two points: $(K+U_g+U_s)_5 = (K+U_g+U_s)_4 + W_{other}$ so: $\frac{1}{2}mv_5^2 + 0 + 0 = \frac{1}{2}mv_4^2 + mgh + 0 + 0$. Leaving that symbolic for one more step, we see that every surviving term in the equation has the same m in it, so multiplying the entire equation by the factor $2/m$ we have:



(5) Merged object strikes ground

$v_5^2 = v_4^2 + 2gh$ or $v_5^2 = (1.585\ m/s)^2 + (2)(9.8\ m/s^2)(0.5\ m) = 12.31$ from which $|v_5| = 3.51\ m/s$. That's how fast the combined object is moving when it hits the floor.