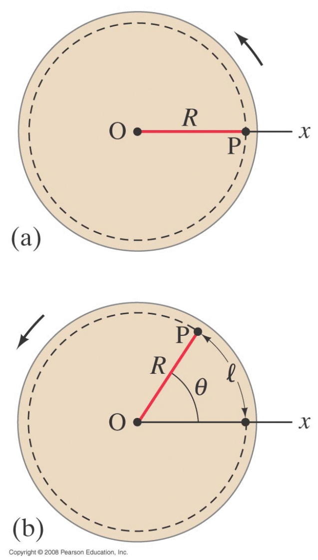
PH2213 Fox : Lecture 23
Chapter 10 : Rotational Motion, Angular Equations of Motion

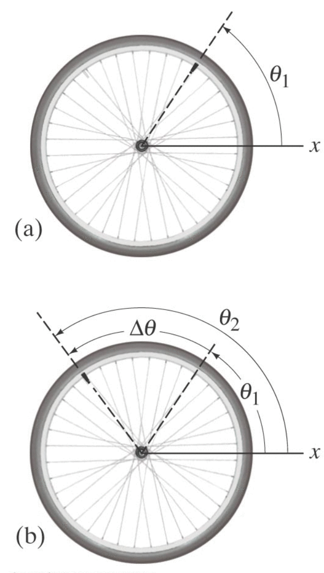
(Short introduction to rotational equations of motion done after the end of the last lecture from chapter 9.)

The last bits of chapter 9 showed that external forces acting on some object affect how it's center of mass (point) moves, but those forces may also cause the object to rotate about that point.

We will restrict ourselves to rotations about a single axis, and also limit ourselves to objects that do not deform, stretch, or distort in any way.

We'll be dealing with what's called: **rigid body rotation about a single axis**.





(a) As an object rotates, every point on that object rotates through the exact same angle θ.

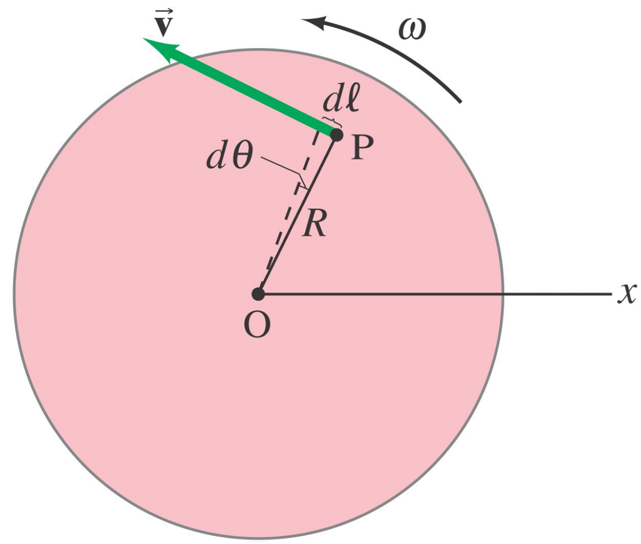
A point located a distance R from the axis of rotation will trace out a distance (arc-length) of l as the object rotates through the angle θ, so we will use this to **define** that angle: $\theta = l/R$ or $l = R\theta$. An angle defined this way is in 'units' of **radians**.

$$\text{Conversion: } 2\pi\ (radians) = 360^o$$

(b) If the object rotates from θ_1 at t_1 to θ_2 at t_2, it's moved through an angle of $\Delta\theta = \theta_2 - \theta_1$ in a time interval of $\Delta t = t_2 - t_1$ and we define the **average angular velocity** as $\omega_{avg} = \Delta\theta/\Delta t$.

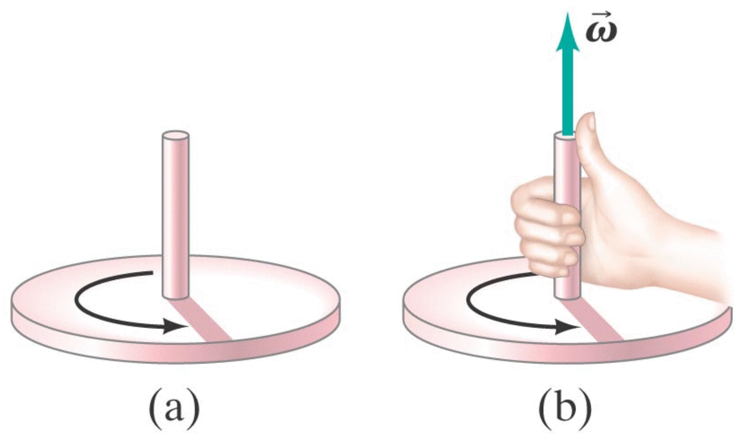
Shrinking the time interval down, we get the calculus limit version of instantaneous angular velocity:

$$\omega = d\theta/dt$$



Vector Nature of Angular Quantities

It's called an angular **velocity** (as opposed to angular speed) because it actually is a **vector**, with the vectorness part defined to be the direction of the axis about which the rotation is occurring.



Right-hand Rule

The figure illustrates one of several 'right-hand rules' we'll encounter. Pointing your **thumb** in the direction of the **axis**, your **fingers** will then 'curl around' in a direction that defines **positive**.

For almost everything we'll do in rotational motion, we'll be using a coordinate system where the (X, Y) plane is in the board (or paper), with the $+Z$ axis pointing up out of the board towards you. **Using that axis, positive angles (and angular velocity, etc) will be in the counter-clockwise direction.**

Angular Equations of Motion

$\omega_{avg} = \Delta\theta/\Delta t$ so $\Delta\theta = \omega_{avg}\Delta t$ which we can write as $\theta = \theta_o + \omega_{avg} t$ (analogous to the 1-D linear equation of motion $x = x_o + v_{avg} t$.

We can define an **angular acceleration** as $\alpha_{avg} = \Delta\omega/\Delta t$ and $\alpha = d\omega/dt$ and go through the same steps we did with 1-D linear equations of motion to derive corresponding **angular** equations of motion:

Analogies between Rotational and Linear Motion	
Rotational Motion About a Fixed Axis	Linear Motion
Angle θ (radians) Angular speed $\omega = d\theta/dt$ Avg Angular speed $\omega_{avg} = \Delta\theta/\Delta t$ Angular acceleration $\alpha = d\omega/dt$	Position x (meters) Linear speed $v = dx/dt$ Avg Linear speed $v_{avg} = \Delta x/\Delta t$ Linear acceleration $a = dv/dt$
For CONSTANT α	For CONSTANT a
$\omega = \omega_o + \alpha t$ $\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$ $\theta = \theta_o + \frac{1}{2}(\omega_o + \omega)t$ $\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$	$v = v_o + at$ $x = x_o + v_o t + \frac{1}{2}at^2$ $x = x_o + \frac{1}{2}(v_o + v)t$ $v^2 = v_o^2 + 2a(x - x_o)$

Useful Conversion: $1\ cycle = 1\ rev = 360^o = 2\pi\ radians$

It's hard to get a gut feeling for what a particular angular speed in radians/second actually means, but we can convert ω into period and frequency:

An object rotates through a full 360^o (i.e. $\Delta\theta = 2\pi$ radians) in one PERIOD, so:

- $\omega = 2\pi/T$ or $T = 2\pi/\omega$
- $\omega = 2\pi f$ or $f = \omega/(2\pi)$