Chapter 24 Examples : Capacitance, Dielectrics, Electrical Energy Storage

Key concepts:

- For a given geometry, if we compute (via the methods of the previous chapter) the potential (voltage) V between two points, we find that the voltage and charge are proportional. The proportionality constant essentially only depends on the **geometry** and **physical properties** of the objects involved, so can be done once up front.

  This proportionality relationship is traditionally written in the form: $\boxed{Q = CV}$ where C is called the **capacitance** of the object.

- Capacitance for simple geometries:

  (a) Two parallel plates of area A, separated by a distance d where $d << A$: $C = \epsilon_o \frac{A}{d}$

  (b) Two (long) cylinders (one inside the other) of length L and with outer radius r_o and inner radius r_i:
  $C = \frac{2\pi\epsilon_o L}{ln(r_o/r_i)}$

  (c) Two concentric spherical shells, with outer radius r_o and inner radius r_i: $C = 4\pi\epsilon_o \frac{r_o r_i}{r_o - r_i}$

- Units: note that the standard metric units of ϵ_o, which appears in all these equations, can be shown to be $(farad)/(meter)$, so in C computations, it's convenient to use the form: $\epsilon_o = 8.854 \times 10^{-12} \ F/m$

- Capacitors connected in parallel: $C_{eq} = \Sigma C_i$

- Capacitors connected in series: $\frac{1}{C_{eq}} = \Sigma \frac{1}{C_i}$

- Energy stored in a capacitor: $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$

- Dielectric: real capacitors almost always have some material separating the conductors, which alters the electric field between them, in turn altering the capacitance of the geometry. If C_o is the 'pure' calculation of the capacitance (the equations above), then the actual capacitance is $C = KC_o$ where K is called the **dielectric constant** of the material being used between the conductor elements of the capacitor.

  The presence of this K alters some of the equations above:

  $C = KC_o$
  $E = E_o/K$ (electric field within the dielectric)

  See table 24-1 for typical values, which run from 1 (vacuum) up to a few hundred.

Cylindrical Capacitor (A)

A cylindrical capacitor consists of a solid inner conducting core with radius 0.250 cm, surrounded by an outer hollow conducting tube. The two conductors are separated by air, and the length of the cylinder is 12.0 cm. The capacitance is 36.7 pF.

(a) Calculate the outer radius of the hollow tube.

The capacitance only depends on the geometry of the device. For a cylindrical capacitor, $C = (2\pi\epsilon_o L)/ln(r_o/r_i)$. Here we know everything in this expression except for r_o, the radius of the outer cylinder, so rearranging the equation: $Cln(r_o/r_i) = 2\pi\epsilon_o L$ or $ln(r_o/r_i) = 2\pi\epsilon_o L/C$ from which: $r_o/r_i = e^{2\pi\epsilon_o L/C}$ and finally: $\boxed{r_o = r_i e^{2\pi\epsilon_o L/C}}$.

Let's compute the exponent first (converting all the quantities to standard metric units of meters, Farads, etc):

$2\pi\epsilon_o L/C = (2)(\pi)(8.854 \times 10^{-12})(0.12)/(3.67 \times 10^{-11}) = 0.182$

So: $r_o = (0.250 \ cm)e^{0.182} = 0.300 \ cm$. (This is only slightly larger than the radius of the inner conductor, so the gap between them is small: only $0.300 - 0.250 = 0.050 \ cm$.)

(b) If the capacitor is charged to 125 V, what will be the charge per unit length λ on the capacitor?

λ is defined to be the charge per length, so $\lambda = Q/L$. For any capacitor, $C = Q/V$ so $Q = CV$ and we can substitute that into our equation for λ to get: $\lambda = (CV)/L$ for a cylindrical capacitor.

Using the numbers we have here: $\lambda = (3.67 \times 10^{-11})(125)/(0.120) = 3.82 \times 10^{-8} \ C/m$ or 38.2 nC/m.

(NOTE: a **coaxial cable** is basically a long cylindrical capacitor. We derived that for such a capacitor, $\lambda = CV/L$ which we can write as $\lambda = (\frac{C}{L})V$. This is sometimes useful since the capacitance for coax cables is often given in terms of the capacitance per unit length (C/L) instead of C directly.)

Cylindrical Capacitor (B)

Suppose we have 10 meters of coaxial cable. The diameter of the inner wire is 1 mm and the diameter of the outer cylinder is 6 mm.

(a) Determine the capacitance of this cable.

A coax cable is basically a wire in the middle, with a woven mesh of wire as the outer cylinder.

For this geometry, $C = 2\pi\epsilon_o \frac{L}{ln(r_o/r_i)}$.

Note that the capacitance depends on the radii of the inner and outer cylinders as the log of their ratio, so we don't need to bother converting from diameter to radius, or from millimeters to meters.

$C = 2\pi(8.854 \times 10^{-12} \; F/m)(10 \; m)/ln(6) = 3.10 \times 10^{-10} \; F$ or $310 \; pF$.

(b) Determine the capacitance per unit length.

This would be C/L so rearranging the expression above for C we have: $C/L = 2\pi\epsilon_o \frac{1}{ln(r_o/r_i)}$ and we could do that from scratch or just use the value of C we just found to determine C/L for this coax cable: $C/L = 31 \; pF/m$.

Real world: Coax cable is usually rated in terms of various parameters per foot or per meter since the parameters depend on the length and only you know what actual length of cable you're going to use. If you search for coaxial cable capacitance, common values range from 10 to 30 pF/m which is in the ballpark of what we found here. In real coax cable there will also be some dieletric material filling the gap between the inner and outer conductors which will produce a higher capacitance than what we found here.

(c) If we put a 3000 volt potential across the coax conductors, how strong would the electric field be near the inner conductor (where it will be the highest)?

When we attach a voltage across the two conductors, that will cause a charge to form (opposite charges on each conductor) of $Q = CV$. The charge per length λ will be Q/L and recall that the electric field from a line of charge is $E = \frac{\lambda}{2\pi\epsilon_o r}$.

Let's do this symbolically since a lot of constants will end up cancelling out.

$Q = CV$ so $\lambda = \frac{Q}{L} = \frac{CV}{L}$ or $\lambda = 2\pi\epsilon_o \frac{V}{ln(r_o/r_i)}$

The electric field at some r from an infinite line of charge is $E = \frac{\lambda}{2\pi\epsilon_o r}$ and substituting in the expression we just found for λ yields $E = 2\pi\epsilon_o \frac{V}{ln(r_o/r_i)} \times \frac{1}{2\pi\epsilon_o r}$ or finally $\boxed{E = \frac{V}{r \; ln(r_o/r_i)}}$.

For our cable, the outer to inner radius ratio was 6 and the **diameter** of the inner wire was 1 millimeter. We're looking for the electric field right near this inner wire, making $r = 0.5 \; mm = 5 \times 10^{-4} \; m$.

In our case then, $E = \frac{3000 \; volts}{(0.0005 \; m)ln(6)} = 3,350,000 \; volts/m$. Unfortunately, an electric field that strong will ionize air and cause the charge to arc off the inner wire.

Adding a dielectric material allows the electric field to be this large (and larger) without causing arc-ing. It's also the reason that coax cables are also rated with the highest voltage they should be subjected to (usually well below the point where arcing will occur...). Note that the voltages involved in typical home usage (cable TV, etc) are only a few tens of volts.

Spherical Capacitor

A spherical capacitor contains a charge of 3.30 nC when connected to a potential difference of 220 V. Its plates are separated by vacuum and the inner radius of the outer shell is $4.00cm$.

(a) What is the capacitance?

From the definition of capacitance: $C = Q/V$ so here, $C = (3.30 \times 10^{-9})/(220) = 1.50 \times 10^{-11} \ C$ or $15.0 \times 10^{-12} \ C$ which is 15.0 pF.

(b) What is the radius of the inner sphere?

As in the previous problem, the capacitance C depends only on the geometry. For a spherical capacitor, $C = 4\pi\epsilon_o \frac{r_o r_i}{r_o - r_i}$. Here, we know the capacitance from part (a), and we know the inner radius of the outer sphere: $r_i = 0.04 \ m$.

Rearranging the equation for the capacitance, $C/(4\pi\epsilon_o) = \frac{r_o r_i}{r_o - r_i}$ but we can write this as $kC = \frac{r_o r_i}{r_o - r_i}$ or $kC(r_o - r_i) = r_o r_i$. Expanding: $kCr_o - kCr_i = r_o r_i$. Collecting the terms involving r_i together: $kCr_o = kCr_i + r_i r_o = r_i(kC + r_o)$ which gives us finally: $r_i = kCr_o/(kC + r_o)$. (Note we can write this as: $r_i = r_o \times \frac{kC}{kC + r_o}$, and whatever the numbers the fraction on the right will be less than 1, so r_i will be less than r_o, as needed since r_i is the radius of the inner sphere, and r_o is the outer sphere.)

Here, $kC = (8.988 \times 10^9)(1.5 \times 10^{-11}) = 0.1348$ so $r_i = (0.1348)(0.04)/(0.1348 + 0.04) = 0.005392/0.1748 = 0.0308$. Everything we did was in standard metric units, so this will have units of meters, so $r_i = 0.0308 \ m$ or 3.08 cm. (And this did come out smaller than r_o so is at least possible.)

(c) What is the electric field just outside the surface of the inner sphere?

To find the electric field: we have a spherical conductor, with Q spread out evenly across its surface. These spherically symmetric charge distributions act as if all the charge were located at the center of the sphere, so the electric field can be written as $E = kq/r^2$ where r is measured from the center of the sphere and r is greater than or equal to the radius of the sphere. We determined in part (b) that the radius of this inner sphere is 0.0308 m and the total charge on this inner sphere is $q = 3.30 \times 10^{-9} \ C$ so the electric field just outside this inner sphere is: $E = kq/r^2 = (8.988 \times 10^9)(3.30 \times 10^{-9})/(0.0308)^2 = 31,300 \ V/m$.

(NOTE: if we move out a bit farther, so that we are just outside the **outer sphere** of this capacitor, the electric field drops to zero since the charge on the inner and outer spheres are the same, just of opposite sign, so from Gauss's Law, the total 'enclosed charge' will be zero.)

Capacitor Network

In the figure, $C_1 = 6.00\mu F$, $C_2 = 3.00\mu F$, and $C_3 = 5.00\mu F$. The capacitor network is connected to an applied potential V_{ab}. After the charges on the capacitors have reached their final values, the charge on C_2 is found to be $40.0\mu C$.
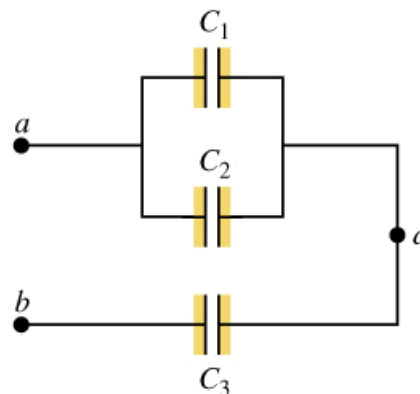
(a) What are the charges on capacitors C_1 and C_3?

(b) What is the applied voltage V_{ab}?

First, so we know what's going on here, if we 'unwrap' this figure, we see that C_1 and C_2 are connected in parallel, and then that combined entity is connected in series with C_3.

(a) C_1 and C_2 are connected in parallel, so the voltage across either one has to be the same. $V_1 = V_2 = Q_1/C_1 = Q_2/C_2$ but we apparently know how much charge was on C_2 so $V_1 = V_2 = (40.0 \times 10^{-6}C)/(3.00 \times 10^{-6}F) = 13.33\ V$.

Looking at C_1 now: $C_1 = Q_1/V_1$ so $Q_1 = C_1V_1 = (6.00 \times 10^{-6}C)(13.33\ V) = 80.0 \times 10^{-6}\ C$ or $80.0\mu C$.

Looking at C_3: this capacitor is connected IN SERIES with the combined equivalent capacitor that C_1 and C_2 make, so the charge on C_3 must be the same as the charge on the combination of C_1 and C_2. (See section 24.2, figure 24.8.) Thus $Q_3 = Q_1 + Q_2 = (80.0\mu C) + (40.0\mu C) = 120.0\ \mu C$.

(b) Determining the applied voltage (one version):

These three capacitors make a single equivalent capacitor that has the same charge as was on the last plate of C_3 (see examples 24.5 and 24.6 on capacitor networks). C_1 and C_2 are connected in parallel, which we can replace with an equivalent capacitor of $C = C_1 + C_2 = 9.00\mu F$. This equivalent capacitor is then in series with C_3, which results in an overall capacitance of this network of $\frac{1}{C} = \frac{1}{9.00\ \mu F} + \frac{1}{5.00\ \mu F}$ from which $C = 3.214\mu F$. So now we have the overall capacitance of this network ($3.214\mu F$), and we know the charge on each plate of this equivalent capacitor ($120.0\mu C$) so we can find the voltage that must have been applied to this network: $C = Q/V$ so $V = Q/C = (120.0\mu C)/(3.214\mu F) = 37.33\ V$. (Note I didn't bother converting the two 'micro' symbols since they will end up canceling each other out anyway.)

(b) Determining the applied voltage (another version):

Before the voltage is connected, everything in the circuit is neutral: there isn't any charge anywhere. When connected, the loose electrons move around ultimately leaving a charge of 40 μC on C_1, 80 μC on C_2 and as argued in part (a), 120 μC on C_3.

The voltage across points a and d in the circuit is the voltage across either C_1 or C_2 which we found to be $13.33\ V$.

We can find the voltage across C_3: $Q = CV$ so $V = Q/C = (120\ \mu C)/(5\ \mu F) = 24\ volts$.

The voltage across both segments of the circuit then will be $13.33\ V$ plus $24.0\ V$ or $37.33\ V$.

A parallel-plate vacuum capacitor has 8.38 J of energy stored in it. The separation between the plates is 2.30 mm. If the separation is decreased to 1.15 mm, what is the energy stored:

(a) if the capacitor was **disconnected** from the potential source before the separation of the plates was changed?

(b) if the capacitor **remained connected** to the potential source while the separation of the plates was changed?

This is an interesting problem since at first glance it doesn't appear that we have enough information to solve it. The capacitance of a parallel-plate capacitor is $C = \epsilon_o A/d$ where A is the (unknown) area of each plate and d the separation distance. The potential energy stored in a capacitor can be written in various forms: $U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$, but we don't directly have C, V, or Q: all the things that show up in various pairs for the equations involving U.

Here we are varying the separation distance, so $C = \epsilon_o A/d$ is certainly changing, and the value of C does affect the amount of energy stored, so clearly varying d will ultimately affect the amount of energy stored U, but how? Note that as we change the value of d, the capacitance will vary according to $1/d$. Cutting d in half would cause C to double, for example. Any decrease in d causes C to increase.

(a) Here, we attached a voltage source, which caused some charge Q to build up on the plates of the capacitor. Now we disconnect the voltage source, leaving Q still there. We now vary the separation distance d, but that doesn't affect the charge: Q has no place to go since the plates aren't connected in any way. So in this situation, we have some fixed value of Q, but d is varying. Since Q is fixed, let's use the first form for U: $U = \frac{Q^2}{2C}$. Since Q is constant for this case, U is proportional to $1/C$. We just argued above that C is proportional to $1/d$ so that means that U must be proportional to $1/(1/d)$ or U is **directly proportional** to d. Keeping the charge constant, if we cut d in half, we also cut U in half. Since U is directly proportional to d, then U/d is constant and we can use this ratio to solve the problem.
$U_{new}/d_{new} = constant = U_{old}/d_{old}$ or $U_{new} = U_{old} \times (d_{new}/d_{old})$. In this problem, $d_{old} = 2.30$ mm and $d_{new} = 1.15$ mm so $U_{new} = (8.38\ J)\frac{1.15}{2.30} = 4.19\ J$.

(b) Here, we are doing something different: we are keeping the capacitor connected to the voltage source, so it's V that remains constant as we vary the distance between the plates. C depends only on the geometry of the situation, so we still have C varying as $1/d$. Since V is constant, let's use the form: $U = \frac{1}{2}CV^2$. Here we see that U is directly proportional to C, which is inversely proportional to d, thus overall U is proportional to $1/d$ instead of d as we had in part (a). Since U is proportional to $1/d$, then Ud is constant, so $U_{new}d_{new} = U_{old}d_{old}$ or $U_{new} = U_{old} \times (d_{old}/d_{new})$. Using the numbers we have in the problem, $U_{new} = (8.38\ J)\frac{2.30}{1.15} = 16.76\ J$.

For the capacitor network shown in the figure , the potential difference across ab is 36 V. Determine:

(a) the total charge stored in this network,

(b) the charge on each capacitor,

(c) the total energy stored in the network,

(d) the energy stored in each capacitor,

(e) the potential difference across each capacitor.
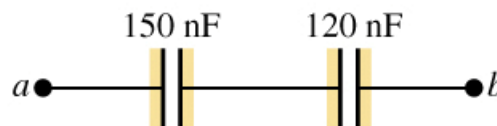
We have two capacitors in series here. Looking from left to right in the figure, we'll have charges of $+Q$ and $-Q$ on the first capacitor's plates, and charges of $+Q$ and $-Q$ on the second capacitor's plates. These two in series make some single equivalent capacitor with the same Q and $-Q$ across its plates. Since they are in series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$. A little algebra lets us write this as $C = \frac{C_1 C_2}{C_1 + C_2}$ so here $C = (150 \ nF)(120 \ nF)/(150nF + 120nF) = 66.67 \ nF$.

(a) Now that we know the complete C for this network, $C = Q/V$ so $Q = CV = (66.67 \times 10^{-9})(36.0) = 2.40 \times 10^{-6} \ Coul$ or $2.40 \mu C$.

(b) From the first paragraph, for these capacitors in series, we have the same Q as we just calculated on each capacitor.

(c) The total energy stored is $U = \frac{1}{2}CV^2 = (0.5)(66.67 \times 10^{-9})(36.0)^2 = 43.2 \times 10^{-6} \ J$ or $43.2 \ \mu J$.

(d) To find the energy stored in each capacitor individually, we have various forms for U. At this moment, we don't (yet) know the voltage drop across each capacitor separately, so one option is to use $U = Q^2/(2C)$ since we DO know the charge and capacitance of each capacitor.

For the first capacitor: $U = (2.40 \times 10^{-6})^2/(2 \times 150 \times 10^{-9}) = 1.92 \times 10^{-5} \ J$ or $19.2 \ \mu J$.

For the second capacitor: $U = (2.40 \times 10^{-6})^2/(2 \times 120 \times 10^{-9}) = 2.40 \times 10^{-5} \ J$ or $24.0 \ \mu J$.

(Adding these together, we get $43.2 \ \mu J$, which is the same energy we calculated was stored in the entire network, in part (c).)

(e) To find the potential drop (i.e. the voltage) across each capacitor individually, $C = Q/V$ so $V = Q/C$ and we do know the charge and capacitance of each capacitor. The voltage drop across the first capacitor will be: $V = (2.40 \times 10^{-6}C)/(150.0 \times 10^{-9}F) = 16.0 \ V$. The voltage drop across the second capacitor will be: $V = (2.40 \times 10^{-6}C)/(120.0 \times 10^{-9}F) = 20.0 \ V$. (And as a check, the voltage drop across the two capacitors combined would be $16 + 20 = 36 \ V$ which is just what we were told it was.)

Capacitor in a computer keyboard

In one type of computer keyboard, each key holds a small metal plate that serves as one plate of a parallel-plate, air-filled capacitor. When the key is depressed, the plate separation decreases and the capacitance increases. Electronic circuitry detects this change in capacitance and thus detects that the key has been pressed. In one particular keyboard, the area of each metal plate is $42.0 \ mm^2$, and the separation between the plates is $0.700 \ mm$ before the key is depressed.

(a) Calculate the capacitance before the key is depressed.

Here we have a parallel plate capacitor, so $C = \epsilon_o A/d$. We were given both of these measurements but in units of square millimeters and we need meters, so $A = 42.0 \ mm^2 \times \frac{1 \ m}{1000 \ mm} \times \frac{1 \ m}{1000 \ mm} = 42.0 \times 10^{-6} \ m^2$. The separation distance $d = 0.7 \ mm = 0.7 \times 10^{-3} \ m$. So finally, $C = (8.854 \times 10^{-12})(42.0 \times 10^{-6})/(0.7 \times 10^{-3}) = 5.312 \times 10^{-13} \ F$. We can write this as $0.5312 \times 10^{-12} \ F$ which is $\boxed{0.5312 \ pF}$.

(b) If the circuitry can detect a change in capacitance of $0.250 \ pF$, **how far** must the key be depressed before the circuitry detects its depression?

When we press down on the key, we push the plates closer together, which will cause the capacitance to go up (since C is proportional to $1/d$). We're told in this part that the capacitance has to go up by $0.250 \ pF$ over its initial value in order for the circuit to be able to detect the change, so we want our new capacitance to be $(0.5312 \ pF) + (0.250 \ pF)$ or $0.7812 \ pF$. What separation distance will result in this new C value? $C = \epsilon_o A/d$ so $d = \epsilon_o A/C$ or $d = (8.854 \times 10^{-12})(42.0 \times 10^{-6})/(0.7812 \times 10^{-12}) = 4.76 \times 10^{-4} \ m$ or $d = 0.476 \ mm$. So here we had to reduce the separation distance from the original $0.700 \ mm$ so the new value of $0.476 \ mm$, which is a CHANGE of $0.224 \ mm$.

We have a 2-plate capacitor constructed of metal squares that are 4 cm on a side, separated by 1 mm.

(a) What is the capacitance of this device?

$C = \epsilon_o A/d = (8.854 \times 10^{-12})\frac{(0.04)^2}{0.001} = 1.42 \times 10^{-11}$ F.

(b) If we attach a 24 V battery across this capacitor, how much charge will it hold?

$Q = CV = (1.42 \times 10^{-11})(24) = 3.4 \times 10^{-10}$ J

(c) How much energy is stored in this capacitor?

We have several options here, $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$, and we know all of the variables so could use any of them. They all yield $U = 4.1 \times 10^{-9}$ J.

(d) Suppose we keep the battery connected and separate the plates so they are 2 mm apart now (instead of the original 1 mm separation, so we're pulling the plates apart here). How much does the energy stored in the capacitor change? Did we have to do work to separate the plates, or do we extract energy from the capacitor by separating the plates?

We can work this from scratch or take a shortcut. Since $C = \epsilon_o A/d$, by doubling the separation distance d, we've cut the capacitance in half. The new C is half the previous one. $Q = CV$ so if we keep the same voltage but cut C in half, we've also cut Q in half.

The battery is still connected, so the voltage V is the same. Let's use $U = \frac{1}{2}CV^2$ then. Since the capacitance was cut in half (and V remained constant), then U was also cut in half. The energy stored in the capacitor has been reduced, so we have the possibility of extracting some energy here.

This is an interesting and counter-intuitive situation. Apparently the energy stored (the electrical potential energy) **decreases** as the separation distance increases. **That implies the force between the plates is repulsive instead of attractive**, even though the plates have opposite signs and should be attracting each other.

(e) Suppose instead that we detach the battery from the original capacitor, leaving the charge there. Now we move the plates from being 1 mm apart, to 2 mm apart. How much does the energy stored change now? What else changes?

In this case, the charge doesn't change (it doesn't have any place to go). We're still cutting C in half by doubling the distance between the plates. Since Q is constant, let's use $U = \frac{1}{2}\frac{Q^2}{C}$. Here we see that cutting C in half will cause U to double. Increasing the separation distance d causes the potential energy stored to go up. Force is the negative gradient of the potential, so in this case apparently the plates are attracting each other (as expected, since we have two oppositely charged things). We have to do work (apply an external force) to pull these plates farther apart.

Forces in capacitors

The previous example illustrated how the potential energy stored in a (parallel plate) capacitor changes as the separation distance between the plates changes.

In the lecture, we showed that if the charge is held constant (i.e. an isolated capacitor no longer connected to a battery), the (attractive) force between the two plates will be $F = \frac{Q^2}{2\epsilon_o A}$. Using $C = \epsilon_o A/d$ and $Q = CV$ we can morph this into some other useful forms, including $F = \frac{1}{d}(\frac{1}{2}CV^2)$. The part in parenthesis interestingly enough is the energy stored in the capacitor.

Suppose we have a 1 μF capacitor charged to $V = 1000$ $volts$, and the separation distance between the plates is 1 mm. How much force is there between the plates of this capacitor?

$F = \frac{1}{d}(\frac{1}{2}CV^2) = \frac{1}{0.001}(\frac{1}{2}(1 \times 10^{-6})(1000)^2)) = 500$ N.

That's a considerable amount of force considering how small the capacitor is, so in 'real' capacitors, some (insulating) material must be be inserted between the plates to keep them apart. Adding that material alters the capacitance though, and we'll discuss this more in the next example.

Adding a material between the plates of a capacitor has the effect of altering the value of C. We compute C values by just doing an integral involving the geometry of the situation, during which we assumed a vacuum between the plates. If we call that result C_o, then the capacitance when the material is inserted can be written as $C = KC_o$ where K is called the dielectric constant of the material. See table 24-1 in the book for some representative values.

Suppose we use the same capacitor from earlier, where $C = 1\mu F$ with a separation distance of 1 mm, charged to 1000 $volts$. How much energy does this capacitor contain? $U = \frac{1}{2}CV^2 = (0.5)(1 \times 10^{-6})(1000)^2 = 0.5$ J.

This capacitor will hold a charge of $Q = CV = (1 \times 10^{-6})(1000) = 1 \times 10^{-3}$ $coulombs$.

- Suppose we leave this capacitor attached to the voltage source and slip some material between them that has a dielectric constant of $K = 2$. What happens to C, Q, V, and U?

  $C = KC_o$ so the new capacitance will be $C = (2)(1 \times 10^{-6}$ $F) = 2 \times 10^{-6}$ F.

  $Q = CV$ and we're holding the voltage constant, so doubling the capacitance will double the amount of charge. Q will be 2×10^{-3} $coul$ now.

  V remains the same (still attached to the 1000 $volt$ source).

  We have various expressions for U, but since V is constant here, let's use $U = \frac{1}{2}CV^2$. We've doubled the value of C so this capacitor is now storing twice as much energy. Adding the material has caused the potential energy to go up, so we had to do work to push this material between the two plates.

  Put another way, the capacitor is trying to expel the material. (That actually happens in lesser-quality capacitors, where you'll occasionally see the case bulge or split open with some of the dielectric material leaking out.)

- What if we take the capacitor and detach it from the battery, leaving the original Q present. What are the new values of C, Q, V, and U now?

  C just depends on the capacitor itself; inserting the $K = 2$ material has doubled the value of C.

  $Q = CV$ so $V = Q/C$. The charge remains the same, so doubling the value of C means the voltage across the plates has been cut in half.

  Since Q is constant, let's use $U = \frac{1}{2}\frac{Q^2}{C}$ to examine the potential energy. We've doubled C, so we've cut U in half here. Adding the material reduces the potential energy of the capacitor. We did negative work putting this material in place (or put another way, we would have to hold the material back from being pulled into the capacitor).

Force on Dielectrics

Let's consider a little parallel-plate capacitor functioning in a circuit, so that we have some voltage V across the plates. What force (if any) will the dielectric material between the plates 'feel'?

One of our forms for the potential energy stored in a capacitor was $U = \frac{1}{2}CV^2$ but introducing a dielectric material between the plates causes the capacitance to increase. If C_o is the ideal capacitance (i.e. when there is NO dielectric material present) then the actual capacitance will be $C = KC_o$.

The potential energy in the presence of the dielectric then will be $U = \frac{1}{2}(KC_o)V^2$ or $U = K \times (\frac{1}{2}C_oV^2)$.

Adding the dielectric means that when the externally applied voltage is the same, the energy stored in the capacitor is higher.

Looked at the other way around, removing the dielectric decreases the potential energy stored. Forces act in a way to decrease U (F being the negative gradient of the potential energy). In this type of capacitor then, any time there is a voltage present across the capacitor (which is pretty much all the time in a working circuit) it's constantly trying to push the dielectric material **out**.

This is a common problem with real-world capacitors, especially cheaply-made ones. Some years ago, several computer manufacturers had to deal with motherboards with leaking or exploding capacitors.