Chapter 25 Examples : Electric Currents and Resistance

Key concepts:

- **Current** : I : rate at which charge is flowing past a point : $I = dQ/dt$. (Units of coulombs/sec, also called an **ampere** or just **amp**, abbreviated using the symbol **A**.) (For historical reasons, we pretend that this is positive charge flowing in one direction, when in reality it's negative charges flowing in the opposite direction.)

- **Ohm's Law** : current related to voltage: $V = IR$, defining the **resistance** (R) of a circuit or entity. R has units of volts/amp, also called the **ohm** abbreviated using the symbol Ω.

- **Resistivity** (typically used in wire-like scenarios) : the resistance of a wire is proportional to the wire's length (L) and inversely proportional to its cross-sectional area (A), so: $R = \rho\frac{L}{A}$. ρ is called the **resistivity** of the material the wire is made of. See table 25-1 for the resistivity of some common materials. **Insulators** typically have very large resistivities. Glass has a resistivity on the order of 10^9 to 10^{12} $\Omega \cdot m$, for example.

  The resistivity of a material usually changes slightly with temperature. To a first order approximation, this is usually represented as:
  $\rho = \rho_o[1+\alpha(T-T_o)]$. The reference temperature is usually taken to be the 'standard temperature and pressure' value of $T_o = 20^oC$. (Table 25-1 includes both ρ_o and α for some materials.)

- **Power** : $P = IV$ and since $V = IR$ this can also be written in various other convenient forms: $P = I^2R$ and $P = V^2/R$

- **Alternating current**

  Electrical power is usually delivered to users as alternating current instead of direct current. The voltage source varies with time as $V = V_o \sin(2\pi f t)$ where f is the frequency of the source (in Hertz). Also written as $V = V_o \sin(\omega t)$. In the US and Canada, $f = 60\ Hz$ is typical; in other parts of the world, $50\ Hz$ is more common.

  Since $V = IR$ then $I = V/R = \frac{V_o}{R}\sin(\omega t)$

  Since $P = I^2R$ the power fluctuates as $P = I_o^2 R \sin^2(\omega t)$. The **average** power would be $P_{avg} = \frac{1}{2}I_o^2 R$ or equivalently $P_{avg} = \frac{1}{2}V_o^2/R$, which can also be written as $P_{avg} = I_{rms}V_{rms} = I_{rms}^2 R = V_{rms}^2/R$ since these forms 'look' like the DC versions. The rms value is the square root of the average of the square of a quantity so $I_{rms} = \frac{1}{\sqrt{2}}I_o = 0.707I_o$ and $V_{rms} = \frac{1}{\sqrt{2}}V_o = 0.707V_o$. These rms values are sometimes called the **effective** values of the current and voltage.

  In the US, the '120 volts' is actually the rms value of the voltage. The true signal coming out of the wires has a magnitude of $V_o = \sqrt{2}V_{rms} = 170\ volts$.

- **Current Density and Drift Velocity**

  Current density: $\vec{j}$: current per unit area. In a wire, this is the current divided by the cross sectional area of the wire: $I = jA$. If the density is not uniform, $I = \int \vec{j} \cdot d\vec{A}$.

  Drift velocity (actual speed of the **electrons** making up the current): $I = -neAv_d$ or $j = -nev_d$. (Typical v_d in household currents is fractions of a millimeter per second...) n is the density of free charges (electrons really) in the material.

Current and drift speed

A 5.00 A current runs through a 12-gauge copper wire (diameter 2.05 mm) and through a light bulb. Copper has 8.5×10^{28} free electrons per cubic meter. (a) How many electrons pass through the light bulb each second? (b) What is the current density in the wire? (c) At what speed does a typical electron pass by any given point in the wire? (d) If you were to use wire of twice the diameter, which of the above answers would change? Would they increase or decrease?

This is a pretty direct application of the definitions of the requested quantities.

(a) The current I is defined as the amount of charge flowing per time, so here we have 5.00 A which is 5.00 C/s. In one second, then, 5.00 C of charge flows past any point in the circuit. This charge is made of individual electrons, each carrying a charge of 1.602×10^{-19} C, so 5.00 C represents $(5.00\ C)/(1.602 \times 10^{-19}\ C/electron) = 3.12 \times 10^{19}$ electrons.

(b) The current density is defined as $j = I/A$ where A is the cross-sectional area of the wire. The diameter of the wire is 2.05 mm or 2.05×10^{-3} m and the cross-sectional area is $A = \pi r^2$ or $A = \pi(d/2)^2 = \frac{\pi}{4}d^2$ so $A = (\pi)(2.05 \times 10^{-3})^2/4 = 3.301 \times 10^{-6}\ m^2$. The current density then is: $j = I/A = (5.00\ A)/(3.301 \times 10^{-6}\ m^2) = 1.51 \times 10^6\ A/m^2$.

(c) The drift speed v_d is related to the current density through: $j = nev_d$, so $v_d = J/(ne)$. Ignoring signs, $v_d = \frac{1.51\times10^6\ A/m^2}{(8.5\times10^{28}m^{-3})(1.602\times10^{-19}\ C)} = 1.11 \times 10^{-4}\ A\ m/C$ but amps is coulombs per second, so this is $1.11 \times 10^{-4}\ m/s$ or $0.111\ mm/s$.

(d) If we increase the diameter of the wire, the cross-sectional area will increase. They didn't say anything about the current changing, so if we still have the same 5.00 A of current, the number of electrons flowing in 1 second won't change. The current density $j = I/A$ so as A increases, j will decrease. The drift speed $v_d = j/(ne)$ so it will also decrease, since j decreased.

Resistance of a wire

In household wiring, copper wire 2.05 mm in diameter is often used. Find the resistance of a 24.0 m length of this wire.

The resistance is defined in terms of the resistivity of the material and its geometry as $R = \rho L/A$ where L is the length of the wire and A its cross-sectional area. From table 25.1, the resistivity of copper (at room temperature) is $\rho = 1.72 \times 10^{-8}\ \Omega\ m$. The length of the wire is $L = 24.0\ m$ and the cross-sectional area will be $A = \frac{\pi}{4}d^2$ where d is the diameter $d = 2.05 \times 10^{-3}\ m$ from which $A = 3.301 \times 10^{-6}\ m^2$.

So finally: $R = \rho L/A = (1.72 \times 10^{-8}\ \Omega\ m) \times (24.0\ m)/(3.301 \times 10^{-6}\ m^2)$ or $R = 0.125\ \Omega$.

Note: wire is sometimes labelled with it's resistance per meter (or per foot) so you can just multiply the length of wire you're using by that R/L factor to get the actual resistance in Ohms of your particular section of wire. The wire here was 24 m long, so the resistance per length would be $R/L = (0.125\ \Omega)/(24.0\ m) = 5.21 \times 10^{-3}\ \Omega/m$.

Ohm's Law and power lines

A copper transmission cable 100 km long and 10.0 cm in diameter carries a current of 125 A. (a) What is the potential drop across the cable? (b) How much electrical energy is dissipated as thermal energy every hour?

(a) The voltage, current, and resistance are related by $V = IR$. We have the current flowing $I = 125\ A$ but don't directly have the resistance of the wire, but we can find it from $R = \rho L/A$ since we know that the wire is made of copper ($\rho = 1.72 \times 10^{-8}\ \Omega\ m$) and is $L = 100 \times 10^3\ m$ long and has a diameter of 0.1 m which implies a cross-sectional area of $A = \frac{\pi}{4}d^2 = 7.854 \times 10^{-3}\ m^2$. The resistance of this long section of wire then is: $R = \rho L/A = (1.72 \times 10^{-8}\ \Omega\ m) \times (10^5\ m)/(7.854 \times 10^{-3}\ m^2) = 0.219\ \Omega$.

NOTE: this is comparable to the resistance of the much shorter and thinner wire in the previous problem. Since $R = \rho L/A$, the **much** larger area A of this transmission wire (which reduces R) mostly makes up for the much longer wire length (which would increase R).

The voltage drop across this 100 km long wire then is $V = IR = (125\ A)(0.219\Omega) = 27.4\ V$. (This may sound like a lot, but these transmission wires typically carry voltages of tens or even hundreds of **thousands** of volts.)

(b) The power emitted (as heat mostly) from the wire will be $P = VI = (27.4\ V)(125.0\ A) = 3425\ W$ which is 3425 J/s (joules/second). In one hour (3600 s), the amount of heat generated by this wire will be $(3425\ J/s)(3600\ s) = 1.23 \times 10^7\ J$. (Spread over 100 km of wire, though. Each meter of wire is only emitting 0.03425 W of heat, so probably wouldn't even feel warm.)

Another wire; different information given

An electrical conductor designed to carry large currents has a circular cross section 2.50 mm in diameter and is 14.0 m long. The resistance between its ends is 0.104Ω. (a) What is the resistivity of the material? (b) If the electric field magnitude in the conductor is 1.28 V/m, what is the total current? (c) If the material has 8.5×10^{28} free electrons per cubic meter, find the average drift speed under the conditions in part (b).

(a) The resistance of a wire depends on the resistivity of the material and the geometry of the wire: $R = \rho L/A$. Here we know R and the geometry of the wire, but wish to find the resistivity, so $\rho = RA/L$. The cross-sectional area of the wire is $A = \frac{\pi}{4}d^2$ where $d = 2.50 \times 10^{-3}\ m$ from which $A = 4.909 \times 10^{-6}\ m^2$, so $\rho = RA/L = (0.104\ \Omega)(4.909 \times 10^{-6}\ m^2)/(14.0\ m) = 3.65 \times 10^{-8}\ \Omega\ m$. (Looking at table 25.1, whatever this material is, it's comparable to the other 'good' conductors.)

(b) The electric field down the length of the wire is $E = V/L$ so $V = EL = (1.28\ V/m)(14.0\ m) = 17.92\ V$. We thus have a 17.92 volt drop across the length of this wire. The current flowing is related to this voltage drop and the resistance of the wire: $V = IR$ so $I = V/R = (17.92\ V)/(0.104\ \Omega) = 172.3\ A$.

Alternately: $I = JA$ and J is related to the electric field: $E = \rho J$ so $J = E/\rho = (1.28\ V/m)/(3.65 \times 10^{-8}\ \Omega\ m) = 3.51 \times 10^7 A/m^2$. Then: $I = JA = (3.51 \times 10^7\ A/m^2) \times (4.909 \times 10^{-6} m^2) = 172.3\ A$.

(Either way, this 14.0 m wire is emitting $P = VI = 3090\ W$, which is about the same as the 100 km wire in the previous problem, so this short wire would likely be very hot.)

Power

A typical cost for electric power is 12 cents per kilowatt-hour. (a) Some people leave their porch lights on all the time. What is the yearly cost to keep a 75 W bulb burning day and night? (b) Suppose your refridgerator uses 400 W of power when it's running, and it only runs 8 hours each day. What is the yearly cost of operating the refridgerator?

This is entirely a units-conversion problem.

(a) Porch light: here we have a 75 W bulb burning continuously for a year. The total energy (joules) used in this time will be 75 J/s times the number of seconds in a year. Accounting for leap years, the length of a year is approximately 365 and 1/4 days so $1\ year = (365.25\ days)(24\ hrs/day)(3600\ s/hr) = 3.16 \times 10^7\ s$. (Note this is very close to π times 10^7 seconds, which is a convenient short-cut for quick estimates with less than a 1 percent error.)

In one year, then, the porch light will use $(75.0\ J/s)(3.16 \times 10^7\ s) = 2.37 \times 10^9\ J$ of energy.

The obscure part here is the units at which power usage is billed: the kilowatt-hour. Watts is a unit of power, which is energy per time, and hours is time, so the 'kW-hr' unit is actually a unit of energy. $1\ kW \cdot hr = (1000\ W)(1\ hr) = (1000\ J/s)(3600\ s) = 3,600,000\ J = 3.6 \times 10^6\ J$. The rate, then, can be written as $(\$0.12)/(3.60 \times 10^6\ J)$.

Finally, the cost for the porch light will be: $(2.37 \times 10^9\ J) \times \frac{\$0.12}{3.60 \times 10^6\ J} = \$79.00$.

(b) The refridgerator only runs for 1/3 of this time, but uses energy at $(400)/(75) = 5.333$ times the rate, so in a year will use $(5.333)/3 = 1.78$ times as much energy, or $(1.78)(\$79.0) = \$140.60$.