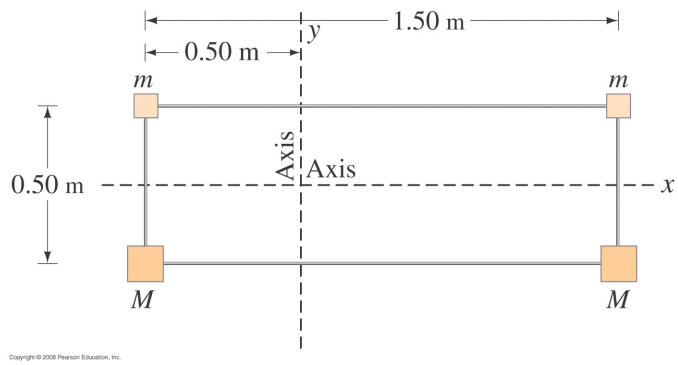
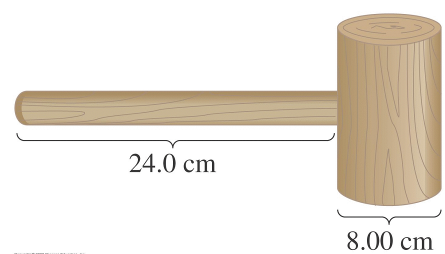
Chapter 9 Section 09 : Center of Mass

HW10-42 (modified) : Determine the X and Y coordinates of the center of mass of the collection of point-masses shown in the figure. The two small objects have masses of $m = 2.2\ kg$ and the two larger objects have masses of $M = 3.2\ kg$. (The array is rectangular and is split through the middle by the The X axis.)



Based on the original version of this problem in the book, also determine the moment of inertia of this object if it's being rotated about the X axis. Then find the moment of inertia of the object if it's being rotated about the Y axis. What is the object's moment of inertia if we rotate this object about the Z axis (an axis coming up out of the page towards you, passing through the origin - i.e. the point where the X and Y axes intersect)?

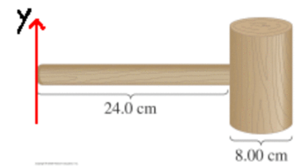
HW09-66 (version 1) : A mallet consists of a uniform cylindrical head of mass 2.60 kg and **diameter** 0.0800 m attached at its center to a uniform cylindrical handle of mass 0.500 kg and length 0.240 m as shown in the figure. How far from the left end of the handle is the center of mass of the mallet located?



(Hint: remember for 'composite objects' you can replace each 'part' with a point mass located at the CM of that part, then use the usual point-mass CM equations to determine the CM of the overall object. From symmetry arguments, where would the CM of the handle be? Where would the CM of the head be?)

Chapter 10 : Rotational Motion

HW09-66 (version 2) : A mallet consists of a uniform cylindrical head of mass 2.60 kg and **diameter** 0.0800 m attached at its center to a uniform cylindrical handle of mass 0.500 kg and length 0.240 m as shown in the figure. Determine the **moment of inertia** of this object for rotations about the Y axis shown.



Hint: remember for 'composite objects' I is just the sum of the moments of inertia of its parts. The handle is a thin rod rotating about one of its ends. The head is a cylinder rotating about an axis that is **parallel** to a Y axis passing through its center of symmetry, so use the parallel axis theorem $I = I_{cm} + Md^2$ to adjust the head's moment of inertia given in the table to the axis we actually need it rotating about.

HW10-07 (modified) : A grinding wheel 0.35 m in **diameter** rotates at 2400 RPM.

(a) Calculate its angular velocity in rad/s.

(b) What are the linear speed and radial acceleration of a point on the outer edge of the grinding wheel?

Some additional parts:

(c) If the mass of the grinding wheel is 0.8 kg, how much energy is stored in the wheel?

(d) If the motor attached to the grinding wheel spins it up from rest to 2400 RPM in 1.5 sec, how much torque must the motor be putting out?

HW10-08 (modified) : A bicycle with tires 68 cm in **diameter** travels 8.6 km.

(a) How many revolutions do the wheels make?

(b) If the bike was travelling at a constant speed of 20 mph, what was the angular speed of the tire in rad/s? In RPM?

HW10-19 (modified) : A centrifuge accelerates uniformly from rest to $15,000\ RPM$ in $220\ s$.

(a) Through how many revolutions did it turn in this time?

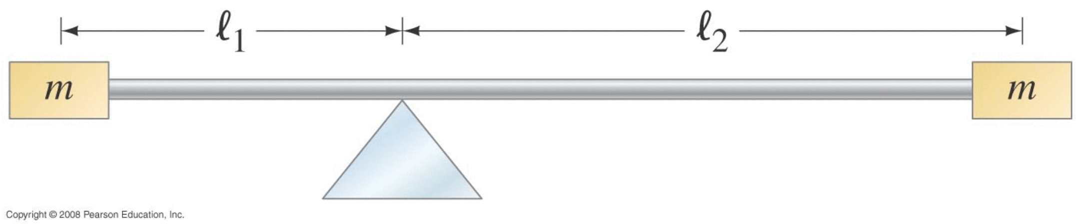
If the moment of inertia of the rotating parts of the centrifuge is $I = 0.15\ kg\ m^2$:

(b) How much work was involved in spinning up the centrifuge to its operating speed? (Think $K_f = K_i + W_{Motor}$ here. The centrifuge started at rest and ended up with some rotational kinetic energy: the motor had to do that much work.)

(c) How much torque did the motor put out during the spin-up?

HW10-28 (modified) : (The original HW version of this was entirely symbolic so I just added numbers here, and some additional questions.)

Two blocks, each of mass $m = 10\ kg$, are attached to the ends of a **massless** rod which pivots as shown in the figure. (For purposes of this problem, treat the blocks as point masses.) The rod is $1\ m$ long, with $l_1 = 30\ cm$ and $l_2 = 70\ cm$. Initially the rod is held in the horizontal position and then released (at rest).



(a) Calculate the magnitude and direction (clockwise or counterclockwise) of the net torque on this system when it is first released.

(b) Determine the instantaneous angular acceleration of the rod just after it's released. ($\sum \tau = I\alpha$, so you'll need to determine the moment of inertia of the rotating 'object' about the pivot point here: the massless rod won't contribute, but the two point-masses on the ends will).

(c) Calculate the magnitude and direction (up or down) of the **linear** acceleration of each block when the rod is first released.

HW10-43 : A potter is shaping a bowl on a potter's wheel rotating at a constant angular velocity of 1.6 rev/s. The frictional force between her hands and the clay is 1.8 N.

(a) Determine the frictional torque on the wheel if the **diameter** of the bowl is 9.0 cm.

(b) How long would it take for the potters wheel to stop if the only torque acting on it is due to the potter's hands (i.e. the frictional torque present here)? The moment of inertia of the wheel and bowl (i.e. the 'rotating object' here) is 0.11 $kg\ m^2$.

HW10-65 : A merry-go-round has a mass of 1240 kg and a radius of 7.50 m. Treat this object as a rotating disk (cylinder). The merry-go-round takes 20 seconds to accelerate uniformly from rest to a final rotation rate of 1 revolution per 7 seconds.

(a) How much net work was required to bring the MGR from rest up to it's final operating speed?

(b) How many rotations did the MGR make during the 20 second spin-up phase?

(c) How much torque was the MGR motor putting out during the 20 second spin-up phase?

HW10-68 : A 2.30 m long pole is balanced vertically on its tip. It starts to fall and its lower end doesn't slip. What will be the speed of the upper end of the pole just before it hits the ground? (Hint: use conservation of energy. The initial rotational speed is essentially $\omega = 0$; what will ω be the instant before the pole hits the ground? Relate this to what we did with the swinging meter-stick in class. The object is rotating about the point where the bottom of the pole is touching the ground, so if you use that point as your axis of rotation, all the kinetic energy is in the form of K_{rot}.)

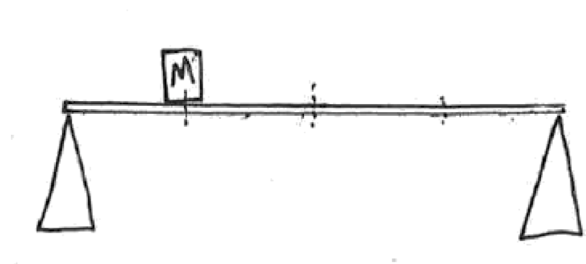
HW10-69 : Calculate the translational speed of a uniform solid cylinder (released at rest) when it reaches the bottom of an incline 6.5 m high. (I.e., it's initial location on the ramp is 6.5 m higher up than the bottom of the ramp. Note that they didn't give us the radius of the cylinder or its mass, so you'll have to carry those along symbolically. Once you've set up the CoE equation for this scenario, you'll find that both R and M end up cancelling out in the resulting equation. Hint: from the chart, the moment of inertia of a uniform solid cylinder is $I = \frac{1}{2}MR^2$.)

Chapter 11 : (Just cross-product)

See the examples11.pdf and the hw11 pdf's on Canvas. Basically all we took from this chapter was the mechanics of how to do a cross-product between two vectors. (HW11-31 in hw11-key.pdf is particularly useful to review.)

Chapter 12 : Static Equilibrium (sections 1 and 2)

HW12-07 (modified) : (The book version didn't give the length of the beam since ultimately it doesn't affect the solution, but to simplify the math I've added the beam length. You might want to try it just leaving the length as some unknown value L and see how that ultimately ends up cancelling out of the final equation.)



A 4 m long 110 kg horizontal beam is supported at each end (right at each end). A 320 kg object rests a quarter of the way in from the left end of the beam. (So it's 1 meter from the left end, and 3 meters from the right end.) How is the weight distributed between the two supports? That is, what vertically-upward force is each support exerting?

(Newton's laws are available, but don't give a solution since we have two many unknowns. You'll have to use $\sum \tau = 0$ about some point(s) to create additional equations(s) to allow a solution to be found. Hint: what happens if you apply $\sum \tau = 0$ about an axis located at the left end of the beam?)

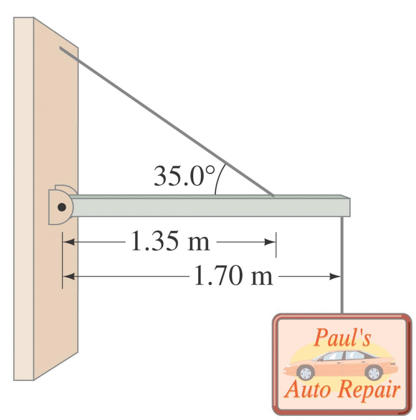
HW12-09 : A 75 kg adult sits at one end (say the left) of a 9.0 m long horizontal board. A 25 kg child sits on the other end (the right-most end).

(a) If we ignore the mass of the board, where should the support pivot be placed so that the board is balanced?

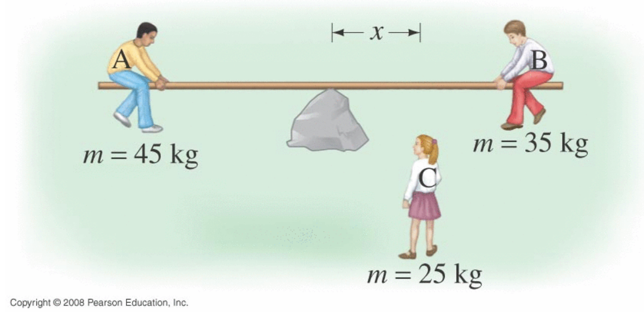
(b) If the board is uniform and has a mass of 18 kg, where does the pivot point need to be now?

(Basically we're looking for the location about which $\sum \tau = 0$. A wordy argument can probably be used for (a), but an equation will be needed for part (b).)

HW12-14 : A shop sign weighing 215 N hangs from the end of a uniform 135 N beam as shown in the figure. Find the tension in the supporting wire (that wire making the 35^o angle) and the horizontal and vertical forces exerted by the hinge on the beam at the wall.
(There are three unknowns here, the F_x and F_y forces at the hinge, and the F_T tension in the cable, so Newton's Laws alone won't solve this. What happens if you apply $\sum \tau = 0$ about the hinge point on the wall? About the point where the wire is connected to the horizontal beam the sign is hanging from?)

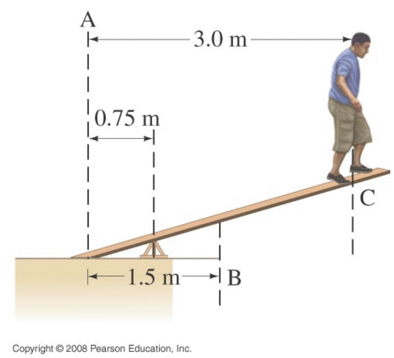


HW12-15 : Three children are trying to balance on a seasaw, which includes a fulcrum rock acting as a pivot **right at the center of the board**, which is 3.2 m long. Two playmates are already on either end of the board. Child A on the left end has a mass of 45 kg. Child B on the right end has a mass of 35 kg. **(The seasaw has a mass - why can we ignore that here?)** Where should child C (with a mass of 25 kg be located to balance the seesaw?



(We're looking for the position relative to the pivot point, so try applying $\sum \tau = 0$ about that point.)

HW12-22 : You are on a pirate ship and being forced to walk the plank. You are standing at the point marked C. The plank is nailed to the deck on the left end, and rests on the support 0.75 m from the left end. The center of mass of the uniform plank is located at point B (which is 1.50 m away from the nail). Your mass is 65 kg, and the plank has a mass of 45 kg.

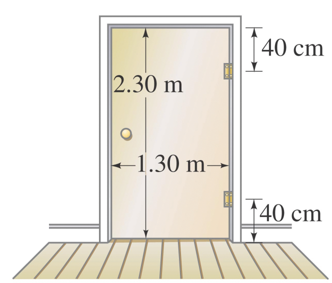


(a) What is the minimum downward force the nails must exert on the left end of the plank to hold it in place?

(NOTE: assume the fulcrum and nail forces are vertical.)

(There's an actual pivot point here that the object would rotate about, so what happens if you apply $\sum \tau = 0$ about that point? Note they've given us a lot of horizontal measurements here - what directions are the forces in? Those measurements can be used to find the 'lever arm' distances relative to that axis of rotation...)

HW12-24 : A door 2.30 m high and 1.30 m wide has a mass of 13.0 kg. A hinge 0.40 m from the top, and another hinge 0.40 m from the bottom each support half the doors weight. Assume that the center of mass of the door is at the geometric center of the door, and determine the horizontal and vertical forces exerted by each hinge on the door.



(They told us that each hinge is supporting half the weight of the door, so we can get the vertical components of each hinge force easily. You'll need $\sum \tau = 0$ to get the horizontal forces each hinge is exerting on the door though. They're each horizontal, but we don't know their signs or magnitudes, so start off with two unknowns, $F_{x,top}$ and $F_{x,bottom}$ each pointing to the right, say. Where's a good place to calculate the torques about? What happens if you calculate all the torques about the point where the top hinge is located? What would the torque due to the weight of the door be? Be on the lookout for places where we can use the $\tau = Fl$ (lever arm) shortcut too here.)

- **HW10-42**
  $x_{cm} = +0.25\ m$, $y_{cm} = -0.463\ m$
  $I_x = 0.675\ kg\ m^2$, $I_y = 6.75\ kg\ m^2$, $I_z = 7.425\ kg\ m^2$

- **HW09-66, version 1** : center of mass would be 0.2542 m from left end

- **HW09-66, version 2** : moment of inertia for composite object:
  $I_{handle,y} = \frac{1}{3}ML^2 = 0.0096\ kg\ m^2$; $I_{head,cm} = \frac{1}{2}MR^2 = 0.00208\ kg\ m^2$ so $I_{head,y} = I_{head,cm} + Md^2$ with $d = 24 + 4 = 28\ cm = 0.28\ m$ so $I_{head,y} = 0.00208 + 0.20384 = 0.20592\ kg\ m^2$. Finally $I_{hammer} = I_{handle,y} + I_{head,y} = 0.21552\ kg\ m^2$.

- **HW10-07 (modified)** :
  (a) $\omega = 80\pi\ rad/s$ (about 251.33 rad/s)
  (b) At outer edge: $|v| = r\omega = 43.982\ m/s$, $a_r = \omega^2 r = 11054\ m/s^2 = 1128\ g's$
  (c) $I = \frac{1}{2}MR^2 = 0.01225\ kg\ m^2$ so $K_{rot} = \frac{1}{2}I\omega^2 = 386.9\ J$
  (d) Use equations of motion to find $\alpha = 167.55\ rad/s^2$ then $\tau = I\alpha = 2.0525\ N\ m$

- **HW10-08 (modified)** :
  (a) arclength $l = r\theta$ so $\theta = 25,294\ rad = 4025.6\ rotations$
  (b) convert v to m/s first, then: $v = r\omega$ so $\omega = v/r = 26.29\ rad/s$

- **HW10-19 (modified)** :
  (a) 27,500 revolutions
  (b) $185,055\ J$ (unrealistically high for an actual centrifuge)
  (c) $\tau = 1.071\ N\ m$

- **HW10-28 (modified)** :
  (a) $\sum \tau = -39.2\ N\ m$ (negative, so clockwise)
  (b) $\alpha = -6.7586\ rad/s^2$ (negative, so clockwise)
  (c) $|a|_{left} = 2.028\ m/s^2$ (upward) and $|a|_{right} = 4.731\ m/s^2$ (downward)

- **HW10-43** :
  (a) $|\tau| = 0.081\ N\ m$
  (b) $\tau = I\alpha$ yields $\alpha = (-)0.73636..\ rad/s^2$ (negative since slowing down); equations of motion yield $t = 13.65\ s$ to reach $\omega_{final} = 0$.

- **HW10-65** : Work: $14,049\ J$; Rotations: 1.4286 revs; Torque: 1565 $N\ m$

- **HW10-68** : CoE yields $\omega = 3.5753\ rad/s$; $v = R\omega = 8.22\ m/s$

- **HW10-69** : CoE with both K_{cm} and K_{rot} yields $v_{cm} = 9.216\ m/s$

- **HW12-07 (modified)** : $F_{left} = 2891\ N$; $F_{right} = 1323\ N$

- **HW12-09** : (a) ignoring the board weight: $x = 2.25\ m$ from the left end
  (b) including the board weight: $x = 2.593\ m$ from the left end

- **HW12-14** : $F_T = 620.215\ N$; wall force: $F_x = 508.05\ N$ and $F_y = -5.74\ N$ (Note that negative value for F_y : where the beam is touching the wall, the other forces and torques are actually trying to pull the left end of the beam upward at that point, so the hinge is having to pull downward slightly at that point...)

- **HW12-15** : child needs to sit at $x = 0.64\ m$ (that far to the right of the fulcrum point)
- **HW12-22** : $F_{nail} = 2352\ N$ (which is about 3.7 times the person's weight, so these are some pretty serious nails)
- **HW12-24** : (using a coordinate system with +X to the right and +Y towards top of page)
  Top connection point: $F_x = +43.58\ N$ (i.e. to the right); $F_y = \frac{1}{2}Mg = 63.7\ N$ (given)
  Bottom connection point: $F_x = -43.58\ N$ (i.e. to the left); $F_y = \frac{1}{2}Mg = 63.7\ N$ (given)