

PH2213 : Advanced Examples from Chapter 10 : Rotational Motion

NOTE: these are somewhat advanced examples of how we can apply the methods from this chapter, so are beyond what will be on the final exam but may prove useful for students taking statics and dynamics classes later.

Key Concepts

Methods in this chapter give us tools for analyzing rotational motion. They are basically the rotational analogs of what we did in earlier chapters on linear motion.

Translational motion involved vector position \vec{r} , velocity \vec{v} and acceleration \vec{a} and we derived numerous 'equations of motion' relating various combinations of these (and the time t). Similarly, the complete description of angular motion also involves vectors. An angle of 30° is meaningless without knowing what axis that angle is a rotation about. So the axis (which could be pointing anywhere, and is thus a 3-D vector) is part of the concept of angular position $\vec{\theta}$, angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$. This can get **very** complicated, so we restrict ourselves to rotations that are confined to a plane and usually define the axis of rotation to be the Z axis.

Key Equations

For an object rotating in the XY plane (i.e. rotating about the Z axis), and angle, angular speed, and angular acceleration are defined to be about the Z axis with the convention that the positive direction is counter-clockwise about that axis.

NOTE: the standard unit for doing calculations involving rotation is the **radian**. One complete rotation is $2\pi \text{ rad}$ so $2\pi \text{ rad} = 360^\circ$ which allows to convert between them as needed.

Angular equivalents to some common 1-D equations of motion:

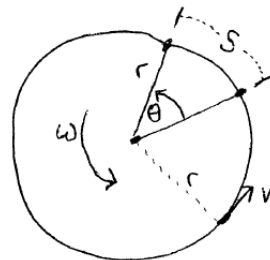
$$\omega = d\theta/dt$$

$$\alpha = d\omega/dt$$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_o + \alpha t$$

$$\omega^2 = \omega_o^2 + 2\alpha(\Delta\theta)$$



Relating Linear and Angular Kinematics

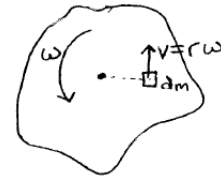
$$v = r\omega$$

$$a_{tan} = dv/dt = r d\omega/dt = r\alpha$$

$$a_{rad} = v^2/r = r\omega^2$$

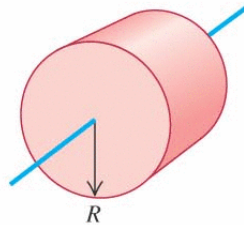
Moment of Inertia and rotational kinetic energy

If an object is rotating, every piece of it is rotating at the same ω even though each piece has different linear speeds. We can accumulate all the kinetic energies in each moving piece though and show that: $K = \frac{1}{2}I\omega^2$ where: $I = \sum m_i r_i^2$ for collections of point masses, or $I = \int r^2 dm$ for solid objects.



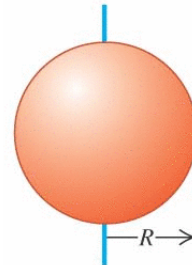
(f) Solid cylinder

$$I = \frac{1}{2}MR^2$$



(h) Solid sphere

$$I = \frac{2}{5}MR^2$$



See page 260 for the moments of inertia of some objects that have common geometric shapes.

Parallel Axis Theorem : If we compute the value of I for some object rotating about an axis through its center of mass and now desire to rotate it about some other axis - but one that is parallel to the original one and just shifted some distance d to the side, we don't need to recompute I from scratch. The new moment of inertia I is related to the original one I_{cm} by: $I = I_{cm} + Md^2$ where M is the total mass of the object.

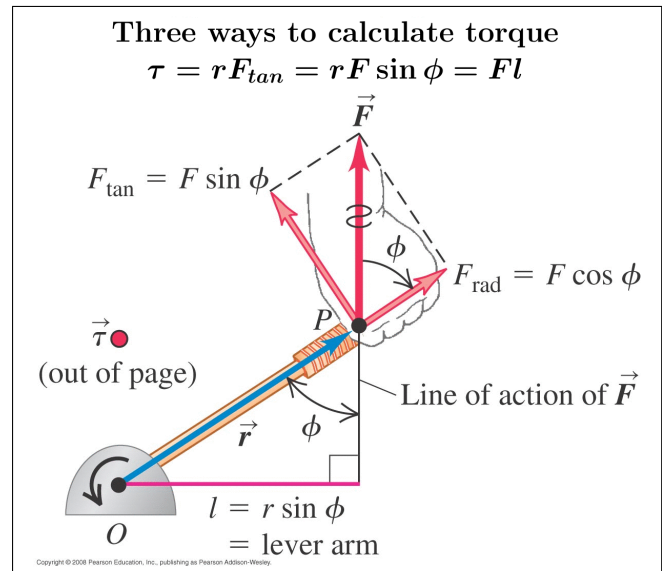
Gravitational Potential Energy : for an extended object, we can add up the U_g of each little dm element of the object and show that $U_g = Mgy_{cm}$: that is, it is the same as if all the mass were located at the center of mass of the object.

Torque : A force \vec{F} applied at a location \vec{r} from the axis of rotation O produces a torque of $\vec{\tau} = \vec{r} \times \vec{F}$. The magnitude will be $|\vec{\tau}| = rF \sin \phi$; direction or sign from right-hand rule. (See figure at right for alternate ways to compute.)

Equilibrium: $\sum \vec{\tau} = 0$

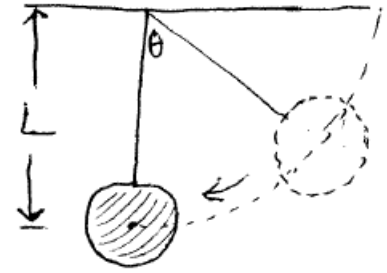
Rotational Versions of Newton's Laws, energy, and power: $\sum \vec{\tau} = I\vec{\alpha}$ $W = \vec{\tau} \cdot \vec{\theta}$ $P = \vec{\tau} \cdot \vec{\omega}$

Torque due to weight of an extended object: acts as if all the mass were located at the CM of the object.



1. Real Pendulum

Let's revisit our pendulum in light of what we've learned in this chapter. In the earlier sample problems involving pendulums, we treated the mass M at the end of the string of length L as a point. Suppose it isn't: suppose the mass is a solid sphere of some radius so that we do have to account for its moment of inertia. How does that affect the speed of the sphere as it passes through the lowest point on the arc?



From earlier chapters, if we hang an object of mass M on the end of a string of length L and start things off at an angle θ , we found using conservation of energy (or work and energy) that the speed at the bottom of the arc will be $v = \sqrt{2gL(1 - \cos \theta)}$

For purposes of this problem, suppose the distance from the point of rotation to the center of mass of the object is $L = 1.5 \text{ m}$. The object itself has a mass of $M = 10 \text{ kg}$ and a radius of 20 cm (i.e. 0.2 m). And suppose we've started things off with the ball at rest hanging out at $\theta = 30^\circ$.

Then according to our 'old' way of doing this problem, the mass will swing past the bottom of the arc with a speed of $v = \sqrt{(2)(9.81)(1.5)(1 - \cos 30)} = 1.986 \text{ m/s}$.

Doing this problem more correctly, we don't have a point mass swinging around a semi-circular arc, we actually have an extended physical object rotating about some point. In fact we have a solid sphere, rotating about an axis that is offset from the center of mass of the sphere by a length $L = 1.5 \text{ m}$. The moment of inertia of a solid sphere rotating about an axis through its center of mass is $I_{cm} = \frac{2}{5}MR^2 = (0.4)(10)(0.2)^2 = 0.16 \text{ kg m}^2$ but here we're rotating about a point 1.5 m away so using the parallel axis theorem, $I = I_{cm} + ML^2 = (0.16) + (10)(1.5)^2 = 22.66 \text{ kg m}^2$.

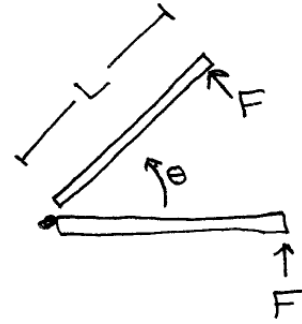
We can work through this one just like we did with the old point-mass pendulum: basically some initial gravitational potential energy is being converted into kinetic energy at the bottom of the arc. Using that bottom point as $y = 0$ we found that the elevation of the initial point was $h = L(1 - \cos \theta) = (1.5)(1 - \cos 30) = 0.20096 \text{ m}$, which means we have an initial $U_g = mgh = (10)(9.81)(0.20096) = 19.714 \text{ J}$

At the bottom now, this energy has been converted into the energy of the sphere rotating about the point on the ceiling, so $K = K_{rot} = \frac{1}{2}I\omega^2$ or $19.714 = \frac{1}{2}(22.66)(\omega)^2$ and finally $\omega = 1.319 \text{ rad/s}$. As the ball passes by this lowest point, at that instant the pendulum has an angular speed of 1.319 rad/s .

Generically, $v = r\omega$ so at the bottom the sphere is moving at a speed of $v = (1.5 \text{ m})(1.319 \text{ rad/s}) = 1.978 \text{ m/s}$. Recall from earlier that our 'point-mass' pendulum had a speed of $v = 1.986 \text{ m/s}$ at the bottom, so taking into account the extended nature of the sphere results in a pendulum that swings slightly slower. It's only 4/10'th of a percent slower in this case, but if you're designing a clock this adds up to about 6 minutes a day. (Note: the sphere in this problem was much larger than what you'd see in an old grandfather clock, so the real impact of this extended-object effect would be smaller but might still enough to throw the clock off a few seconds each day, so would still need to be taken into account when designing the clock.)

2. Opening a Door (1)

Suppose we open a door by pushing with a constant force at the edge of the door in such a way that we keep the force perpendicular to the door, as shown in the figure. How fast will the door be moving when it has swung through 90 degrees?



This force is producing a torque about the axis of rotation. If this torque is constant, we can use our (angular) equations of motion to determine how fast it is rotating at some later point.

Suppose the force has a magnitude of 10 N and we can model the door as a thin rectangle that is rotating about one of its edges. $I = \frac{1}{3}ML^2$ for this geometry, where L here would be the width of the door. (The table in the book does not include this shape specifically, but since $I = \int r^2 dm$, it only matters how far out each dm mass element is from the axis of rotation. The door rotating about one edge is equivalent to a rod rotating about one end. Or you can visit wikipedia and find the same equation...)

Suppose the door has a mass of $M = 21\text{ kg}$ and a width of $L = 1\text{ m}$.

Then $I = \frac{1}{3}(21\text{ kg})(1\text{ m})^2 = 7\text{ kg m}^2$.

This force is always being applied perpendicular to the door right at the edge, so the torque, $\tau = F_{\perp}r$ will be the same as the door opens. $\tau = (10\text{ N})(1\text{ m}) = 10\text{ N m}$. This means we have a constant torque and can use $\tau = I\alpha$ to find the (constant) angular acceleration of the door: $(10) = (7)\alpha$ or $\alpha = 1.428\text{ rad/s}^2$.

Since we have constant angular acceleration, we can use $\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$ to find the angular speed when the door has opened by 90 degrees (i.e. $\Delta\theta = \pi/2\text{ rad}$): $\omega^2 = (0)^2 + (2)(1.428)(\pi/2)$ or $\omega = 2.12\text{ rad/s}$.

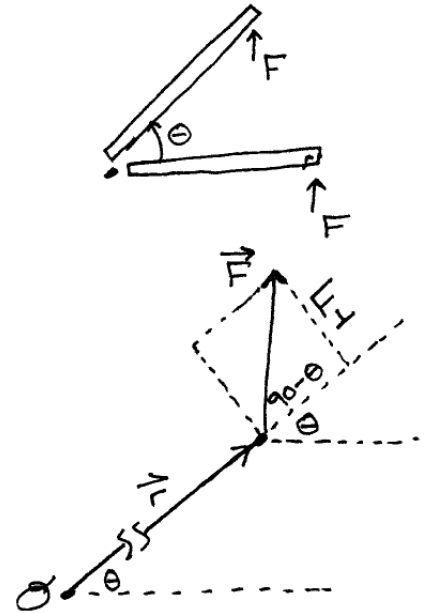
The linear speed of the outer edge of the door at this point would be:

$$v = r\omega = (1\text{ m})(2.12\text{ rad/s}) = 2.12\text{ m/s}.$$

3. Opening a Door (2)

Let's try opening the door a different way, where this time we just keep pushing 'forward' with a constant force, but still applying the force at the outer edge of the door. In this case (see figure), the force is no longer perpendicular to the door (except right at the start). That means that F_{\perp} is no longer constant, so the torque won't be constant either and we can't use our angular equations of motion to solve for the speed of the door at $\theta = 90^\circ$ this time.

We can use work and energy methods to solve this scenario. The kinetic energy of the door when it has opened is equal to the initial kinetic energy plus any work done: $K_2 = K_1 + \sum W$. Here, K is just the rotational kinetic energy of the door, so K has the form of $\frac{1}{2}I\omega^2$. The work done by the pushing force will be $W = \int \vec{F} \cdot d\vec{l}$ or in angular terms: $W = \int \tau d\theta$.



$\tau = F_{\perp}r$ so we need to figure out how F_{\perp} is changing as the angle changes. Using the lower part of the figure, we can propagate the angle θ around to see that $F_{\perp} = F \sin(90 - \theta)$ but we can write that as $F_{\perp} = F \cos \theta$.

Putting this back into our work equation: $W = \int \tau d\theta$ becomes $W = \int_0^{\pi/2} (F \cos \theta)(L)d\theta$ or just $W = FL \int_0^{\pi/2} \cos \theta d\theta$. That is an integral we can do though. The antiderivative of \cos is just \sin so $W = FL \sin \theta$ evaluated between the limits of 0 and $\pi/2$. Finally then: $W = FL = (10 \text{ N})(1 \text{ m}) = 10 \text{ N m}$.

Putting this back into our work-energy equation: $\frac{1}{2}I\omega^2 = \frac{1}{2}I\omega_o^2 + \sum W$ becomes: $\frac{1}{2}(7 \text{ kg m}^2)\omega^2 = 0 + 10 \text{ N m}$ leading to $\omega = 1.69 \text{ rad/s}$. A point at the outer edge of the door will be moving with a speed of $v = r\omega = (1.0 \text{ m})(1.69 \text{ rad/s}) = 1.69 \text{ m/s}$.

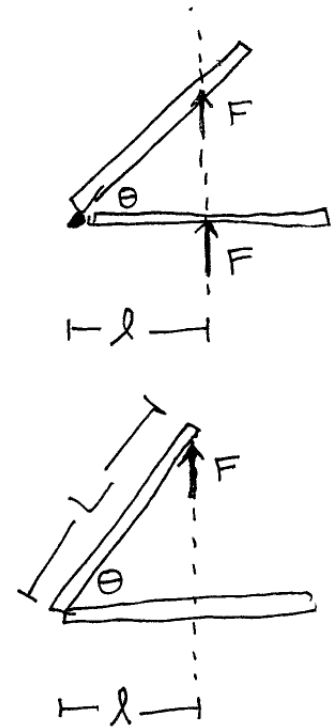
Note that this is not as fast as we had in the previous case.

4. Opening a Door (3)

Suppose this time, we apply a force whose line of action does not change as the door opens. This type of force is fairly common and might represent a plunger-type gadget that is pushing forward to open something. It just keeps pushing forward in a straight line, regardless of how the door is moving.

Note that the point of contact between the plunger and the door changes as the door moves, but the lever arm remains the same. That means we have a constant torque and therefore a constant acceleration so we can use our equations of motion and bypass the work integral.

We have to be careful though: since the lever arm is fixed here, as we see in the lower figure eventually the door will swing open far enough that the plunger is **no longer in contact** with the door and will therefore no longer be exerting any force (torque) on it.



All the work (energy) that the plunger puts into the door happens between the start and that critical angle where the plunger ceases to be in contact with the door. From there out, the torque drops to zero so the angular acceleration also drops to zero and the door will just continue moving at whatever ω it had where the plunger stopped touching the door.

Looking at the lower figure, the critical angle (let's call it θ_c) will be where $\cos \theta_c = l/L$ where L is the width of the door and l (something smaller than L) is how far out from the axis of rotation the plunger is acting.

How fast will the door be moving at this critical angle? $\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$, where we can find α from $\tau = I\alpha$. We have several expressions for computing the torque but since the lever arm is fixed here, $\tau = Fl$ is the most convenient. $\tau = I\alpha$ becomes $Fl = I\alpha$ or $\alpha = Fl/I$

If the door starts at rest then, the angular speed at the critical angle will be $\omega^2 = 0 + \frac{2Fl}{I}\theta_c$.

We can get to this point more easily by looking at work and energy. $K_2 = K_1 + \sum W$ becomes $\frac{1}{2}I\omega^2 = 0 + \int_0^{\theta_c} \tau d\theta$ where $\tau = Fl$ is constant so $\frac{1}{2}I\omega^2 = Fl \int_0^{\theta_c} d\theta$ or just $\frac{1}{2}I\omega^2 = Fl\theta_c$ which we can rearrange into $\omega^2 = \frac{2Fl}{I}\theta_c$ - the same thing we got using our angular equations of motion.

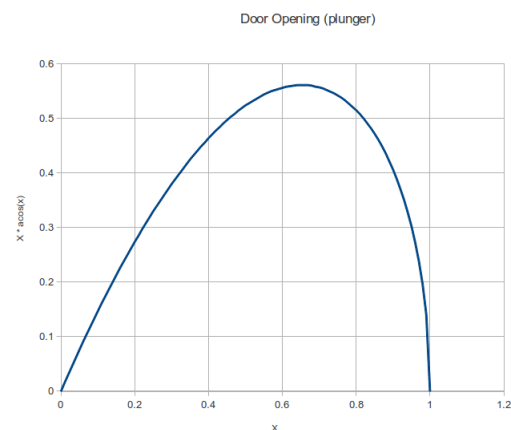
Suppose the width of the door is $L = 1 \text{ m}$ and we apply the same 10 N force we've been using in the other examples but this time we apply it using this plunger method right in the middle of the door, so $l = 0.5 \text{ m}$. Then $\cos \theta_c = 0.5/1.0$ gives $\theta_c = \pi/3$. Finally $\omega^2 = \frac{(2)(10)(0.5)}{7} \frac{\pi}{3} = 1.496$ or $\omega = 1.22 \text{ rad/s}$. That's the angular speed the door will have at the point where the plunger ceases to be in contact with the door so it will retain this speed from there on. The linear speed at the edge of the door from here out would be $v = r\omega = (1 \text{ m})(1.22 \text{ rad/s}) = 1.22 \text{ m/s}$.

Suppose we apply the force at a different location, say $l = 0.75 \text{ m}$ or three-quarters of the way out from the axis of rotation. Now $\theta_c = \cos^{-1}(0.75/1.00) = 0.7227$ and $\omega^2 = \frac{(2)(10)(0.75)}{7} (0.7227) = 1.549$ or $\omega = 1.24 \text{ rad/s}$. That's slightly faster than the first case where we positioned the plunger at $l = 0.5 \text{ m}$.

We can try various different positions and find that the final speed of the door changes. Where should we position the plunger to give the door the maximum possible speed?

The farther out we place the plunger, the larger the torque will be since $\tau = Fl$. But the farther out the plunger is, the less it will remain in contact with the door (meaning that θ_c is smaller). These two factors are working against one another. Where should we place the plunger so that the door reaches the maximum speed? Our equation for the final speed of the door was $\omega^2 = \frac{2Fl}{I} \cos^{-1}(l/L)$ which we can write as $\omega^2 = (\frac{2FL}{I})(\frac{l}{L}) \cos^{-1}(\frac{l}{L})$.

If we let $x = l/L$ then this is $\omega^2 = (\text{constant})(x \cos^{-1} x)$ so the maximum speed will occur where the expression $x \cos^{-1} x$ reaches a maximum. (This expression is graphed in the figure to the right, and clearly does have a maximum.) We've encountered this sort of thing in calculus though. We can differentiate this expression and set it to zero to find the value of x that maximizes the expression. Unfortunately that leads to an equation we can't solve exactly. We can use a computer program to find an approximate solution though, which is $x = 0.652$. So if we want this plunger to open the door as fast as possible, we could position it so that it pushes on a point that is that fraction of the door's width out from the axis of rotation. (For our 1 meter wide door, that would be at $l = 0.652 \text{ m}$.)



At this particular value for l , how fast will the door be moving? $\omega^2 = (\frac{2FL}{I})(\frac{l}{L}) \cos^{-1}(\frac{l}{L})$ so $\omega^2 = \frac{(2)(10)(1)}{7}(0.652) \cos^{-1}(0.652) = 1.603$ or $\omega = 1.27 \text{ rad/s}$.

Looking back over all these door-opening examples, the maximum speed by far occurred in the first scenario, where we applied the force at the edge of the door and rotated the force as the door opened so that it was always exactly perpendicular to the door. This produced the maximum possible torque, and this torque remained active through the entire opening process, leading to the maximum possible amount of work done (i.e. maximum possible final rotational kinetic energy of the door). Applying the same magnitude of force in any other way led to a slower final speed.

5. Opening a Door (4)

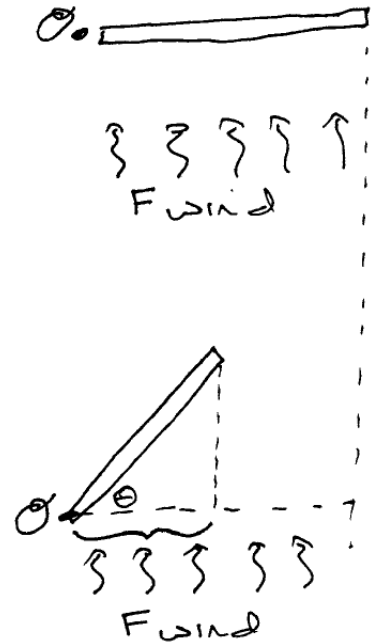
Just to complete this section, let's consider a force that is spread out over the door like wind. We can show that the torque generated is the same as if the force were acting at a point at the geometric center of the door, so this is similar to Case 2. That means the work integral will involve the cosine of the angle of the door.

As the door opens though (lower figure), less of the wind is actually striking the door - in fact the actual force hitting the door will involve the cosine of the angle as well. In addition, the drag coefficient changes as the angle changes, which (roughly) introduces yet another cosine factor.

Ultimately, we end up with an integral that (ignoring all the random constants that appear) involves $\int \cos^3 \theta d\theta$.

If you are taking Calculus 2 this semester, this is exactly the sort of integral you've been doing recently: trig functions in various oddball combinations and powers.

Most of those integrals are just for practice, but some really do appear in real-world scenarios like this.



6. Rotating Basketball - with Friction

Suppose we have a basketball sitting on the floor and we spin it, giving it some initial angular speed, about an axis that is perpendicular to the floor. (So the ball is just sitting there and spinning in place - it's not rolling anywhere.) We want to make a rough estimate of how long it takes for friction to slow the ball down to a stop. A regulation basketball weighs 22 ounces (which represents a mass of about 0.62 kg), and has a circumference of 28.5 inch (which turns into a radius of 4.536 inches or 11.52 cm or finally 0.1152 m).

Let's say the frictional force between the material the ball is made of and the floor is 0.50 . (I have no idea what it might be, so this is a wild guess.)

Let's suppose we start the ball spinning so that it makes just 10 complete revolution per second. Each revolution represents 2π radians, so that means the ball has turned through $10 \times 2\pi$ radians in 10 seconds, which is an angular speed of $\omega = (10 \times 2\pi)/1 \text{ s} = 62.8 \text{ rad/s}$.

Where the ball touches the floor, it flattens out slightly, which means we have a little circle on the floor over which the frictional force is distributed. It can be shown that this will create a torque of magnitude $\tau = \frac{2}{3}fR$ where f is the usual frictional force we can calculate from $f_k = \mu_k n$ and R is the radius of the little circle that represents the contact surface between the ball and the floor. Looking closely at this contact point, it looks like this area is very small - maybe a square centimeter or so, which means the radius is probably around 0.5 cm or 0.005 m . The frictional force here is $f_k = \mu_k n = \mu_k mg = (0.5)(0.62)(9.8) = 3 \text{ N}$. Finally, the torque this friction is generating will be about $\tau = \frac{2}{3}f_k R = \frac{2}{3}(3 \text{ N})(0.005 \text{ m}) = 0.01 \text{ N m}$.

$\tau = I\alpha$ so we'll need the moment of inertia of the basketball. It's essentially a hollow sphere of mass $M = 0.62 \text{ kg}$ and radius $R = 0.1152 \text{ m}$ so using the table of moments of inertia, for such an object $I = \frac{2}{3}MR^2 = \frac{2}{3}(0.62)(0.1152)^2 = 5.5 \times 10^{-3} \text{ kg m}^2$ so $\tau = I\alpha$ becomes $(0.01) = (5.5 \times 10^{-3})(\alpha)$ or $\alpha = 1.82 \text{ rad/s}^2$.

We kind of ignored all the signs here, but this angular acceleration should be negative: the friction is slowing down the rotation of the ball. $\omega = \omega_o + \alpha t$ and we have the initial rotational speed of the ball $\omega_o = 62.8 \text{ rad/s}$, the final rotational speed: $\omega = 0$ and we now have $\alpha = -1.82 \text{ rad/s}^2$ so: $0 = 62.8 - 1.82t$ or $t = 35 \text{ s}$.

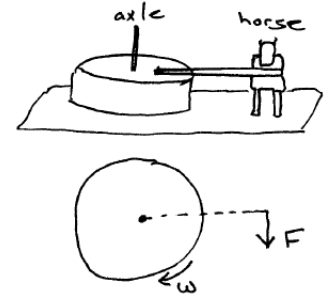
If we do this experiment for real and we find the ball spins for a longer period of time before coming to rest, then a couple of factors could account for this: maybe the coefficient of friction between the ball and the floor is smaller than the 0.5 we assumed; or maybe the contact area between the ball and the floor is smaller than what we guessed.

Consider what happens if we deflate the ball a bit and try to spin it. Now the contact area will be much larger, which means the frictional torque will be larger which in turn will cause the angular acceleration (well, deceleration) to be of larger magnitude, and the stopping time will be shorter.

For a metal ball (ball-bearing maybe), the contact surface is **very** small, which means the frictional torque will be very small and the ball can spin for a long time before stopping.

7. Work and Power in Rotational Motion

Suppose we have a horizontal, flat, stone cylinder with some axle through the center that is being rotated by a horse attached to it via a harness, basically creating an old-fashioned device to grind grain. We want to determine how much power the horse is putting out to turn the wheel. Let's say the horse's force is acting at a distance of 2 m out from the axle, the horse is walking at a speed of 1.5 m/s , the coefficient of kinetic friction between the (rotating) stone and the flat rock underneath it is 0.4 , the rotating rock cylinder has a diameter of 2 m (i.e. a radius of 1.0 m) and a mass of 200 kg .



From the rotational equivalents of our various work and power equations, $P = \tau\omega$ so we need to determine how much torque the horse is exerting, and the angular speed of the grinding stone. The latter is easy enough. The horse is walking at a constant speed of 1.5 m/s along a circular path with radius 2.0 m so $v = r\omega$ or $\omega = v/r = (1.5\text{ m/s})/(2.0\text{ m}) = 0.75\text{ rad/s}$.

How much torque is the horse exerting though? It must be generating enough torque to just match the torque due to rotational friction. **For a force evenly distributed over a circular contact area**, we can break the contact area into little area elements dA and integrate these torques over the entire contact area and show that the total torque friction generates is $\tau_{fric} = \frac{2}{3}f_k R$ where f_k is the overall force of friction present (i.e. $f_k = \mu_k n$) and R is the radius of the disk. Here, $n = mg = (200\text{ kg})(9.8\text{ m/s}^2) = 1960\text{ N}$ so $f_k = \mu_k n = (0.4)(1960\text{ N}) = 784\text{ N}$, and $\tau = \frac{2}{3}(784\text{ N})(1.0\text{ m}) = 522.7\text{ N m}$.

Power : The horse must be generating exactly the same amount of torque in the opposite direction, so already we can determine how much power the horse is putting out: $P = \tau\omega$ so $P = (522.7\text{ N m})(0.75\text{ rad/s}) = 392\text{ watts}$ or slightly over a half a horsepower. That's reasonable. A horse can apparently put out about 15 HP for brief intervals, or 1 HP pretty much indefinitely.

Force : How much force is the horse exerting on the harness? From the diagram the force is exactly perpendicular to the radius, so $\tau = F_{tan}r$ where r is the distance from the horse's force to the axis of rotation (which was given to be 2 m above), so: $(522.7\text{ N m}) = (F)(2.0\text{ m})$ or $F = 261\text{ N}$.

Work : We saw above that to keep the grinding wheel moving with the given speed, the horse is putting out 392 watts, which is 392 joules of energy every second. Let's look at the initial spin-up, where the wheel starts from rest and then reaches its 'running speed'. Suppose it takes 5 seconds for this to occur. During this time, there is an angular acceleration, which means the horse must be putting out more force than what we computed above. It has to counteract the frictional torque present and more in order to accelerate the wheel up to its running speed. We're going from rest to an angular speed of 0.75 rad/s in an interval of 5 seconds, so this corresponds to an angular acceleration of $\alpha = \Delta\omega/\Delta t = (0.75\text{ rad/s})/(5\text{ s}) = 0.15\text{ rad/s}^2$.

How much torque does the horse have to generate to do this? $\sum \tau = I\alpha$. Let τ be the torque the horse is creating. We still have the 523 N m of frictional torque, so $\tau - 523 = (I)(0.15)$. The moment of inertia of the wheel is $I = \frac{1}{2}MR^2 = (0.5)(200\text{ kg})(1.0\text{ m})^2 = 100\text{ kg m}^2$ so $\tau - 523 = (100)(0.15)$ or $\tau = 523 + 15 = 538\text{ N m}$. Just a little more.

$W = \tau\theta$ so what angle did the wheel turn through during these 5 seconds? It's starting from rest and accelerating at 0.15 rad/s^2 so $\theta = \theta_o + \omega_o t + \frac{1}{2}at^2$ or $\theta = 0 + 0 + (0.5)(0.15)(5)^2 =$

1.875 *rad* (about 107 degrees, or just a bit more than a quarter of the way around the circular path). $W = \tau\theta = (538 \text{ N } m)(1.875 \text{ rad}) = 1009 \text{ J}$.