Chapter 21 Examples : Electric Charge and Electric Field

Key concepts :

- Charge can be either negative or positive, and is measured in units of Coulombs (C)
- For normal matter, all the way down to individual protons and electrons, all charges are integer multiples of $e = 1.602 \times 10^{-19}\ C$
- Each electron has a charge of $-1e$ and each proton has a charge of $+1e$
- Coulomb's Law : the magnitude of the force between two point charges q_1 and q_2 is given by $F = k\frac{q_1 q_2}{r^2}$. Charges of the same sign repel each other; charges of opposite sign attract.
- Electric field from a point charge Q: $\vec{E} = \frac{kQ}{r^2}\hat{r}$
- $\vec{E}$ can be used to determine the force that some new charge q will feel in its presence: $\vec{F} = q\vec{E}$
- Matter can usually be characterized as being somewhere between a **conductor** and an **insulator**.
  - Conductors are materials with a lot of loosely-bound electrons which can flow easily through the material
  - Insulators are the opposite: the electrons in their atoms and molecules are tightly bound and do not flow easily

General Suggestions throughout the course :

- Draw good pictures!
- Common constants are given in tables at the very beginning and end of the book.
- Common results from trig and calculus are given in the appendices.
- Watch out for units. All our equations involve standard metric units (seconds, Newtons, meters, kg, etc). Is the information in a problem given in these units? Are the answers requested in these units or something else?
- Watch out for how they give and ask for information.
  - Do they provide a diameter or circumference when you really need a radius?
  - A full Coulomb is a **lot** of charge, so charges are often given in problems as
    - milli-coulombs : $1\ mC = 1 \times 10^{-3}\ C$
    - micro-coulombs : $1\ \mu C = 1 \times 10^{-6}\ C$
    - nano-coulombs : $1\ nC = 1 \times 10^{-9}\ C$
- Recall Newton's Laws: forces are vectors, so watch out for directions and angles, if appropriate. You may need to convert the force magnitude into x and y components based on the geometry of the problem.

Charge in a Lightning Bolt : Lightning occurs when there is a flow of electric charge (principally electrons) between the ground and a thundercloud. The maximum rate of charge flow in a lightning bolt is about $20,000\ C/s$ and the bolt lasts for $100\ \mu s$ or less.

(a) How much charge flows between the ground and the cloud in this time?

The amount of charge will be the rate multiplied by the time interval. A microsecond is a millionth of a second, so:

$$Q = (20,000\ C/s) \times (100 \times 10^{-6}\ s) = 2.0\ C$$

(b) How many electrons flow during this time?

The number of electrons would be this amount of charge divided by the charge per electron:

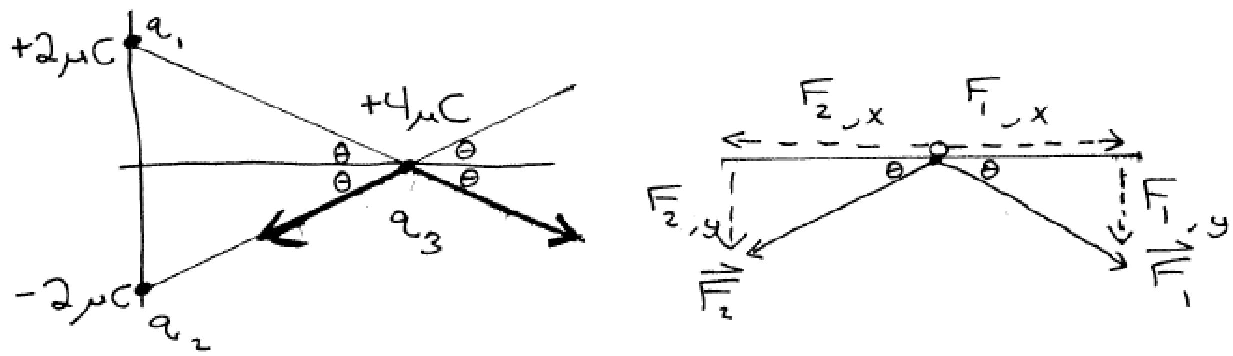
$$n_e = \frac{Q}{e} = \frac{2.0\ C}{1.602 \times 10^{-19}\ C/electron} = 1.25 \times 10^{19}\ electrons$$

(That sounds like a lot, but as we'll see in a later problem, even a microscopic amount of matter will contain that many electrons...)

Electric Force (1) : A point charge of $q_1 = +2.0\mu C$ is located at $x = 0, y = 0.3\ m$ and a second point charge of $q_2 = -2.0\mu C$ is located at $x = 0, y = -0.3\ m$. What are the (a) magnitude and (b) direction of the total electric force that these charges exert on a third point charge of $q_3 = +4.0\mu C$ located at $x = 0.4\ m, y = 0$? (Use a coordinate system with $+X$ horizontal to the right and $+Y$ vertically upward on the page.)

The situation as described is shown in the left figure below, with the directions of the force vectors acting on the third charge. (Remember: like charges repel each other, and opposite charges attract.) Since the magnitudes of q_1 and q_2 are the **same** and they are at the **same** distance from q_3, the **magnitudes** of the two force vectors will be **identical**. Also, from the geometry here, all those angles labelled θ are the same angle.

The right side of the figure highlights the force vectors themselves. Since the magnitudes and angles are the same, we can see that the x components of these two forces will **cancel each other out**, and the y components will be the same value, producing an overall force that is directed in the negative y direction. (So right away we know the angle of this net force: it will be straight down, which would be an angle of 270^o in the usual polar convention of measuring angles counter-clockwise around from the $+X$ axis.



Note: the labels on the forces are different from what the book would use. The subscript just refers to the charge that is exerting that force on charge number 3.

The magnitude of each of the forces can be found from $F = k\frac{q_a q_b}{r^2}$. We have the coordinates of all the charges and in this case they form nice right triangles and the radius will just be the hypotenuse of the triangles, or $r = \sqrt{.3^2 + .4^2} = .5m$.

The magnitude of each force then would be: $F = (9 \times 10^9)\frac{(2\times10^{-6})\times(4\times10^{-6})}{0.5^2} = 0.288\ N$.

The magnitude of the y component of each force then would be $(0.288\ N)\sin\theta = (0.288) \times \frac{0.3}{0.5} = 0.1728\ N$.

Each force has this same y component and both are directed downward so the overall force in the y direction will be of magnitude $0.3456\ N$ and in the $-Y$ direction.

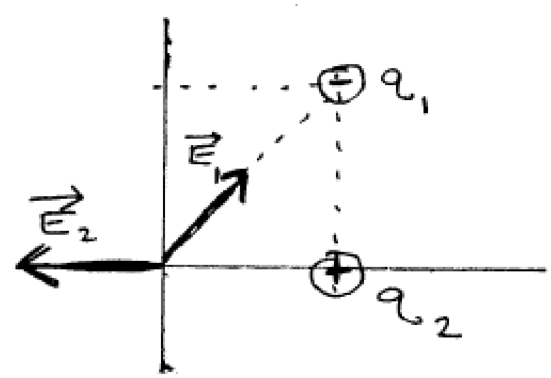
The total force in the X direction we already argued to be zero, so the overall magnitude will be $F = \sqrt{F_x^2 + F_y^2} = 0.3456\ N$, with a direction of 270^o CCW from the X axis (in the usual polar convention). Depending on how the question was worded, other answers might be -90^o (still in the usual polar convention), or perhaps 90^o **clockwise** from the $+X$ axis...

Electric Field (1) : Given a point charge $q_1 = -4.00nC$ at the point $x = 0.6\ m, y = 0.8\ m$ and a second point charge $q_2 = +6.00nC$ at the point $x = 0.6\ m, y = 0\ m$, calculate the magnitude and direction of the net electric field at the origin due to these two charges.

Here we need to compute the vector electric fields due to each charge and add them (as vectors) to produce the net electric field at the desired location (the origin). We don't have any nice symmetries we can take advantage of here, so we'll just have to brute-force it.

The electric field due to a point charge is equal to $\vec{E} = k\frac{q}{r^2}\hat{r}$. where q is the charge, r is the distance from our point of interest to the charge, and $\hat{r}$ is a unit vector pointing outwards **from that charge**.

- q_1 : $\hat{r}$ is pointing outward from that charge, but the charge itself is negative so the electric field is pointing inward towards the negative charge.
- For q_2, we have $\hat{r}$ pointing out away from the charge and the charge itself is positive, so overall the electric field due to q_2 is pointing away from that charge.

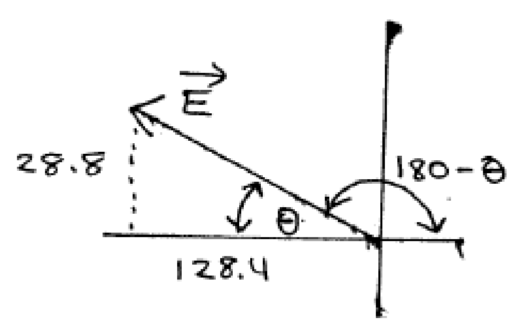


The two $\vec{E}$ involved are sketched in the figure.

Calculating $\vec{E}_1$: This field will have a magnitude of $|E| = |kq/r^2|$. Given the coordinates of the charge, we see that the distance from the desired location (the origin) is the hypotenuse of that triangle, so $r = \sqrt{(0.6)^2 + (0.8)^2} = 1.00\ m$. $|E_1| = (9 \times 10^9)(4.0 \times 10^{-9})/(1.0)^2 = 36.0\ N/C$. Let θ be the angle between the $+X$ axis and the vector $\vec{E}_1$. The $\hat{\imath}$ component of $\vec{E}_1$ will be $|E_1|\cos\theta = (36.0\ N/C)\frac{0.6}{1.0} = 21.6\ N/C$. The $\hat{\jmath}$ component of $\vec{E}_1$ will be $|E_1|\sin\theta = (36.0\ N/C)\frac{0.8}{1.0} = 28.8\ N/C$. Overall, then, $\boxed{\vec{E}_1 = (21.6\hat{\imath} + 28.8\hat{\jmath})\ N/C}$.

Calculating $\vec{E}_2$: This field will have a magnitude of $|E| = |kq/r^2|$. Given the coordinates of the charge, we see that the distance from the desired location (the origin) to the charge is just $0.6\ m$, so: $|E_2| = (9 \times 10^9)(6.0 \times 10^{-9})/(0.6)^2 = 150.0\ N/C$. From the figure, this field should be pointing entirely in the negative X direction, so in vector notation this would be: $\boxed{\vec{E}_2 = -150\hat{\imath}\ N/C}$.

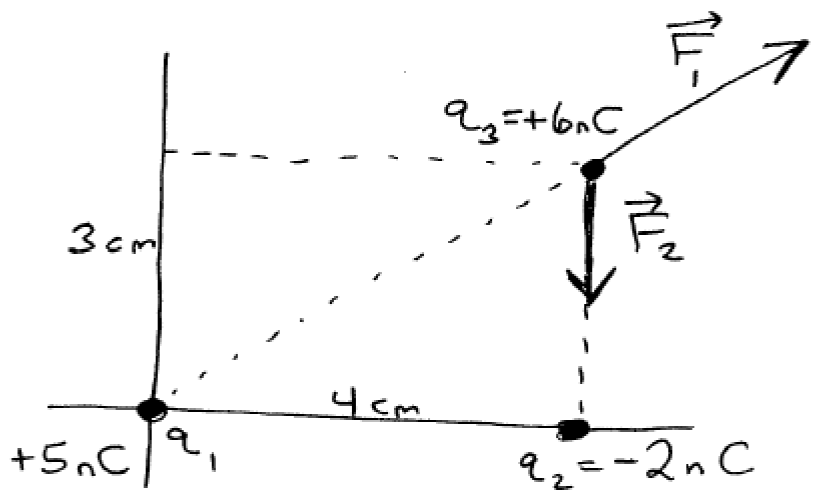
Total Electric Field : We can now add these vectors to find that $\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\hat{\imath} + 28.8\hat{\jmath})\ N/C$. This is roughly sketched in the figure. We can find the angle θ inside the triangle from: $\tan\theta = 28.8/128.4$ or $\theta = 12.6^o$. The usual polar convention is to measure angles CCW around from the $+X$ axis though, so the 'right' angle answer here would be $180 - 12.6 = 167.4^o$.



Electric Force (2) : A charge $q_1 = +5.00\ nC$ is placed at the origin of an xy-coordinate system, and a charge $q_2 = -2.00\ nC$ is placed on the positive x-axis at $x = 4.00\ cm$. A third particle, of charge $q_3 = +6.00\ nC$ is now placed at the point $x = 4.00\ cm$, $y = 3.00\ cm$.

Find the x and y components of the total force exerted on the third charge by the other two, then convert to magnitude and direction.

The geometry of the situation is shown in the figure, along with the directions of the forces that will be acting on the third charge. It will be repelled by q_1 since they are both positive charges, and will be attracted towards q_2 since they are of opposite sign. We can compute the magnitude of each force from $F = k\frac{q_a q_b}{r^2}$, resolve those into components, and then combine the two forces to get the total force acting (as a vector), which then can finally be converted into a magnitude and direction. Whew.

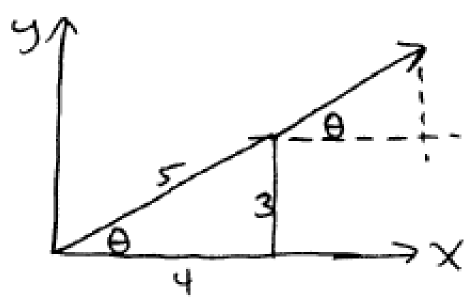


$\vec{F}_1$, the force between the two positive charges, will have a magnitude of $|F_1| = k\frac{q_1 q_3}{r^2}$ where r is the distance between those two. We know the coordinates of all the charges here, so we recognize that distance as the hypotenuse of a triangle with sides of 3 cm and 4 cm so here $r = \sqrt{3^2 + 4^2} = 5\ cm = 0.05\ m$. The magnitude of the force between charges 1 and 3 then is:

$$F_1 = (9 \times 10^9)\frac{(5\times10^{-9})(6\times10^{-9})}{0.05^2} = 1.08 \times 10^{-4}\ N.$$

We need to resolve this into components. $F_{1x} = F_1 \cos\theta$ and we see that the angle we need is the same as the marked interior angle of the triangle, so $F_{1x} = (1.08 \times 10^{-4}\ N)(\frac{4}{5}) = 8.64 \times 10^{-5}\ N$. Similarly, $F_{1y} = F_1 \sin\theta = (1.08 \times 10^{-4}\ N)(\frac{3}{5}) = 6.48 \times 10^{-5}\ N$. Overall then:

$$\vec{F}_1 = (8.64 \times 10^{-5}\hat{\imath} + 6.48 \times 10^{-5}\hat{\jmath})\ N.$$



$\vec{F}_2$ is easier to deal with, since it is directed exactly downward (the negative $\hat{\jmath}$ direction). The magnitude will be $|F_2| = k\frac{|q_2 q_3|}{r^2} = (9 \times 10^9)\frac{(2\times10^{-9})(6\times10^{-9})}{0.03^2} = 1.2 \times 10^{-4}\ N$, so $\vec{F}_2 = (-1.2 \times 10^{-4}\hat{\jmath})\ N$.

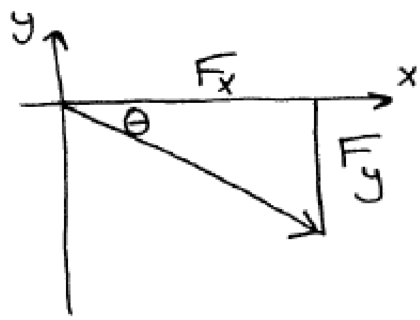
The problem asked us to compute the x and y components of the total force, so:

$$F_x = F_{1x} + F_{2x} = 8.64 \times 10^{-5} + 0 = 8.64 \times 10^{-5}\ N.$$

$$F_y = F_{1y} + F_{2y} = 6.48 \times 10^{-5} - 1.2 \times 10^{-4} = -5.52 \times 10^{-5}\ N.$$

continued...

We need to convert these components into an overall magnitude and direction now. The magnitude will just be $F = \sqrt{F_x^2 + F_y^2} = 1.02 \times 10^{-4}\ N$. To find the angle, we'll sketch out what this total force looks like. It has a positive X component and a negative Y component, so it appears as in the figure. Using just the magnitudes of these components and treating the triangle as just a plain old triangle: $\tan\theta = \frac{|F_y|}{|F_x|} = \frac{5.52\times10^{-5}}{8.64\times10^{-5}} = 5.52/8.64 = 0.6489$ from which $\theta = 32.6^o$, which we could describe as 32.6 degrees below the X axis. (In proper polar notation, the angle would be either -32.6^o or $360-32.6 = +327.4^o$.



Electric Force (3) : Replacing Gravity with Electric Force :

A silly example, but imagine a parallel universe where there is no gravity, but where the electric force has the same properties as in our universe. In this parallel universe, the sun carries charge Q, the earth carries charge $-Q$ and the electric attraction between them keeps the earth in orbit. The earth in the parallel universe has the same mass, the same orbital radius, and the same orbital period as in our universe. Calculate the value of Q. Consult appendix F as needed.

The earth is moving in a circular path, so there is a centripetal acceleration of $a_c = v^2/R$ directed towards the Sun (R being the distance from the Earth to the Sun). We can find the speed v from the fact that it takes the Earth 1 year to draw out a circle of radius R, so $v = \frac{2\pi R}{T}$. We'll leave everything symbolic for now until we get to the end. If there is an acceleration, there must be a force of magnitude ma producing it. For this problem, we're told to assume that this force is coming from the electrical attraction of the oppositely charged Earth and Sun, so from Coulomb's Law: $F = k\frac{q_1 q_2}{r^2}$ becomes $F = k\frac{Q^2}{R^2}$ directed inward, just as the acceleration is. In terms of magnitudes, then, $F = ma$ becomes:

$$k\frac{Q^2}{R^2} = M_e v^2/R = M_e \times (\frac{2\pi R}{T})^2)/R.$$

Shuffling terms around and cancelling where we can, we end up with:

$$Q = \sqrt{\frac{4\pi^2 M_e R^3}{kT^2}}.$$

From the tables at the end of the book the mass of the earth is $M_e = 5.97 \times 10^{24}\ kg$, the distance from the earth to the sun is $R = 1.5 \times 10^{11}\ m$, and the period is $T = 365.3\ days \times \frac{86400\ s}{1\ day} = 3.16 \times 10^7\ s$, leading to $Q = 2.98 \times 10^{17}\ C$.

Electric Force (4) : Unbalanced Charge

Suppose we have two aluminum spheres, each with a mass of 0.025 kg separated by 80 cm. The atomic mass of aluminum is 26.982 $gm/mole$ and the atomic number of **Al** is 13.

(a) First, how many electrons are there in each sphere?

The atomic number of aluminum is 13, which means that each atom has 13 protons (in the nucleus), 13 electrons, and various numbers of neutrons (in the nucleus), producing various different isotopes of **Al**).

So we know each atom has 13 electrons. How many atoms are there in each sphere? The atomic mass is given as 26.982 $gm/mole$ (that's one of the numbers given in a periodic table), which means that 1 mole of aluminum atoms has a mass of 26.982 $grams$ (or 0.026982 kg). One mole represents one Avogadro's number worth of something: i.e. $N_A = 6.022 \times 10^{23}$ things.

Now we can convert the mass of the sphere into how many atoms are present:

$(0.025\ kg) \times (\frac{1\ mole}{0.026982\ kg}) \times (6.022 \times 10^{23} \frac{atoms}{mole}) = 5.58 \times 10^{23}\ atoms$

Each of these atoms has 13 electrons, so multiplying by 13 we end up with $7.25 \times 10^{24}\ electrons$.

(b) How many electrons do we need to move from one sphere to the other to produce a (metric) ton of force between them?

Note: one metric ton is the weight of 1000 kg of material here on the surface of the earth, so $W = F_g = mg = (1000\ kg)(9.8\ m/s^2) = 9800\ N$. (For comparison, converting newtons to pounds, this represents a weight of 2204.6 *pounds*, about 10% more than an English-unit ton.)

We're going to move some electrons from one of the spheres to the other, which means that one of them will have a charge of $+Q$ and the other will have a charge of $-Q$. The force between them will be $F = kq_1q_2/r^2 = -kQ^2/r^2$. We desire the magnitude of this force to be 9800 N so $9800 = kQ^2/r^2$. The balls are 80 cm apart (center to center), so $r = 0.8\ m$, and $k = 9 \times 10^9\ N\ m^2/C^2$ so: $Q^2 = (9800)(r^2)/k = (9800)(0.8)^2/(9 \times 10^9)$ from which $Q = 8.35 \times 10^{-4}\ C$.

How many electrons does this represent? Each electron has a charge of $e = 1.602 \times 10^{-19}\ C$. The total charge Q will be the number of electron n times this, so:

$$n = Q/e = (8.35 \times 10^{-4} C)/(1.602 \times 10^{-19} C/electron) = 5.2 \times 10^{15}\ electrons.$$

That sounds like a huge number (and it certainly is) but in the previous part we found that each sphere held 7.25×10^{24} electrons, so actually this is a very tiny fraction of the number of electrons present: less than one part in a billion, in fact.

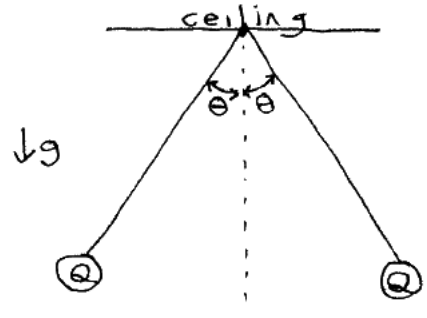
An electrical imbalance of just one part in a billion has resulted in a (metric) ton of force between the two small spheres, illustrating just how strong the electric force is compared to gravity.

Electric Force (5) : Static Electricity: Charged Party Balloons

Suppose we take two balloons and rub them to generate some excess charge, then hang them from the ceiling as shown in the figure. We see that they hang at a particular angle. Each balloon has a tiny mass, so if we look at all the forces acting on one of them, we have gravity pulling downward, the electric force repelling them, and some tension in the string.

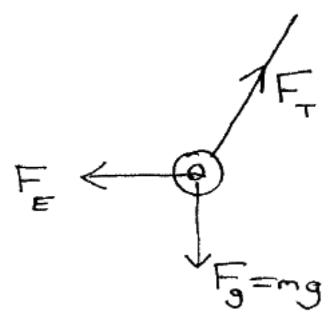
Suppose we charge up each balloon with the same charge Q, hang them on strings that are 1 meter long, each balloon has a mass of 1 gram, and we observe that the strings make an angle of 30 degrees with respect to the vertical.

How much charge Q is on each balloon?



Let's apply Newton's Laws to the balloon on the left. We draw the three forces acting on it and since the balloon is now just hanging there motionless, we know that $\Sigma \vec{F} = 0$ which means that separately the X and Y components of the forces must add to zero.

In the X direction, we have the electric force to the left, a component of the tension to the right, and nothing from gravity since it's force is vertically downward. So: $-F_e + T\sin\theta = 0$ or $\boxed{F_e = T\sin\theta}$.



In the Y direction, we have a component of tension upward and gravity downward, so $T\cos\theta - mg = 0$ or $\boxed{mg = T\cos\theta}$.

If we just divide the first boxed equation by the second, the tension variable will cancel out: $\frac{F_e}{mg} = \frac{T\sin\theta}{T\cos\theta}$ or $\frac{F_e}{mg} = \tan\theta$.
We can rewrite this as $F_e = mg\tan\theta$.

The electric force will be $F_e = kq_1q_2/r^2$ or here $F_e = kQ^2/r^2$ so substituting in that expression we have $kQ^2/r^2 = mg\tan\theta$ or solving for Q (what we're looking for) we have:

$\boxed{Q^2 = mgr^2\tan\theta/k}$ which we'll be able to use to determine the charge, since we know all the quantities on the right-hand side.

r is the distance between the balloons (well, their centers, since with spherically symmetric charge distributions, we can pretend that all the charge is located at their centers). How far apart are they? If the length of the string is L, we see that $r = 2L\sin\theta$ or we can observe that since each string has swung out by 30 degrees, the two strings are 60 degrees apart. That gives us a neat **short cut** here. From symmetry, we see that all three of the angles must be 60 degrees, meaning we have an **equilateral triangle** and thus the two balloons are L (here 1 meter) apart. So $r = 1$ *meter* here.

Substituting in all the variables now: $Q^2 = (0.001)(9.8)\tan(30^o)(1)^2/(9\times10^9) = 6.287\times10^{-13}$ or finally $Q = 7.93\times10^{-7}$ C or a bit under 1 microcoulomb.

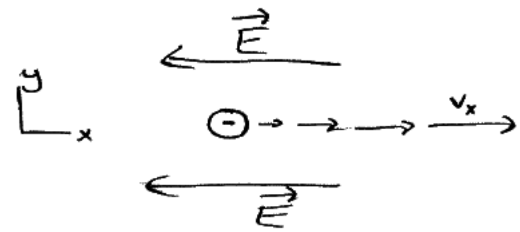
Motion of Charged Particles : Cathode ray tube

Back in the old days before flat-screen TV's, they were built using a device called a cathode ray tube, or CRT. (These are still used in some lab equipment such as oscilloscopes.)

Electric fields are used in two places: first, a material is heated up so that it releases electrons and an electric field is used to accelerate the electrons to a very rapid speed. These electrons then pass through two electric fields that are oriented perpendicular to one another that will deflect the electrons so that they strike a particular spot on the screen, causing it to light up. This process happened rapidly enough that the entire screen could be 'written' to 60 times each second, producing a moving picture.

Let's look at two parts of this device.

(a) Electron Gun : in this part, electrons essentially at rest are accelerated to some speed v over some distance Δx.
The electron is accelerating here, so there needs to be a force to cause that, and an electric field will create that force.



The electron is going from rest to a final speed v over a distance Δx. Assuming a constant acceleration, we have $v^2 = v_o^2 + 2a_x\Delta x$ or $a_x = v^2/(2\Delta x)$.

This implies a force of $F = ma = \frac{m_e v^2}{2\Delta x}$.

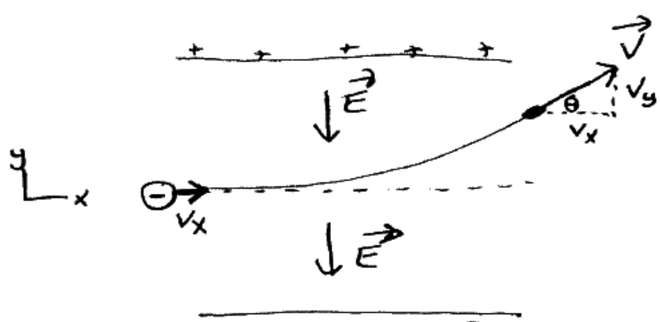
The electric field will create a force of $\vec{F} = q\vec{E}$ so here $\boxed{E_x = \frac{m_e v^2}{2q\Delta x}}$.

The charge on the electron is negative, so the electric field here in pointing to the left in the figure.

In a real CRT, the final speed is about $v = 0.1c = 3 \times 10^7\ m/s$ and this speed is reached in just a few centimeters, so let's say $\Delta x = 0.05\ m$. The mass of an electron is $m_e = 9.11 \times 10^{-31}\ kg$ and the charge on the electron is $q = -1.602 \times 10^{-19}\ C$.

Putting all that together, we end up with an electric field of $E_x = -51,200\ N/C$. (We'll see in a later section how this electric field is produced by applying a fairly high voltage across a couple of metal plates in the electron gun.)

(b) Deflecting the electrons
These fast electrons are now passed through a pair of metal plates that create an electric field between them (see example 21-13 and example 21-16). Suppose we want to bend the path of the electrons so that they fly off at a 30^o angle relative to their original direction. What electric field is needed to do this?



The electric field in the Y direction will cause an acceleration in the Y direction: $F_y = qE_y$ but $F_y = ma_y$ so $a_y = qE_y/m_e$.

As the electron passes through this field, there is no force in the X direction, so it's velocity in the X direction will remain unchanged. We do have a force in the Y direction though so the electron will start to build up a velocity in the Y direction: $v_y = v_{oy} + a_y t$. Since it's not accelerating in the X direction, we can determine the time t that the electron is in this field: $\Delta x = v_x \Delta t$ so the time will be $\Delta t = \Delta x / v_x$.

The initial y velocity was zero, so we can write the final y velocity as $\boxed{v_y = a_y \Delta t = a_y \Delta x / v_x}$.

When the electron leaves this apparatus, we need it to be moving at the 30^o angle shown in the figure, so $\tan\theta = v_y / v_x$ or $v_y = v_x \tan\theta$.

Look like we have all the pieces now: $v_y = a_y \Delta x / v_x$ but also $v_y = v_x \tan\theta$ so putting these together, we get: $a_y = \frac{v_x^2}{\Delta x}\tan\theta$.

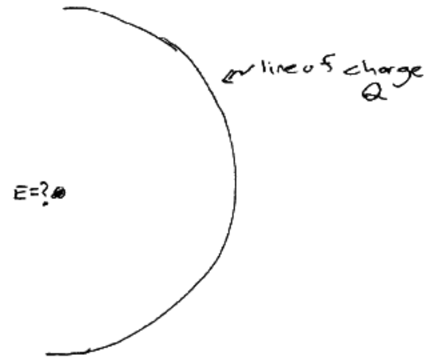
But $a_y = F_y / m_e = qE_y / m_e$ which gives us:

$E_y = \frac{m_e v_x^2}{q\Delta x}\tan\theta$

If we assume these plates are about 5 cm in size and we want a 30^o deflection, (and using $v_x = 3 \times 10^7\ m/s$ as in the first part) we find we need an electric field of about $E_y = 59,100\ N/C$.

(As noted in the first part, we'll see this field is created by putting a few thousand volts across the plates involved. These old school TV's could be dangerous to work on since such high voltages were involved...)

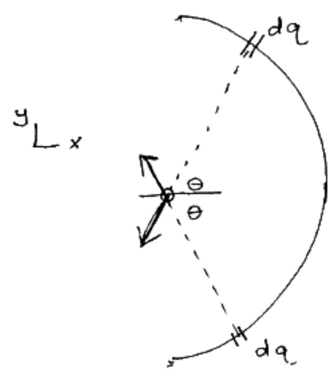
Electric Field Calculation : At the center of a semi-circle of charge



The book computes the electric field for various geometries to illustrate the use of **calculus**, so here's another example.

Suppose we take a thin plastic wire place some positive charge Q uniformly along the wire, then bend it into a semi-circular shape of radius R as shown in the first figure.

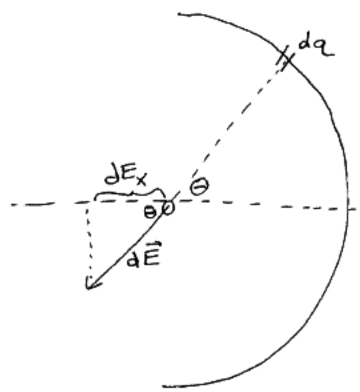
What will the electric field be right at the center of the circle?



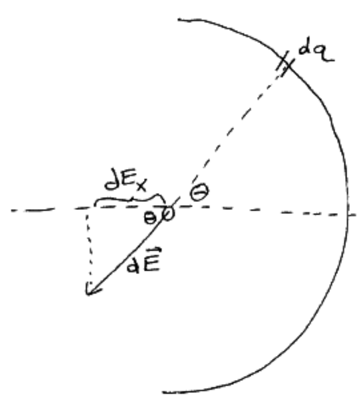
We're going to be breaking the wire into a bunch of little elements that we'll integrate over, so first let's pair them up, as shown in the second figure.

Let's use a coordinate system with the origin at the center of the circle, and with $+X$ pointing to the right and $+Y$ pointing up towards the top of the page.

For each little charge element dq that's some angle θ above the X axis, there's another identical element at the same angle below the X axis. The electric fields from these are shown, and we see that the Y components will cancel out, leaving just the X component. Ultimately then, we'll need to do an integral over just the dE_x components from each little dq element in the object.



So what is the X component of the electric field created by a given little element, located at some angle θ, for example? The electric field will have a magnitude of $dE = kdq/r^2$ and the X component of that vector will be $(dE)\cos\theta$, and given the direction of the $d\vec{E}$ vectors here, it'll be negative. The total electric field magnitude at that point then will be $E_x = -\int \cos\theta dE_x$ or $E_x = -\int \frac{k}{R^2}\cos\theta dq$.



We have a theta here already, so let's fiddle with things so we can integrate over theta. The charge dq in each little element will be the linear charge density λ times the physical length of that element: $dq = \lambda dl$, and the little arc-length dl we can relate to the angle element as $dl = Rd\theta$. Ultimately then: $dq = \lambda R d\theta$.

We can rewrite the integral now as: $E_x = -\int \frac{k}{R^2}\cos\theta\lambda R d\theta$

Pulling out all the constants: $E_x = -\frac{k\lambda}{R}\int\cos\theta d\theta$ and we've arrived at an integral we can do. The integral of cosine is just sine and what are the limits of integration? Looking at any of the figures, starting at the bottom of the arc, we're integrating from $\theta = -\pi/2$ to $\theta = +\pi/2$ so $\int\cos\theta d\theta = \sin\theta$ and evaluating this at the integration limits we have $\sin(\pi/2) - \sin(-\pi/2) = 1 - (-1) = 2$.

We're left now with $E_x = -\frac{k\lambda}{R}(2)$ or $E_x = -2\frac{k\lambda}{R}$.

The linear charge density λ is the charge per length, so if we want to put this in terms of the original charge Q, we can do so. The length of the wire was half the circumference of a circle, so $\lambda = Q/(\pi R)$.

Making that substitution, we arrive at the final result, that the electric field at the center of the circle will be $E_x = -\frac{2k}{\pi}\frac{Q}{R^2}$.

(Note when we did a full circle 'ring' of charge, we found $E = 0$ right at the center, but here we've only got half the circle so we do end up with an electric field at that point.)

Note that all these calculations gave us the electric field at **just that one single point**: the point right at the center of the semicircle. We'd have to go through another entire integral process if we wanted to find the field at any other point, and those integrals would be painful...