Chapter 23 Examples : Electric Potential

Key concepts:

- Electric potential is defined as the electric potential energy **per unit charge**. Since potential energy has a variable reference level, we define the electric potential **difference** between two points 'a' and 'b' as the difference in potential energy of a test charge placed at those points, divided by that test charge:

  $V_{ba} = V_b - V_a = \frac{1}{q}(U_b - U_a)$

- Potential differences are measured in joules per coulomb, also called **volts**.

- The change in potential energy of a charge q when it moves through a potential difference V_{ba} is $\Delta U = qV_{ba}$.

- $V_{ba} = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l}$

  This comes from the corresponding definition of potential energy change as a charge q moves from position a to position b:

  $U_{ba} = U_b - U_a = -\int_a^b \vec{F} \cdot d\vec{l} = -q\int_a^b \vec{E} \cdot d\vec{l}$

- Point charge: if we define the electric potential to be 0 when infinitely far away from the point, then $V = kQ/r$.

- Electric potentials from **distributions of charges** can be found by summing or integrating V over all the (point) charge elements. **NOTE** that unlike $\vec{E}$, the potential V is a **scalar** so we don't have to worry about components.

- Ring of charge (along axis perpendicular to the plane of the ring, passing through the center of the ring): $V = kQ/\sqrt{x^2 + R^2}$

- Charged circular disk (same axis as above): $V = \frac{Q}{2\pi\epsilon_o R^2}(\sqrt{x^2 + R^2} - x)$

- **Relating the electric field to the gradient of the potential**

  $\vec{E} = -\vec{\nabla}V = -(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k})$

- Note: the electrical potential energy U_E becomes just another term in our various work and conservation of energy equations: $(K + U_E + \cdots)_b = (K + U_E + \cdots)_a + W_{other}$

The electric potential (voltage) is the electrical potential energy per charge: $V = U/q$. Compared to gravity, this would be as if we defined something called 'gravitational potential' equal to the gravitational potential energy per mass: U_g/m. Near the earth, $U_g = mgh$ so this 'gravitational potential' near the earth would be $U_g/m = gh$. Recall from PH2213 that $g = GM_e/R_e^2$ so essentially this 'potential' construct condenses all the geometry and everything else we need to compute the actual potential energy, except for the mass of the object we're interested in. This doesn't have much utility in the case of gravity, but **does** in the case of electricity since if we connect a series of conductors, they'll all be at the same potential (voltage level).

Electric Potential and Surface Charge Density on a sphere

A conducting sphere of diameter 32 cm is charged to 680 V (relative to $V = 0$ at infinity, the usual reference point for V).

(a) What is the surface charge density σ on the sphere?

From example 23-4: $V = kQ/r$ so $Q = Vr/k = 4\pi\epsilon_o rV$, which we could use to find the **charge** on the sphere, then divide by the surface area of the sphere to determine the charge density.

Since we're ultimately looking for **charge density** σ, let's keep this symbolic for one more step: $\sigma = Q/A = Q/(4\pi r^2)$ but above we found that $Q = 4\pi\epsilon_o rV$ so substituting that expression in for Q, we get a lot of cancellation, leaving us with:

$\boxed{\sigma = V\epsilon_o/r}$

Note: this is just one example of why many electrical-related equations end up preferring to use the ϵ_o constant instead of k. If we had left this in terms of k, we'd have an extra 4π in the equation: $\sigma = V/(4\pi kr)$. In this case, the version of the equation that used ϵ_o was 'cleaner', and this happens often enough that many equations will use it instead of k (even though k is easier to remember!)

Applying this to the values we have here: $\sigma = V\epsilon_o/r = (680)(8.85\times10^{-12})/(0.16) = 3.761\times10^{-8}\ C/m^2$

(b) At what distance from the center of the sphere will the potential due to the sphere be only 25 V?

$V = kQ/r$ so we could just plug in the desired voltage and compute Q from the surface charge density and then find the distance r needed.

It's much simpler to just do this as a ratio problem: $V = kQ/r$ so the voltage is inversely proportional to the distance. Equivalently, that means that the product Vr is constant.

At the surface of the sphere, we have a known radius of 0.16 m and a known voltage of 680 V and that product is a constant, so let's use that:

$(680\ volts)(0.16\ m) = (25\ volts)(r)$ or $r = (0.16)(680)/(25) = 4.35\ m$

(c) How much charge is on the sphere?

$\sigma = Q/A$ so $Q = \sigma A = \sigma(4\pi r^2) = (3.761 \times 10^{-8}\ C/m^2)(4\pi(0.16\ m)^2) = 1.21 \times 10^{-8}\ C$ or just 12.1 nanocoulombs. A fairly small amount of charge on the sphere represented a 680 V voltage.

(d) How strong is the electric field just outside the sphere?

From Gauss's law, we have a nice spherical symmetry here, so the electric field will be radial and of magnitude $E = kQ/r^2 = (9 \times 10^9)(1.21 \times 10^{-8})/(0.16)^2 = 4254\ V/m$.

Electric Potential : Connecting two conductors

Suppose we have a spherical conductor of radius r_1 and charge Q. A second (initially uncharged) spherical conductor of radius r_2 is now connected to it by a long wire.

(a) After the connection, what can you say about the potential of each sphere?

They have to be the same; otherwise current would flow until they are the same. **This is an important general concept**: with any set of connected conductors, once the charge has been given time to redistribute (which happens quickly) **all** of them, and every part of them, will be at the **same voltage**.

(b) How much charge was transferred from the first sphere to the second? (Assume they're far enough apart they don't influence each other.)

Let's say ΔQ is the amount of charge that has moved from ball 1 to ball 2. After the connection is made and the charges redistribute, ball 1 will have charge of $Q - \Delta Q$ and ball 2 will have a charge of $0 + \Delta Q$.

Generically $V = kQ/r$ from a point/sphere geometry so let's look at the potential (i.e. voltage) of each sphere after the charge has redistributed:

$V_1 = k(Q - \Delta Q)/r_1$ and $V_2 = k(\Delta Q)/r_2$.

From the first part of the problem though, now that the two spheres are connected by the conducting wire, the two spheres have to be at the same voltage, so: $V_1 = V_2$ or $k(Q - \Delta Q)/r_1 = k\Delta Q/r_2$. We can cancel out the k right away, leaving: $(Q - \Delta Q)/r_1 = \Delta Q/r_2$.

Rearranging this to solve for ΔQ, we find:

$\Delta Q = Q\frac{r_2}{r_1+r_2}$

If r_1 (the radius of the original charged sphere) is very small compared to r_2 (i.e. a tiny charged ball is brought in contact with a much larger object), the fraction is approaching 1, so nearly all the charge would transfer (big spark). Even if the two objects are of comparable size, the ΔQ will be a significant fraction of the original charge. (The sort of thing that happens when you walk across the floor and touch a car or a metal doorknob...)

(c) How does the new (common) voltage compare to the original voltage?

Initially sphere 1 (of radius r_1 had some charge Q on it, and sphere 2 was uncharged.

The initial voltage on sphere 1 then was: $V_{initial} = kQ/r_1$.

After they are connected, the voltages on the two spheres is the same. Using the formula for sphere 2, we have $V_{new} = k\Delta Q/r_2$ but $\Delta Q = Q\frac{r_2}{r_1+r_2}$ so $V_{new} = k\frac{Q}{r_2}\frac{r_2}{r_1+r_2} = kQ\frac{1}{r_1+r_2}$

We can write this as: $V_{new} = \frac{kQ}{r_1}\frac{r_1}{r_1+r_2}$ or finally as $V_{new} = V_{initial}\frac{r_1}{r_1+r_2}$.

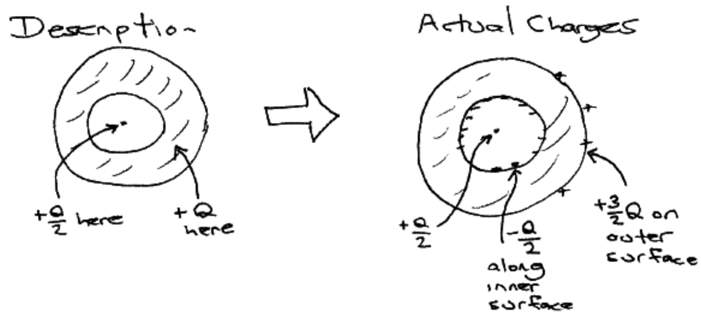
The denominator is always larger than the numerator, so the voltage will always be less after the two spheres are connected. If the initial charged sphere is much smaller than the other one, the new voltage will be much lower than the initial voltage (and we found in the previous part that this is the case where the most charge flows too). This means we have lots of electrons moving across a large potential difference (making a big spark).

Electric Potential : Partially hollowed-out sphere

A hollow spherical conductor carrying a net charge of +Q has an inner radius r_1 and an outer radius $r_2 = 2r_1$. At the center of the sphere is a point charge $+Q/2$. **Determine the electric field and the potential everywhere.**

Before we go any further, where is the charge actually located?

This partially hollowed-out sphere is a conductor. We have $+Q/2$ at the very center, which will induce a charge of $-Q/2$ on the inner surface of the sphere (i.e. spread around at $r = r_1$). We're told that the spherical conductor has a total charge of $+Q$ which means the **outer** surface of the conductor must have a charge of $+1.5Q$ so that, when combined with the $-0.5Q$ induced on it's inner surface, it will have an overall net charge of the $+1.0Q$ that they specified.

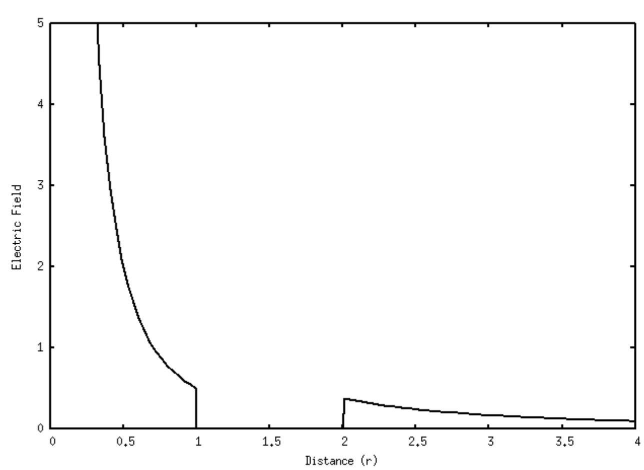


Electric Fields

We have spherical symmetry here, so per Gauss's Law, the field E is the same as that due to a point charge equal to the **net enclosed charge**: $E = kQ_{encl}/r^2$.

- For points completely **outside** the sphere (i.e. for $r > r_2$):
  the total enclosed charge will be: $+Q/2$ from the point charge **plus** the $-Q/2$ on the inner surface plus the $+1.5Q$ on the outer surface or overall: $1.5Q$. Here then: $E = kQ_{encl}/r^2 = 1.5kQ/r^2$.
- **Within the conducting part** of the material (i.e. between r_1 and r_2):
  the electric field will be zero, since we're inside the conductor itself.
- In the **hollow** part of the sphere (i.e. $r < r_1$) :
  the 'enclosed charge' is the $+Q/2$ located at the center, so the electric field in that hollow part will be $E = kQ_{encl}/r^2 = 0.5kQ/r^2$.

Here's a rough graph of what $E(r)$ looks like:



(continued...)

Potentials : what is the electric potential (voltage) as a function of r?

Again, spherical symmetry here so we can use $V = kQ_{encl}/r$.

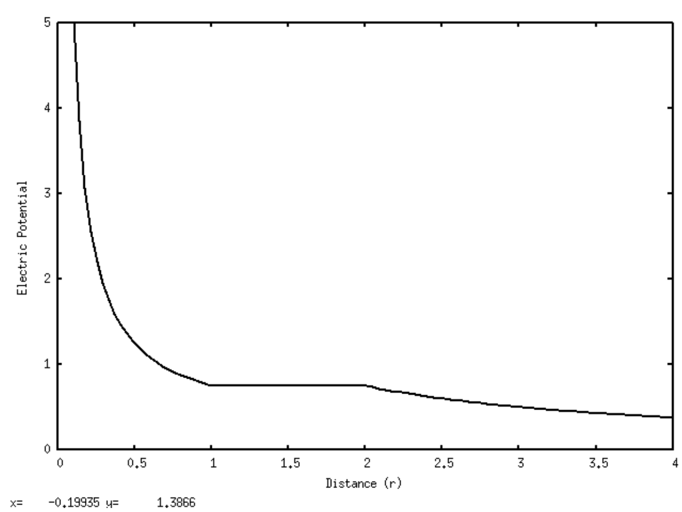
- Outside the sphere ($r > r_2$) :
  The total enclosed charge is $1.5Q$ so $V = 1.5kQ/r$ (or $\frac{3}{8\pi\epsilon_o}\frac{Q}{r}$)
- Within the conductor (i.e. r_1 and r_2):
  the electric field is zero so V will stay the same as it was on the outer surface: $V = 1.5kQ/r_2$. All the points in the thick conducting sphere itself will all be at the same voltage.
- In the hollow part (i.e. $r < r_1$):
  The potential will be $V = kQ_{encl}/r + C$ where C is some constant. Remember that potential always has a reference level. Starting from the outside of the whole thing, we assumed a reference level of $V = 0$ as r goes to infinity.

  Every part of the metal sphere is at the same voltage, and we found that value to be $V = 1.5kQ/r_2$, so we know that at $r = r_1$ the potential must still be $V = 1.5kQ/r_2$. That will let us determine the constant. $Q_{encl} = Q/2$ in this region, so $V(r) = 0.5kQ/r + C$ but $V(r_1) = 1.5kQ/r_2$ (the known voltage of every point of the conductor part of the sphere) so $0.5kQ/r_1 + C = 1.5kQ/r_2$. But in this problem, we have $r_1 = 0.5r_2$ so making that substitution, we have $kQ/r_2 + C = 1.5kQ/r_2$ or finally $C = 0.5kQ/r_2$.

  Inserting that constant: $V = 0.5kQ/r + 0.5kQ/r_2$ or combining terms:

  $V = 0.5kQ(\frac{1}{r} + \frac{1}{r_2})$.

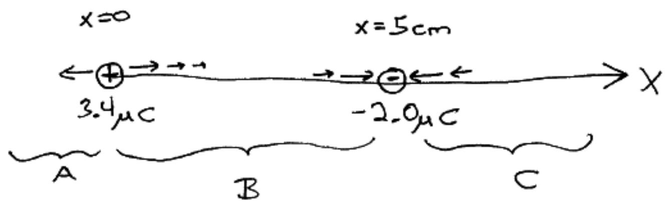
Here's a rough graph of what $V(r)$ looks like:



Electric Field and Potential due to two charges

Two point charges, $q_1 = +3.4\mu C$ and $q_2 = -2.0\mu C$ are placed 5 cm apart on the x axis.

(a) At what points along the x axis is the electric field zero?



Let's consider three regions: **A** which is to the left of the positive charge, **B** between the two charges, and **C** to the right of the negative charge.

In region A, the electric field from the positive charge is pointing to the left, and the electric field from the negative charge is pointing to the right, so there's the possibility of getting them to cancel there **except** in this region, the negative charge is farther away, and of smaller magnitude, so the magnitude of its electric field will be much smaller than the field from the positive charge. There's no way they can cancel each other out in that region.

In region B, the electric field from the positive charge is pointing to the right, and the electric field from the negative charge is also pointing to the right, so there's no way they can cancel each other in this region either.

In region C, the electric field from the positive charge is pointing to the right, and the electric field from the negative charge is pointing to the left, so we might get cancellation somewhere in this region. The negative charge is weaker, but we can be closer to it than to the positive charge, so there will definitely be some point where we can get them to cancel each other.

Let x be the coordinate of the point we're interested in. Then the distance from the positive charge to that point will be $r_1 = x$. The distance from the negative charge to that point will be $r_2 = x - 5$.

In general, the strength of the electric field will be $E = kq/r^2$ so the total electric field at point x will be:

$E = \frac{kq_1}{x^2} + \frac{kq_2}{(x-5)^2}$ and we want to find the x value which makes this zero, so: $\frac{kq_1}{x^2} + \frac{kq_2}{(x-5)^2} = 0$

We can divide out the common k, leaving us with: $\frac{q_1}{x^2} + \frac{q_2}{(x-5)^2} = 0$

We don't really need to convert the charges to coulombs or the distances to meters since all the units will end up cancelling, so we can write this as:

$\frac{3.4}{x^2} + \frac{-2.0}{(x-5)^2} = 0$ or $\frac{3.4}{x^2} = \frac{2.0}{(x-5)^2}$

Cross multiplying: $(3.4)(x-5)^2 = 2x^2$. Expanding everything out and rearranging terms, we arrive at the quadratic equation: $1.4x^2 - 34x + 85 = 0$ which has solutions at $x = 21.5$ cm or $x = 2.8$ cm. The second solution is bogus since we know x must be to the right of the negative charge, that is we know $x > 5$.

So apparently the electric field will be zero only at $x = 21.5$ cm. A charged particle (of any kind) located right there would feel no force.

(b) At what point along the x axis is the potential zero? (Let $V = 0$ at $r = \infty$.)

This is similar to what we did above, but simpler since we don't end up with a quadratic equation. We **do** end up having to be careful relating x to r though, as we'll see.

The potential from a point charge (assuming a reference point of $V = 0$ at $r = \infty$) is $V = kq/r$.

Let's look at the three regions again.

In region A, to the left of the positive charge, we have a positive voltage from the positive charge and a negative voltage from the negative charge, so they might cancel, but in this region the positive charge is closer and stronger, so the magnitude of it's voltage will always be more than that from the negative charge. They won't be able to cancel anywhere in this region.

In region B (between the two charges), we have a positive voltage from the positive charge, and a negative voltage from the negative charge, and we can be closer to either one of them, so we'll definitely find a point here where they cancel.

In region C (to the right of the negative charge) we have a positive voltage from the positive charge, a negative voltage from the negative charge, and although the negative charge is weaker, we could potentially be close to it so we should find a location there too.

Let's look at region C first

Here we're looking for some x that is larger than 5 cm where the voltages add to zero. $V = kq/r$ but we have to be careful. The r in that equation is the **distance** from the charge to the point of interest. The symbol r is always positive (or zero), never negative.

Over here in region C, $x > 5$ so we can write the distance from the first charge as $r_1 = x$ and we have to write the distance from the negative charge as $r_2 = x - 5$. That guarantees that in this region (where $x > 5$) we end up with an r value that is positive.

The sum of the voltages being zero means $kq_1/r_1 + kq_2/r_2 = 0$ or just $q_1/r_1 + q_2/r_2 = 0$ and here we have $\frac{3.4}{x} + \frac{-2}{x-5} = 0$ or $\frac{3.4}{x} = \frac{2}{x-5}$. Cross multiplying, we have $(3.4)(x - 5) = (2)(x)$ or $3.4x - 17 = 2x$ from which $x = 12.1$ cm.

Analysis for Region B

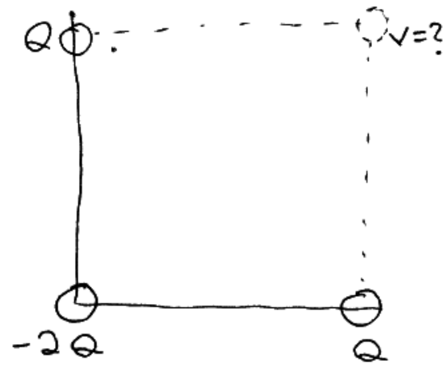
Note that we are now **between** the two charges. We can still write $r_1 = x$ but we have to change things a bit for r_2. Remember, we need r_2 to be a **positive** value representing how far the test point is from the negative charge. This time, we have to write $r_2 = 5 - x$ to achieve this. In this region then, our total voltage being zero becomes: $\frac{3.4}{x} + \frac{-2}{5-x} = 0$.

Cross multiplying again, we have $(3.4)(5 - x) = 2x$ or $17 - 3.4x = 2x$ which results in $x = 3.1$ cm.

Ultimately then, we end up with two locations where the potential (voltage) will be zero.

Note that the locations we found in part (b) have no correlation to those we found in part (a). Remember, the electric field is basically the **gradient** (derivative) of the potential. In part (b), we were looking at the where a function (V) is zero. In part (a), we were looking for where the derivative of that function is zero.

Electric Potential due to a collection of point charges



Three point charges are arranged at the corners of a square of side L as shown. What is the potential at the fourth corner point?

The potential (i.e. voltage) at the point of interest will be the sum of the potentials due to each of the three charges.

For a point charge, $V = kq/r$ and this is a **scalar** quantity, not a vector. Voltage doesn't have components.

The voltage due to the two positive charges Q is easy to compute, since each of those is located a distance of $r = L$ from the point of interest. Each of those will contribute a voltage of $V = kq/r = kQ/L$ to the sum.

The $-2Q$ charge at the origin is located a distance of $r = L\sqrt{2}$ from the point of interest, so it will contribute a voltage of $V = kq/r = k(-2Q)/(L\sqrt{2})$ to the sum.

The total voltage then at the fourth corner will be:

$V = \frac{kQ}{L} + \frac{kQ}{L} + \frac{-2kQ}{L\sqrt{2}}$.

Factoring out some common terms:

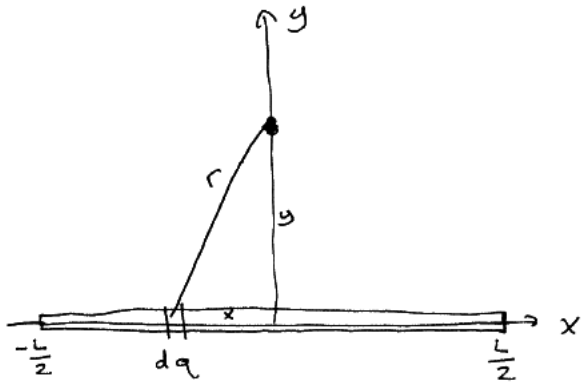
$V = \frac{2kQ}{L}(1 - \frac{1}{\sqrt{2}})$ or about $V = 0.586kQ/L$.

Electric Potential from a short line of charge

A thin rod of length L is centered on the x axis as shown in the figure. The rod carries a uniformly distributed charge Q. Determine the potential V as a function of y for points right along the y axis. Use the usual convention of letting $V = 0$ at infinity.

For a charge distribution, the voltage is the sum of the voltages due to each little charge element: $V = \int dV$.

Here, we'll split the line into little dx pieces, each having a charge of $dq = \lambda dx$ where $\lambda = Q/L$.



The voltage at the point of interest due to one of these little point charge elements will be $dV = kdq/r$ or $dV = k\lambda dx/r$. The distance r we can write in terms of x and y though: $r = \sqrt{x^2 + y^2}$ so the integral becomes $V = \int dV = \int k\lambda \frac{dx}{\sqrt{x^2+y^2}}$ where the limits of integration will be from $x = -L/2$ to $x = +L/2$.

Factoring out constants, we have $V = k\lambda \int \frac{1}{\sqrt{x^2+y^2}} dx$ but fortunately that's an integral that's been done and is in the table of integrals at the end of the book:

$\int \frac{1}{\sqrt{x^2+y^2}} dx = ln(x + \sqrt{x^2 + y^2})$

Let's define $a = L/2$ so the answer comes out a little cleaner. Evaluating this integrand at the limits from $x = -a$ to $x = +a$, gives us:

$ln(a + \sqrt{a^2 + y^2}) - ln(-a + \sqrt{a^2 + y^2})$ which we can combine into:

$ln(\frac{\sqrt{a^2+y^2}+a}{\sqrt{a^2+y^2}-a})$

That's just the integral part; we need to put back the constants we pulled out, namely $k\lambda$ but $\lambda = Q/L = Q/(2a)$ so we might write the final result as:

$V(y) = \frac{kQ}{2a} ln(\frac{\sqrt{a^2+y^2}+a}{\sqrt{a^2+y^2}-a})$

(NOTE: what if we have an infinitely long wire with some linear charge density λ on it? What happens to this expression as the parameter $a = L/2$ goes to infinity? This equation becomes undefined, so we'll need to take a different approach, which we'll see in Chapter 24.)

Electric Field as the Gradient of the Potential (A)

Suppose that we measure the electric potential in a region of space and find that it appears to fit the equation: $V = by/(a^2 + y^2)$, where a and b are some constants.

(a) Determine the electric field vector $\vec{E}$ that must exist.

The electric field is the (negative) gradient of the potential:

$\vec{E} = -\vec{\nabla}V = -(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k})$

In this particular potential, we have no dependence at all on x or z so the derivatives with respect to x and z are zero: this field thus has no $\hat{\imath}$ or $\hat{k}$ components: $E_x = 0$ and $E_z = 0$.

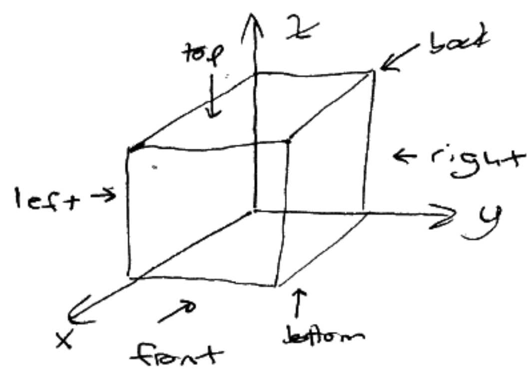
Taking the derivative in the y direction, after a bit of algebra we end up with:

$E_y = -\frac{\partial V}{\partial y} = b\frac{(y^2-a^2)}{(y^2+a^2)^2}$.

Putting these components into vector form: $\vec{E} = 0\hat{\imath} + b\frac{(y^2-a^2)}{(y^2+a^2)^2}\hat{\jmath} + 0\hat{k}$

(b) Let's compute the flux through a cube that is 1 meter along each side, with one corner at the origin, and the edges running along the x, y, and z axes. Suppose that in the equations above, $b = 10\ V/m$ and $a = 0.5\ m$.

This is actually easier than it might look, since we just found that the electric field is entirely in the Y direction, and only depends on Y. Looking at the figure, since $\vec{E}$ is entirely in the $\hat{\jmath}$ direction, the flux $\Phi = \vec{E} \cdot \vec{A}$ through the front, back, top, and bottom sides will all be zero since the field is perpendicular to the those area elements. We only need to calculate the flux for the left and right sides of the box.



On the **left side** of the box, $y = 0$ so the electric field has a constant strength of $E = -b/a^2$ or here $E = -(10)/(0.5)^2 = -40\ N/C$. The electric field is pointing to the left (and is constant along that surface), and the area on that side is also pointing to the left. $\Phi = \vec{E} \cdot \vec{A} = |E|\ |A|\ \cos\phi$ where remember the ϕ angle there is the angle between the directions of the two vectors. They're in the same direction (both pointing to the left) so $\phi = 0$ leaving us with $\Phi = (40\ N/C)(1\ m^2) = +40\ N\ m^2/C$.

Through the **right side**, $y = 1$ so $E = \frac{(10)(1^2-0.5^2)}{(1^2+0.5^2)^2} = 4.8\ N/C$, which is constant again. This field is pointing to the right, and so is the area element, so the flux through this side will be the field times the area (1 square meter) giving $\Phi = (4.8\ N/C)(1\ m^2)\cos 0 = +4.8\ N\ m^2/C$.

Summing these, the total flux through the cube then is $44.8\ N\ m^2/C$.

(c) Continuing the previous part, how much charge must there be inside this cube?

According to Gauss's Law, the total flux through a closed surface equals the charge enclosed divided by ϵ_o so $(44.8) = Q/(8.85 \times 10^{-11})$ or $Q = 3.96 \times 10^{-9}\ C$ or about 4 nanocoulombs.

Electric Field as the Gradient of the Potential (B)

A dust particle with mass 0.050 g and a charge of $+2.0 \times 10^{-6} C$ is in a region of space where the potential is given by $V(x) = 2x^2 - 3x^3$ (with V in volts, and x in meters).

(a) If the particle is released at rest at $x = 0.6$, what will it's initial acceleration be? (Magnitude and direction.)

$\vec{F} = m\vec{a}$ and the force on a charge is $\vec{F} = q\vec{E}$. We can determine the electric field from the gradient of the potential, which will let us determine the force and then finally the acceleration.

$\vec{E}$ is the negative gradient of the potential. Since V has no dependence on y or z, $\vec{E} = -\frac{\partial V}{\partial x}\hat{i}$.

The electric field vector will be pointing in the x direction. Differentiating V with respect to x:

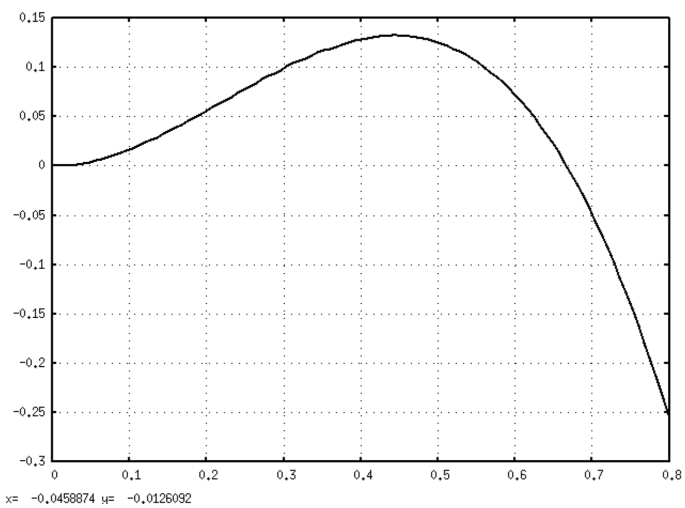
$E(x) = -dV/dx = -(4x - 9x^2) = 9x^2 - 4x$. The dust particle starts at $x = 0.6$ so the electric field at this point is $E = (9)(0.6)^2 - (4)(0.6) = 0.84\ N/C$. (Positive, so the electric field is pointing in the $+X$ direction at this point.)

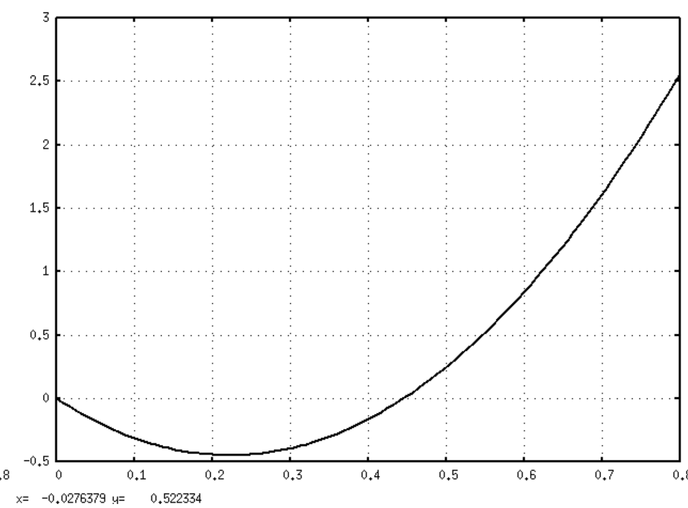
That makes the force: $F = qE = (2 \times 10^{-6})(0.84) = 1.68 \times 10^{-6}\ N$ (positive since the charge is positive and E is pointing in the positive x direction).

$F = ma$ so $(1.68 \times 10^{-6}) = (0.050 \times 10^{-3})(a)$ whence $a = 0.034\ m/s^2$. The positively charged dust particle at this location will feel an acceleration of that value, in the $+X$ direction.

Note: the particle will accelerate in the x direction, but as it moves, it's now located in a different x coordinate, which means the force is different (stronger in this case), so the acceleration will now be different. We don't have a constant acceleration here, so can't really 'finish' this problem, but all they asked for was the instantaneous acceleration when the dust particle was located at a particular spot.

The figures below show the potential V (left) and the electric field E (right) as a function of x:





Potential and Energy (A)

What potential difference is needed to give a helium nucleus ($Q = 2e = 3.2 \times 10^{-19}\ C$) that was initially at rest, a kinetic energy of 125 keV?

We can use conservation of energy here. We need to increase the K of the particle by the given amount, which means we will be decreasing it's potential energy by that amount: $\Delta K = -\Delta U$.

$\Delta U = q\Delta V$. Putting these together: $\Delta V = -\frac{\Delta K}{q}$.

Here we're given the change in kinetic energy: $125,000\ eV$. The helium nucleus has two protons so has a charge of exactly $2e$.

Finally: $\Delta V = -\frac{125{,}000\ eV}{2e} = -62,500\ volts$.

Note this example (and the next) illustrate why energies are often given in units of electron volts when we're dealing with individual elementary particles and atoms. Except for quarks, all matter has charges that are integer units of the basic charge $e = 1.602 \times 10^{-19}\ C$ so it's usually easier to just leave the charge directly in units of e and use energies that are in units of eV (electron volts).

Potential and Energy (B)

An electron starting from rest gains 1.33 keV of kinetic energy in moving from point A to point B.

(a) How much kinetic energy would a proton acquire starting at rest at B and moving to A? (Note it's moving in the opposite direction.)

$\Delta K = -\frac{\Delta V}{q}$ from the previous problem.

The kinetic energy gained only depends on the voltage change and the charge. For the proton, the charge is of the opposite sign but we're moving in the opposite direction, so the ΔK will be exactly the same.

(b) Determine the ratio of their speeds at the end of their respective trajectories.

$K = \frac{1}{2}mv^2$ and from (a) we know they will have the same kinetic energy. Thus: $\frac{1}{2}m_p v_p^2 = \frac{1}{2}m_e v_e^2$

Rearranging: $\frac{v_e}{v_p} = \sqrt{m_p/m_e} = 42.8$ (using the masses on the inside front cover of the book).

Note that a given voltage difference will accelerate electrons to a much higher speed (by a factor of 42.8) than it would accelerate protons (in the opposite direction). The earliest particle accelerators focused on experiments with electrons since they could be brought to extremely high speeds more easily.

Electric Potential and Energy (A)

Two identical ping-pong balls out in space are flying towards each other with identical initial speeds of 10 m/s. Each has a charge of $1\ \mu\ C$, uniformly distributed on their surfaces.

How close will the balls get to each other before coming (momentarily) to a stop? (Or will they collide first?)

Assume each ball has a mass of 2.7 $gram$ and a radius of 2 cm, and when their speeds were measured, they were 3 m apart.

Here we'll use conservation of energy. At the initial position, each of the balls has a kinetic energy of $K = \frac{1}{2}mv^2 = (0.5)(0.0027)(10)^2 = 0.135\ J$, giving a total kinetic energy of twice that, or $0.27\ J$.

They also have some initial electrical potential energy of $U_E = kq_1q_2/r$ so here we have $U_E = (9 \times 10^9)(1 \times 10^{-6})(1 \times 10^{-6})/(3) = 0.003\ J$.

The total mechanical energy then in the original configuration is $0.273\ J$.

The balls both have a positive charge, so there is a repulsive electrical force between them, gradually slowing them down. In energy terms, at some point they'll lose all their kinetic energy, coming to a stop, at which point all the energy in the system will be in the form of just the electric potential energy.

At that point, $K = 0$ and we need $U_E = 0.273\ J$ so at what distance does that occur?

$U_E = kq_1q_2/r$ so $0.273 = (9 \times 10^9)(1 \times 10^{-6})(1 \times 10^{-6})/r$.

Solving for r, we find that the ping pong balls will come to a stop when their centers at located a distance $r = 0.033\ m$ or 3.3 cm apart.

(Now that's the distance between the centers of the two ping pong balls, and unfortunately a regulation ping pong ball has a diameter a bit larger than that, so it looks like they'll end up slightly crashing into each other here...)

This was a somewhat contrived example, of course, but it relates to atomic and nuclear collisions where, for example, we're trying to inject an extra proton (positive charge) into a nucleus (lots of positive charge) in order to create a new element.

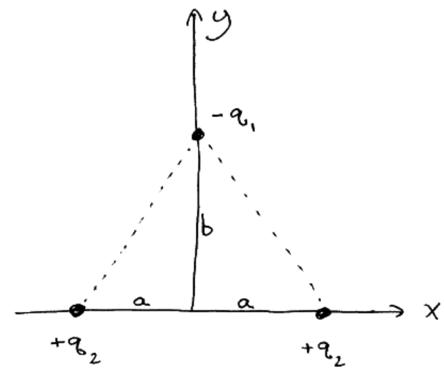
Why did we have to do this experiment out in space?

Let's look at the electric field right at the surface of one of the balls. For this sort of spherically symmetric object, $E = kq/r^2$ and with a radius of $r = 2\ cm = 0.02\ m$ we have: $E_{surface} = (9 \times 10^9)(1 \times 10^{-3})/(0.02)^2 = 2.25 \times 10^{10}\ V/m$ which is *far* higher than the $3 \times 10^6\ V/m$ that will cause air to break down (ionize).

Electric Potential and Energy (B)

A charge $-q_1$ of mass m rests on the y axis at a distance b above the x axis. Two positive charges of magnitude $+q_2$ are fixed on the x axis at $x = +a$ and $x = -a$.

If the $-q_1$ charge is given an initial velocity v_o in the positive y direction, what is the minimum value of v_o such that the charge escapes to a point infinitely far away from the two positive charges?



This is not that bad actually. We're giving the particle some initial velocity, so it has some initial kinetic energy. It also has some electrical potential energy which will be negative. We have to give the particle at least enough kinetic energy to overcome that initial U_E in order for the particle to escape. We could give it more, in which case it will still be moving when it gets infinitely far away from the other two charges, but let's look at the limiting case: what's the smallest amount of kinetic energy we need to give the particle so that it just barely escapes? I.e. it arrives at infinity with no kinetic energy, and no potential energy. I.e. the total mechanical energy at the final position should be zero.

Well, electricity is a conservative force, so that means we need the total mechanical energy at the initial position to be zero as well. We can compute how much (negative) electrical potential energy exists at the initial position; then we just need to add exactly that amount of kinetic energy (i.e. give the particle some velocity we can determine) to make the initial mechanical energy zero.

What is the potential energy U at its current (original) location? Generically $U = kq_1q_2/r$ so apply this with the particular charges and distances we have.

The charge on the left side of the X axis is located a distance of $r = \sqrt{a^2+b^2}$ away from the charge on the Y axis, so there is a potential energy of $U = k(-q_1)(q_2)/\sqrt{a^2+b^2}$.

The charge on the right side of the X axis is located exactly the same distance away, and has the same charge, so it is contributing exactly the same amount of potential energy.

Overall, in the initial configuration, there is present a total electrical potential energy of:
$U = -2kq_1q_2/\sqrt{a^2+b^2}$.

That's negative, so we need to add exactly that same amount of energy to the particle (in the form of it's kinetic energy) in order for it to 'escape' from the two charges glued to the X axis.

It needs to be given an initial kinetic energy of $K = +2kq_1q_2/\sqrt{a^2+b^2}$.

Well, they asked for the initial **velocity**, but $K = \frac{1}{2}mv^2$ so setting this equal to the expression we just found, we can rearrange to find that:

$v_o = \sqrt{\left(\frac{4kq_1q_2}{m\sqrt{a^2+b^2}}\right)}$