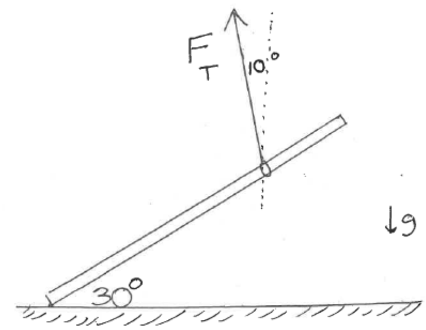
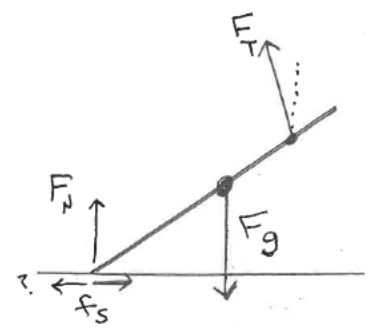
PH2213 Fox : Lecture 28
Chapter 12 : Static Equilibrium

Suppose the pole is being supported by a cable that isn't vertical, but instead makes a 10^o angle with the vertical as shown in the figure here.

> **Example 4** : A 20 m long 1000 kg pole is being lifted as shown in the figure. If the pole is stationary at this point, determine the tension in the cable (which is perpendicular to the pole and connected to it at a point 5 m in from the free end), and the forces at the end of the pole on the ground.
> What does μ_s need to be (at least) to keep the pole from sliding across the ground?



Note that the question asked for forces (plural). There will be the usual normal force (vertically upward here) keeping the pole from moving through the ground, but since the cable force is at an angle, it has a horizontal component, which would cause the pole to (linearly) accelerate to the left unless some other force (static friction to the right) stops that motion.



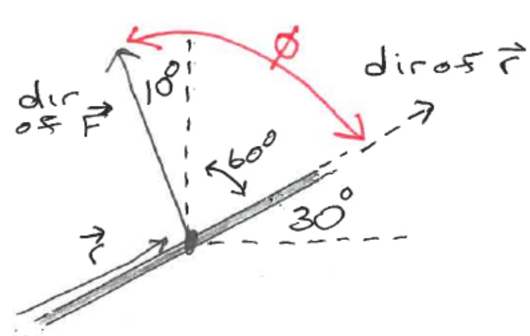
We're starting off with three unknowns here: F_T, F_N and f_s. Two of them are located right where the pole is touching the ground though, so let's do our analysis about an axis that goes through that point.

That makes $\tau_{F_N} = 0$ and $\tau_{f_s} = 0$ since $r = 0$ for both of them.

We already found $\tau_{F_g} = -84870\ N\ m$.

The torque due to the cable tension is trickier since we either have to find an expression that represents the component of F_T perpendicular to r so we can use $\tau = rF_{tan}$), or determine the angle between the directions of $\vec{r}$ and $\vec{F}$ (so we can use $\tau = rF \sin\phi$).

The latter is actually easier here. Here we zoom in on the point where the cable is connected to the pole. I added horizontal and vertical dotted reference lines to show the angles we know: the pole making a 30^o angle relative to the horizontal, and the wire making a 10^o angle relative to the vertical. There's a right angle between the dotted lines, so we know there's a 60^o angle between the pole and the vertical. Overall then, we see that $\phi = 60 + 10 = 70^o$, so $\tau_{F_T} = rF \sin(70^o)$.



The RHR tells us that this torque will be positive.

Collecting terms: $\sum \tau = 0 + 0 - 84870 + (15)(F_T)\sin(70^o)$ which leads to $F_T = 6021\ N$. (That's quite a bit less tension in the cable than we needed in the previous version.)

Propagating angles around to break F_T into X and Y components:

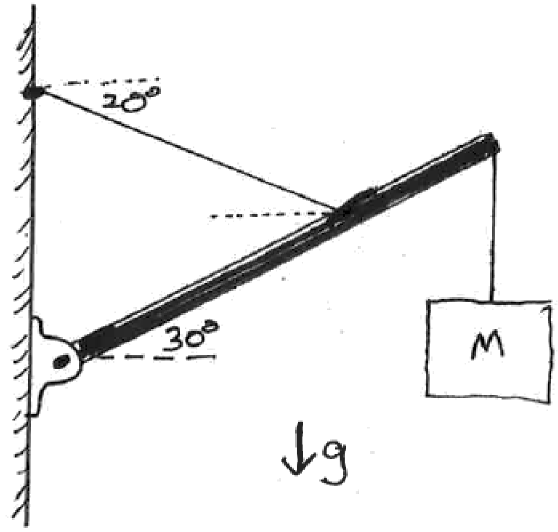
$\sum F_y = 0$ so $(F_T)(\cos 10) + F_N - mg = 0$ so $F_N = 9800 - 5930 = 3870\ N$.

$\sum F_x = 0$ so $f_s - (F_T)(\sin 10) = 0$ so $f_s = (5658)(0.5) = 1046\ N$.

What coefficient of static friction do we need to allow this much static friction to exist?

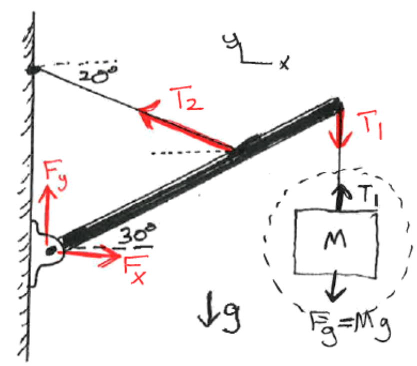
$f_s \leq f_{s,max} = \mu_s F_N$ so $1046 \leq (\mu_s)(3870)$ or $0.270 \leq \mu_s$ or flipping the equation around: $\mu_s \geq 0.270$. The coefficient of static friction has to be at least that large for there to be enough static friction to allow the pole to be in the configuration. If not, f_s can't be large enough, and the horizontal force component of F_T will cause the pole to accelerate to the left, sliding across the ground unsafely.

A shop sign (of mass $M = 20\ kg$) is hanging from the end of a 3 m long pole (of negligible mass) as shown in the figure. The pole makes an angle of 30^o up from the horizontal. A support wire is connected from the pole to the building, making an angle of 20^o below the horizontal, and connecting to the pole 1 m in from the outer end of the pole.,
Determine the tensions in the cables and the force the left end of the pole is exerting on the building.
(Note: the pole itself obviously would have some mass and would contribute to the torque and force calculations, but this example is more about propagating angles.)



Note: the point where the pole is connected to the building is a swivel joint with frictionless ball bearings, so the point won't introduce any torque.)

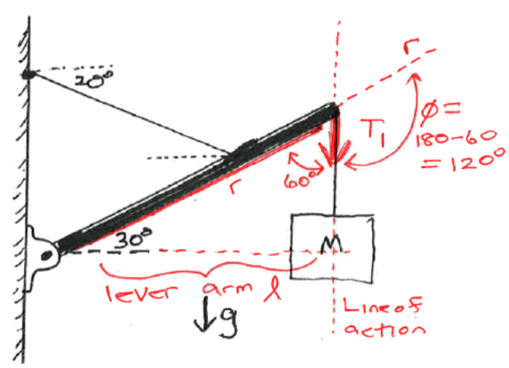
Determine forces acting on object : First, what is our object, then what forces are acting on it. Our object of interest here is the pole. What are **all** the forces acting on **it**? There's the tension in the cable connected to the sign (pulling downward at the outer end of the pole), the tension in the second cable that's been used to connect the pole to the wall to help hold it in place, and some forces in the X and Y direction at the pivot point.



We can immediately determine tension T_1 though by applying Newton's Laws to another object: the hanging sign itself. $\sum F_y = 0$ **on the sign** so $T_1 - mg = 0$ or $T_1 = mg = (20\ kg)(9.8\ m/s^2) = 196\ N$.

We need to select a point to use as our **axis of rotation** for all our torque calculations, so let's put it on top of the pivot. That will automatically make the torque introduced by F_x and F_y (which are both unknown at this point) zero since those forces are right on top of the axis.

Torque due to T_1 : this force of 196 N is acting straight down on the outer end of the pole. (NOTE: it's the tension force T_1 that is acting on the pole, not the force of gravity acting on the sign. We had to take a detour to determine the value of T_1 and it's equal to the weight of the sign, but it's important to be somewhat picky about applying Newton's laws in more complicated scenarios or you can quickly get lost. Only the forces acting ON THE POLE directly are involved.)

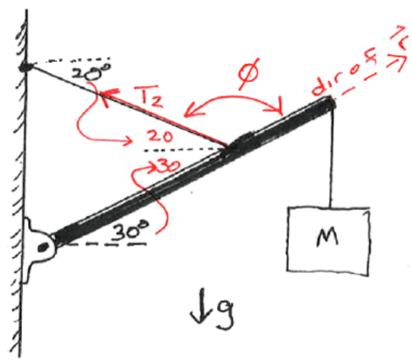


We have multiple ways to determining the magnitude of this torque.

Using $\tau = rF\sin\phi$: ϕ needs to be the angle between the direction of $\vec{r}$ and the direction of $\vec{F}$. In the lower triangle, I've propagated the 30^o angle we were given to the 'other corner' of that triangle, where the angle must be 60^o. The figure indicates that that angle plus ϕ will be 180^o, so ϕ must be

$180 - 60 - 120^o$. This force is being applied $r = 3\ m$ from the axis of rotation, so the magnitude of the torque will be: $\tau = (3\ m)(196\ N)\sin(120^o) = 509.22\ N\ m$. That force alone would cause the pole to rotate clockwise about our axis, so finally $\boxed{\tau_{T_1} = -509.22\ N\ m}$.

Another convenient form is to use the **lever arm** approach. We extend the vector $\vec{T_1}$ to create the **line of action**, then drop a perpendicular to that line from the axis of rotation point. Here, that's the horizontal line labelled **lever arm** which has a length of $l = (3\ m)\cos 30 = 2.5980...\ m$ so $\tau = Fl = (196\ N)(2.5980..\ m) = 509.22\ N\ m$ (same as the other method of course).



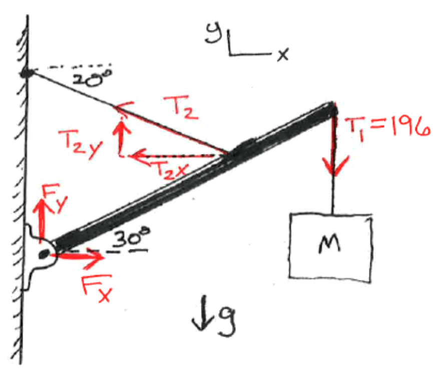
$\boxed{\text{Torque due to } T_2}$: this force, of unknown magnitude T_2 is heading off in the direction shown, and making an angle of 20^o to the horizontal.
Let's try and use the $\tau = rF\sin\phi$ method, so we'll need to figure out the angle between the direction of $\vec{r}$ and the direction of $\vec{T_2}$. I drew a little dotted horizontal line in black starting where T_2 is touching the pole and then propagated the 20 and 30 deg angles into that area. T_2 apparently makes a $20 + 30 = 50^o$ angle with the pole.

The ϕ that we need is marked in the figure, so that angle must be $180 - 50 = 130^o$.

This force acting alone would cause the pole to rotate counter-clockwise, so this is a positive torque:

$\tau_{T_2} = +rF\sin\phi = +(2\ m)(T_2)\sin 130 = +1.532T_2$.

Collecting all these terms now: $\sum \tau = 0$ (about the axis we selected), so: $0 + 0 + (-509.22) + (1.532T_2) = 0$ from which $\boxed{T_2 = +332.4\ N}$.



$\boxed{\text{Wall Force}}$: now that we know all the other forces in the scenario, we can go back and use Newton's Laws to determine the force the wall must be exerting on the left end of the pole.
(Refer back a couple of figures where I show all the forces, in red, acting on the pole.)
In the X direction, we have the wall force F_x and also a **component** of the tension force T_2 (the other force, T_1 being vertical), so:

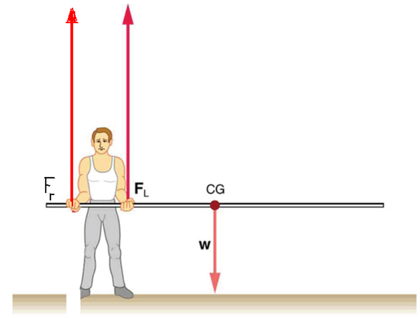
$\sum F_x = 0$ so $F_x + (-332.4\cos(20^o) = 0$ or $F_x = 312.4\ N$.

In the Y direction, we have the wall force F_y, the Y component of T_2 (up), and the 196 N of tension in T_1 (down), so:

$\sum F_y = 0$ so $F_y + (332.4)\sin(20^o) - 196 = 0$ from which $F_y = 82.3\ N$.

That point on the wall is pushing on the left end of the rod with a force of 312.4 N to the right, and 82.3 N up. What angle is that overall force vector making? $\tan\theta = 82.3/312.4$ or $\theta = 13.9^o$. That is **not the same** as the 30 degree angle the pole is making. **That's important to take note of** : when we have 'loose' materials like strings, cords, wires, and so on, the force in those materials needs to be aligned along the material itself, but that's not the case with solid connecting elements (rods, poles, etc).

A pole is being held as shown in the figure. Determine the forces (in terms of the pole weight $W = F_g = Mg$) the person must be exerting.
Pole length: 4 m. Left hand is 1 m from the left end of the pole, and right hand is 40 cm from from the left end of the pole (and thus 60 cm from their right hand).

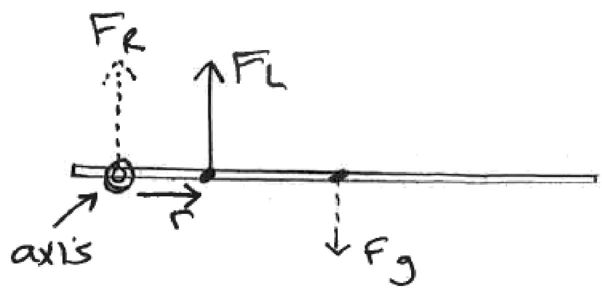


Newton : To apply Newton's Laws, let's define our coordinates to be +X to the right on the page, and +Y vertically upward (i.e. towards to the top of the page). Then $\sum F_y = ma_y = 0$ implies that $F_R + F_L - Mg = 0$ or $\boxed{F_R + F_L = Mg}$. (Remember, we're starting off assuming both F_L and F_R are positive upward and we'll just let the algebra tell us what direction they're really in.)

Torque : The pole isn't rotating about any axis, so let's use one that's going through the point where F_R is being applied. Then:

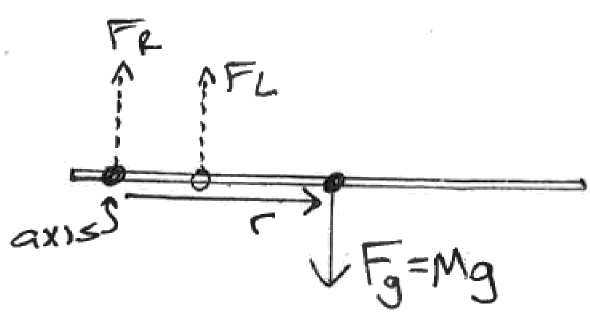
Torque due to F_R (about our chosen axis): $\tau_{F_R} = rF_{tan} = 0$ because r is the distance between the axis and where the force is being applied, and in this case they're at $\boxed{\tau_{F_R} = 0.0}$.

Torque due to F_L (about our chosen axis): $\tau_{F_L} = rF_{tan}$: The axis we've chosen to use is 40 cm = 0.4 m in from the left end of the pole, and this force is being applied 1 m from the left end, so $r = 0.6$ m. This force is perpendicular to $\vec{r}$ as a vector, so F_{tan} is the full value for F_L.



Using the RHR, we see this will be a positive torque. (Or: if we force the pole to rotate about our chosen axis and then apply just this force, which is upward on the page, that will cause the pole to rotate counter-clockwise, making this a positive torque.) Finally: $\boxed{\tau_{F_L} = +(0.6)(F_L)}$.

Torque due to gravity (about our chosen axis): $\tau_{F_g} = rF_{tan}$: The axis we've chosen to use is 40 cm = 0.4 m in from the left end of the pole, and this force is being applied 2 m from the left end (since gravity acts as if all the force were located at the CM of the object, which will be 2 meters in from each end of this 4 meters long pole), so $r = 1.6$ m. This force $F_g = Mg$ is perpendicular to $\vec{r}$ so the entire force of gravity is tangential.



Using the RHR or looking at how this single force would cause the pole to rotate about the axis we chose, the pole would rotate clockwise about that point, so this torque will be negative. Finally: $\boxed{\tau_{F_g} = -(1.6)(Mg)}$.

Collecting terms, $\sum \tau = 0$ so $0 + (0.6)(F_R) - (1.6)(Mg) = 0$. Rearranging, we find that $\boxed{F_R = +2.67(Mg)}$.

Using Newton's Laws ($\sum F_y = 0$) $F_L + F_R - Mg = 0$ so $F_L = Mg - F_R = (Mg) - (2.67Mg)$ or

$$\boxed{F_L = -1.67(Mg)}$$

If the person wants to hold the pole in this fashion, their left hand (the one further from the end of the pole), would need to be exerting a force (upward) that's nearly three times the weight of the pole. Their right hand (the one closer to the end of the pole) would need to exert a force (**downward**) that's nearly double the weight of the pole.