Chapter 07: Work-K

HW07-05: (Probably too easy for a test question, so look at this as a 'warm up' question.) How much work would movers do pushing a 46 kg crate 10.3 m horizontally across a rough floor at a constant speed if the coefficient of kinetic friction is 0.40? (Assume the pushing force is horizontal too.)

 $\mathbf{HW07\text{-}06}$: (Another warm-up question.) A 1200 N crate rests on the floor. How much work is required to move it at constant speed (a) 6.0 m along a floor against a friction force of 230 N, and (b) 6.0 m vertically upward?

(Note they've given you the friction force the person has to match and the weight of the crate in newtons to short-cut this problem.)

HW07-15: A grocery cart with a mass of 16 kg is pushed at constant speed up a 12° ramp by a force F_p which acts at an angle of 17° **below** the horizontal. Find the work done by each of the forces \vec{F}_g , \vec{F}_N and \vec{F}_p on the cart if the ramp is 7.5 m long. (No mention of friction here, so assume there isn't any. Also, note that the speed is constant, so K isn't changing here. Think about how you can use that fact to short-cut the solution and avoid using Newton's Laws entirely.)

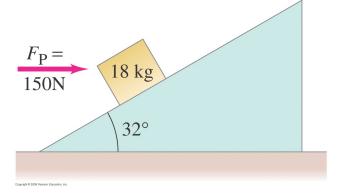
HW07-16: A 380 kg piano slides 4.6 m down along a 30° incline and is kept from accelerating by a man who is pushing back on it (his force is parallel to the incline). Determine: (a) the net work done on the piano, (b) the work done on the piano by the force of gravity, (c) the work done on the piano by the man, and finally (d) the force exerted by the man. (Ignore friction here.)

(Note: avoid using Newton's Laws here. What are the only two forces doing non-zero work on the piano? Use that to determine (c) from (a) and (b); then relate the work done by the man to the force he must be exerting.



HW07-81: A force $\vec{F} = (10.0\hat{i} + 9.0\hat{j} + 12.0\hat{k}) \ N$ acts on a small object of mass 85 grams. If the displacement of the object is $\vec{d} = (5.0\hat{i} + 4.0\hat{j}) \ m$, find the work done by this force on the object. What is the angle between \vec{F} and \vec{d} ? (Just testing your 'dot product' skills with this one. They gave a mass here, but did they need to?)

HW07-66: (a) How much work is done by the horizontal force $F_p = 150 \ N$ on the 18 kg block as the block moves 5.0 m up along the 32^o frictionless incline? (b) How much work is done by the gravitational force on the block during this displacement? (c) How much work is done by the normal force? (d) What is the speed of the block (assume it started at rest) after this displacement?



(The speed isn't constant here obviously since they're asking for it's value, so unlike some of the previous problems there isn't a nice short-cut to solve this one. Remember: solve this entirely using work and energy or via conservation of energy. You don't need Newton's Laws anywhere to solve this.)

Chapter 08: Conservation of Energy

HW08-13: (A warm-up problem.) A sled is initially given a shove up a frictionless 18.0° incline. It reaches a maximum vertical height 1.22~m higher than where it started at the bottom. What was it's initial speed? (Hint: the angle given is not used at all in the solution here! Focus on CoE.)

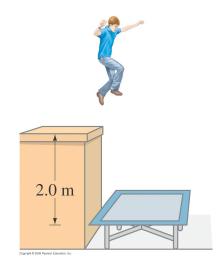
HW08-15: A spring with $k = 88 \ N/m$ hangs vertically next to a vertical ruler. The end of the spring is next to the 15 cm mark on the ruler. If a 2.5 kg mass is now attached to the end of the spring and the mass is now allowed to fall, where will the end of the spring line up with the ruler marks when the mass is at its lowest position?

(Forget the tricky wording about the ruler here and basically just focus on how far the spring will stretch before it brings the object to a momentary stop. This is a conservation of energy problem, so be sure to account for all forms of energy that are present.)

HW08-18: A 62 kg trampoline artist jumps upward from the top of a platform with an initial vertical speed of 4.5 m/s. (a) How fast is he going as he lands on the trampoline (i.e. when he first touches it)? (b) If the trampoline behaves like a spring with $k = 58,000 \ N/m$, how far does he depress the trampoline?

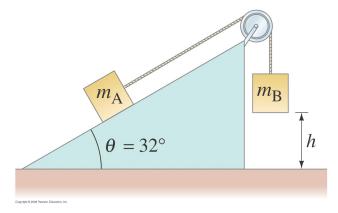
(Again as in the first problem, from when he first touches the trampoline to when he comes to a stop, his vertical coordinate is changing, so be sure to account for all types of mechanical energy here.)

(Ignore any lateral motion here and treat this as if all the motion were perfectly vertical.)

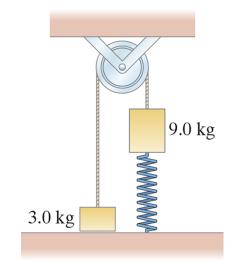


The next two are 'connected object' problems, so apply CoE to each object separately then combine the results to eliminate the 'work done by the tension force' terms in each equation. If you insist on treating the problem as a single 'system', be sure to explain how you're handling the work done by the tension force present - i.e. don't just ignore it. It is a force that is present here, so needs to be addressed one way or another.

HW08-22: Two masses are connected by a string as shown in the figure. Mass $m_A = 3.5 \ kg$ rests on a frictionless 32^o incline, while $m_B = 5.0 \ kg$ is initially held at a height of $h = 0.75 \ m$ above the floor. If they start at rest, how fast are the masses moving when block B hits the floor? Remember, no equations of motion here; don't determine the acceleration. Use conservation of energy to arrive at the answer directly.



HW08-23: The 9.0 kg mass in the figure is held just barely in contact wit a spring with $k=450\ N/m$. When that mass is released, it falls, compressing the spring and pulling the 3.0 kg mass up. How far does the 9.0 kg mass fall before momentarily coming to rest?



HW08-56: How long would it take a 1750 W motor to lift a 335 kg piano to a sixth-story window 18 m above? (Assume the piano is moving at a constant speed.)

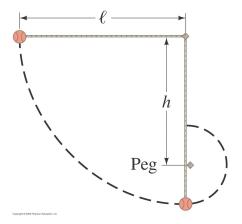
HW08-68: What minimum horsepower must a motor have to be able to drag a 370 kg box along a level floor at a speed of 1.20 m/s if the coefficient of friction is $\mu_k = 0.45$? (Assume the cable connecting the motor to the box is pulling horizontally.)

 $\mathbf{HW08-90}$: Some electric power companies use placement of water to store energy. Water is pumped from a low reservoir to a high reservoir, usually at night when the load on their generators is less. Then during the day the higher elevation water runs through turbines to generate electricity. To store the energy produced in 1.0 hour by a 180 MW electric power plant, how many cubic meters of water will have to be pumped from the lower reservoir to the higher reservoir? Assume the upper reservoir here is 380 m above the lower one.

(Some bits you'll need: water has a mass of 1000 kg for every cubic meter; $1~MW = 1 \times 10^6~W$, so how many joules of energy needs to be stored?)

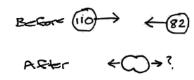
(Another real-world approach being tested is to store energy by raising very heavy blocks to a higher level when the load on their system is low, then allowing them to slowly fall, converting that 'stored' gravitational potential energy back into electrical power when needed.)

HW08-82: (A modified version of this HW problem with numbers instead of symbolic.) A ball is attached to a horizontal cord of length L = 80 cm whose other end is fixed. (a) If the ball is released at rest, what will be its speed at the lowest point of its path? (b) A peg is inserted h = 60 cm directly below the point of attachment of the cord. What will be the speed of the ball when it reaches the top of its circular path around that peg?



Chapter 09: Conservation of Momentum

HW09-03: (A warm-up question, with additional parts beyond what was in the original.) A 110 kg tackler moving at 2.5 m/s meets head-on (and holds onto) an 82 kg halfback moving at 4.4 m/s. (Note the description here implies that they're running in a line directly towards one another.)



- (a) What will be their mutual speed immediately after the collision?
- (b) How much energy was lost in the collision?
- (c) If the collision itself took place in 0.1 sec, determine the magnitude of the force each player exerted on the other during the collision.

(Why do the questions just ask for 'speed' and 'magnitude' of the force and not signed quantities here? You'll need to create a coordinate system to solve this, and the signed values will depend on what coordinate system you picked, so this HW question was forced to ask for just magnitudes.)

HW09-15 (modified): Let's extend the previous question into a 3-D collision. A mass $m_A = 2.0 \ kg$, moving with velocity $\vec{v}_A = (4.0\hat{i} + 5.0\hat{j} - 2.0\hat{k}) \ m/s$, collides with mass $m_B = 4.0 \ kg$ which is initially at rest. Immediately after the collision, mass m_A is observed to be travelling at velocity $\vec{v}_A' = (2.0\hat{i} - 3.0\hat{k}) \ m/s$.

- (a) Find the velocity of mass m_B after the collision. $\vec{v}_B' = (\underline{\hat{i}} + \underline{\hat{j}} + \underline{\hat{j}} + \underline{\hat{k}}) m/s$
- (b) How much energy was lost in the collision? $____$ J
- (c) If the collision took 1 ms, what force did A exert on B during the collision? $\vec{F} = (\underline{\hat{i}} + \underline{\hat{j}} + \underline{\hat{j}} + \underline{\hat{k}}) N$

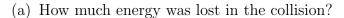
(Different from the first problem: here we do have a coordinate system provided since we have the (x, y, z) components of A's initial velocity.)

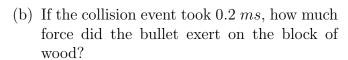
Note that (a) is asking for the vector velocity of object B after the collision (just like the vector velocity of A after the collision was given).

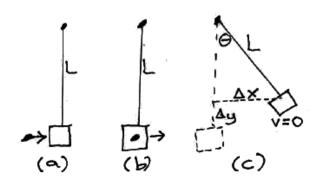
Remember: $K = \frac{1}{2}mv^2$ where v is the **speed** of an object, so you'll need to convert the vector velocities into their magnitudes at that point. For any vector, it's magnitude is related to its components: $|v| = \sqrt{v_x^2 + v_y^2 + v_z^2}$.

(c) Remember the force on object B just involves finding the change in B's momentum as a result of the collision.

HW09-38 (modified): A 28 g rifle bullet traveling at 190 m/s (horizontally) embeds itself in a 3.1 kg block of wood hanging on a 2.4 m long string, which makes the pendulum swing upward in an arc.







(c) Determine the vertical and horizontal components of the pendulum's maximum displacement. (I would have just asked for the angle of the string at the point where the combined object comes to a momentary stop, but the same figure can be used to find the vertical and horizontal displacement.)

NOTE: since this is an error I see often on test answers, it's worth noting that **28** g refers to the **mass** of the bullet in **grams**. In this context that **g** symbol refers to **units in grams** and **not** the gravitational constant $(g = 9.8 \text{ m/s}^2)$. Since 1 kg is 1000 grams (1 kg = 1000 g) then the bullet here has a mass of 0.028 kg.

HW09-40 (modified): (Use a coordinate system with +Y pointing vertically upward for this one.) You drop a 21 g ball from a height of 1.5 m and it only bounces back to a height of 0.85 m.

- (a) What was the impulse on the ball when it hit the floor? (Remember 'impulse' is just another word for $\Delta \vec{p}$.)
- (b) How much energy was lost in the collision?
- (c) If the ball was in contact with the floor for $0.3 \ ms$, how much force did the floor exert on the ball?

HW09-47 (slightly modified): Billiard ball A of mass $m_A = 0.120 \ kg$ moving with speed $v_A = 2.80 \ m/s$ strikes ball B of mass $m_B = 0.140 \ kg$ that was initially at rest. As a result of the collision, ball A is deflected off at an angle of 35^o with a speed of $v_A' = 2.10 \ m/s$.



(Note this is a 2-D collision. Use a coordinate system here where ball A is initially travelling along the +X axis, and the 35^o deflection angle will be counter-clockwise relative to that X axis. Also note that the magic word 'elastic collision' was not mentioned here, so don't assume that mechanical energy will be conserved here - it won't be, and part (b) asks for how much energy ended up being lost in the collision.)

- (a) Determine the speed and direction of ball B just after the collision.
- (b) How much energy was lost in this collision?

HINT: instead of leaving everything in vector notation, it's simpler to break up the conservation of momentum equation into separate X and Y components like I do in the 2-D collisions in the examples 09.pdf file, with +X horizontally to the right, and +Y pointing towards the top of the page, for example.

$$\sum (mv_x)_{before} = \sum (mv_x)_{after}$$
 and separately $\sum (mv_y)_{before} = \sum (mv_y)_{after}$.

This will ultimately give you the x and y components of B's velocity vector after the collision, which can then be turned into its speed and angle (and the **sign** of the X and Y component of B's velocity after the collision will show that it's heading off to the right (positive X velocity) and below the X axis (negative Y velocity component). If $\theta_A = +35^o$ (positive meaning CCW relative to the X axis), then θ_B will be some negative angle (an angle CW relative to the X axis).

HW07-05: 1857.3 J

HW07-06: (a) 1380 J, (b) 7200 J.

HW07-15: $W_{F_q} = -244.5 \ J$, $W_{F_N} = 0 \ J$, $W_{push} = +244.5 \ J$.

HW07-16:

- (a) $K_f = K_i + \sum W$ and K isn't changing here, so the net work ($\sum W$) must be zero.
- (b) work done by gravity: +8565.2 J (use $W_q = -mg\Delta h$),
- (c) work done by person: -8565.2 J,
- (d) person's force: 1862 N (use $W_{person} = \vec{F}_{person} \cdot \vec{d}$, with the person's force in the exact opposite direction as the piano's displacement)

HW07-81: (a) Do the dot product: W = 86 J

(b) Use above result and the fact that $\vec{F} \cdot \vec{d} = Fd\cos\phi$ to find $\phi = 41.84^{\circ}$.

HW07-66: (a) $W_{push} = \vec{F} \cdot \vec{d} = (150 \ N)(5 \ m) \cos 32^o = 636.04 \ J$

- (b) $W_{grav} = -mg\Delta y = -467.39 \ J$
- (c) $W_{F_N} = 0$
- (d) $K_f = K_i + \sum W$ leads to $v = 4.329 \ m/s$

(Remember: don't solve these with Newton's Laws and equations of motion. Learn how to use work and energy to do them!)

HW08-13: 4.89 m/s (initial K becomes final mgh and h was given so never needed the angle)

 $\mathbf{HW08-15}$: The object falls 0.5568 m (stretching the spring that far) before coming to a (momentary) stop.

HW08-18: (a) |v| = 7.7104... m/s, (b) trampoline will depress by d = 0.26278...m (CoE leads to a quadratic equation to solve for d here.)

HW08-22: $|v| = 2.33227... \ m/s$

 $\mathbf{HW08\text{-}23}$: The 3 kg block rises 0.26133.. m and the 9 kg block falls 0.26133.. m.

HW08-56: The piano's kinetic energy isn't changing, but it's gaining mgh = 59094 J of gravitational potential energy between the two point. The motor is putting out 1750 J/s so it takes about 33.8 s. (Which implies the piano is rising at a speed of v = (18 m)/(33.8 s) = 0.533 m/s.)

 $\mathbf{HW08\text{-}68}:\,1958\;W$ or about 2.62 hp

HW08-90: The power plant is putting out 180 $MW = 180 \times 10^6 \ J/s$, so in one hour (3600 seconds) this represents an energy of $6.48 \times 10^{11} \ J$. How much water do we need to raise 380 m to store that much energy? $U_g = mgh$ so $m = (energy)/(gh) = 1.74 \times 10^8 \ kg$. One cubic meter of water represents 1000 kg, so we'd need to raise 1.74×10^5 cubic meters of water.

HW08-82 : (a) $|v| = 3.9598 \ m/s$, (b) $|v| = 2.800 \ m/s$

HW09-03: (a) $|v| = 0.446875 \, m/s$ (and if you set this up with the tackler moving to the right and the halfback moving to the left, you'll find that the halfback has a higher momentum than the tackler, so the 'combined object' will end up moving in the same direction as the halfback was originally moving). (b) $\Delta E = -1118.3 \, J$ (1118.3 J of energy was lost in the collision). (c) $|F| = 3241.6 \, N$ (each player exerts that much force on the other during the collision).

HW09-15 (modified): (a) \vec{v}_B after the collision will be $1.0\hat{j} + 2.5\hat{j} + 0.5\hat{k}$. (b) Energy before collision: 45 J; energy after colliion: 28 J, so here we lost 17 J of energy in the collision. (c) The force on B is the change in its momentum divided by the Δt of the collision: $\vec{F}_{on\ B} = \frac{\vec{p}_{B,after} - \vec{p}_{B,before}}{\Delta t} = (4000\hat{i} + 10000\hat{j} + 2000\hat{j}) N$

HW09-38 (modified): The two object combine in the collision, so the 'object' after the collision has a mass of 3.128 kg. Immediately after the collision, this combined object will have a speed of 1.70077 m/s. (a) Just before collision $E = 505.4 \ J$; just after $E = 4.524 \ J$, so 500.88 J of energy was lost in the collision (nearly all). (b) Looking at the change in the momentum of **just** the block of wood before and after the collision, the bullet apparently exerted a force of about 26362 N on it (with the block exerting an equal and opposite force on the bullet). (c) Using CoE after the collision, the combined object rises $\Delta y = 0.14758 \ m$ from it's original elevation before (momentarily) coming to a stop. Using the $L = 2.4 \ m$ length of the string, the string swings out $\theta = 20.2^o$ from its originally vertical orientation, representing a horizontal displacement of about $\Delta x = 0.829 \ m$ for the block.

HW09-40 (modified): Use CoE to relate the given heights into the velocities the ball had just before and just after hitting the floor. The ball hits the floor with $v = -5.4222 \ m/s$ and bounces back up with a velocity of $v = +4.0817 \ m/s$. (a) The impulse (change in momentum) of the ball when it hits the floor is $J = \Delta p = +0.19958 \ kg \ m/s$. (b) The energies before and after are 0.3087 J and 0.1749 J, representing an energy loss of 0.1338 J. (c) The force on the ball is its change in momentum divided by the time of the collision, or 665.3 N.

HW09-47 (slightly modified): This is a generic 2-D collision, so break the problem up into X and Y components. (a) Using a coordinate system with +X to the right and +Y up towards the top of the page: $(\sum p_x)_{before} = (\sum p_x)_{after}$ leads to ball B having an X components of velocity of $v_x = 0.92553 \ m/s$ after the collision. $(\sum p_y)_{before} = (\sum p_y)_{after}$ (and the left side is zero here) leads to ball B having a Y component of velocity of $v_y = -1.0324 \ m/s$ after the collision. Combining those, this represents a speed of $|v| = 1.38655 \ m/s$ after the collision, and an angle of about 48.1^o below the X axis.

(b) Total energy just before the collision: $0.4704\ J$; total energy just after the collision: $0.2646 + 0.1346 = 0.3992\ J$, so apparently about $0.0712\ J$ was lost. (That only about 15 percent of the original energy, so this is *almost* an elastic collision, but not quite.)