Chapter 29 Examples : Electromagnetic Induction and Faraday's Laws

Key concepts:

Changing magnetic fields induce EMF's (voltages); a result referred to as electromagnetic induction

Magnetic flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Faraday's law of induction: $\xi = -\frac{d\Phi_B}{dt}$

If the circuit contains N loops closely wrapped so the same flux passes through each, the overall EMF induced is $\xi = -N\frac{d\Phi_B}{dt}$

Lenz's law: a current produced by an induced EMF moves in a direction so that the magnetic field created by that current opposes the original change in flux

Coil with N loops rotating at angular speed ω in a constant field B: $\xi = NBA\omega \sin(\omega t) = \xi_o \sin(\omega t)$

Moving conductor: $|\xi| = BLv$

(A/C) Transformer: $V_s = N_s d\Phi_B/dt \qquad V_p = N_p d\Phi_B/dt$ so $V_s/V_p = N_s/N_p$

Assuming nearly 100% efficiency, $P_p = P_s$ so $P = IV$ implies: $I_s/I_p = N_p/N_s$

Changing magnetic flux produces an electric field (general form of Faraday's law): $\int \vec{E} \cdot d\vec{l} = -d\Phi_B/dt$

Electromagnetic Induction(1)

A flat, rectangular coil consisting of 50 turns measures 25.0 cm by 30.0 cm. It is in a uniform 1.20 T magnetic field, with the plane of the coil parallel to the field. In 0.222 s, it is rotated so that the plane of the coil is perpendicular to the field. (a) What is the change in the magnetic flux through the coil due to this rotation? (b) Find the magnitude of the average EMF induced in the coil during this rotation.

(a) The flux $\Phi_B = \int \vec{B} \cdot d\vec{A}$. EACH turn of the coil has this flux passing through it. Initially, $\vec{B}$ and $\vec{A}$ are in the same direction, so the flux through each turn in the coil becomes simply BA. The total flux through all $N = 50$ loops then will be $\Phi_B = NBA$. The area is $A = (0.25\ m)(0.30\ m) = 0.075\ m^2$, so here we start off with a flux of $\Phi_B = (50)(1.20\ T)(0.075\ m^2) = 4.5\ T\ m^2$. (The units of magnetic flux are usually expressed in 'Webers', with $1\ Wb = 1\ T\ m^2$ so we can also write this as $\Phi_B = 4.5\ Wb$.)

(b) Now we suddenly turn the coil so that the plane of the coil is perpendicular to the field. That means that $\vec{A}$ is now perpendicular to $\vec{B}$, which means the magnetic flux through the coil is now ZERO.

The EMF induced when the magnetic flux changes is given by Faraday's Law: $\xi = -\frac{d\Phi}{dt}$. They're vague about exact directions here, so we really can't say anything about the sign of this EMF so going forward I'll just consider its magnitude. Here, the flux is changing from 4.5 Wb to zero in a time interval of 0.222 s, so we can approximate the induced EMF to be $|\xi| \approx \Delta\Phi/\Delta t = (4.5)/(0.222) = 20.3\ V$. (So if we connect this coil up to a voltmeter and flip it quickly, we should see the voltmeter - briefly - read this voltage, then drop back to zero.)

Electromagnetic Induction (2)

In a physics lab experiment, a coil with 200 turns enclosing an area of 12 cm^2 is rotated in 0.04 s from a position where its plane is perpendicular to the earth's magnetic field to a position where its plane is parallel to the field. The earth's magnetic field at the lab location is $6.0 \times 10^{-5}\ T$. (a) What is the total magnetic flux through the coil before it is rotated? After it is rotated? (b) What is the average EMF induced in the coil?

This problem is sort of the reverse of the previous one. We start off with the area vector $\vec{A}$ being perpendicular to the magnetic field, so the initial flux is zero. Then we turn the coil so that the area vector is parallel to the field, making the angle between them either 0 or 180, they don't really specify. So as in the first problem, we'll just ignore the sign and worry about the magnitude of the induced EMF: $|\xi| \approx \Delta\Phi/\Delta t$.

When the area is parallel to the field, the flux will be $\Phi_B = BA$ in each turn, for an overall flux of $\Phi_B = NBA$ through all N turns in the coil. Be careful converting the units of the area. $A = 12\ cm^2$ so we have TWO factors of cm we need to convert to m:

$A = 12\ cm^2 \times \frac{1\ m}{100\ cm} \times \frac{1\ m}{100\ cm} = 12 \times 10^{-4}\ m^2$.

The flux at this point then is: $\Phi_B = NBA = (200)(6 \times 10^{-5}\ T)(12 \times 10^{-4}\ m^2) = 1.44 \times 10^{-5}\ T\ m^2$.

This makes the induced average voltage: $|\xi| \approx \Delta\Phi/\Delta t = (1.44 \times 10^{-5})/(0.04) = 3.6 \times 10^{-4}\ V$.

The Earth's magnetic field is pretty weak; here we're rotating a coil around in this field and turning the coil into a source of voltage. If we increase the size of the coil and/or increase the number of turns of wire in the loop, or speed up the rotation, we might be able to create a device that converts mechanical energy (something has to be causing the loop to rotate) into electrical power directly. How practical would it be to create something like this that would act like a 9 V battery? A 120 V source? (See example 7.)

Electromagnetic Induction (3)

A circular loop of wire with a radius of 12.0 cm and oriented in the horizontal xy-plane is located in a region of uniform magnetic field. A field of 1.5 T is directed along the positive z-direction, which is upward. (a) If the loop is removed from the field region in a time interval of 2.0 ms, find the average EMF that will be induced in the wire loop during the extraction process. (b) If the coil is viewed looking down on it from above, is the induced current in the loop clockwise or counterclockwise?

As with the first two problems, the induced EMF will equal the change in flux divided by the time interval. This time, we'll be careful about the signs.

Let's choose a direction for the area vector $\vec{A}$ so that it is in the same direction as the field. (This implies, by the right-hand rule, that the 'path' around the edge of this circle - i.e. the path through our wire - is counterclockwise, when viewed from above.) Initially, then, we have a flux of $\vec{B} \cdot \vec{A} = (B)(A)\cos 0 = BA$. The area of the coil is $A = \pi r^2 = \pi(0.12)^2 = 0.04524\ m^2$. The flux then at this point is $\Phi_B = BA = (1.5)(0.04524) = 0.06786\ Wb$.

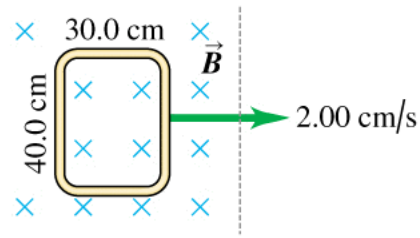
The coil is then withdrawn from the field, so the flux now is zero. The change in flux divided by the change in time (the EMF) is: $\xi \approx -\Delta\Phi/\Delta t = -(0 - 0.06786)/(2 \times 10^{-3}) = +33.9\ V$.

Having $\vec{A}$ pointing up in the $+z$ direction implies that the loop we are taking is a counterclockwise walk around the edge of the circle, so we are saying here that the EMF is increasing in the counterclockwise direction. These things act like batteries instead of resistors, so the current is flowing in the direction of the EMF, so I is moving counterclockwise as well.

(Looking at it from the Lenz perspective, the 'induced' effect should occur in a way that tries to oppose the effect. Here we have a flux that is decreasing, so the induced current will move in a way that tries to increase the flux. A counterclockwise current will do that, since it will produce a magnetic field $\vec{B}'$ that is in the $+z$ direction (same as the direction of $\vec{A}$, which will cause a positive flux...)

Electromagnetic Induction (4)

A rectangle measuring 30 cm by 40 cm is located inside a region of a spatially uniform magnetic field of 1.25 T, with the field perpendicular to the plane of the coil. The coil is pulled out at a steady rate of 2 cm/s, travelling perpendicular to the field lines. The region of the field ends abruptly as shown. Find the EMF induced in this coil when it is (a) all inside the field; (b) partly inside the field; (c) all outside the field.



(a, c) While the loop is entirely inside the field, the flux isn't changing: $\Phi_B = \int \vec{B} \cdot d\vec{A} = BA$ during this interval. So as long as the wire remains entirely inside the field, and doesn't rotate, the flux isn't changing. The induced EMF depends on CHANGES to the flux, so for now the induced EMF will be zero. In part (c), we are entirely outside the field, so the flux is zero and remains zero, so there is no change in flux there, and hence the induced EMF in the loop will be zero.

(b) Here we are partly in the field and partly outside. We can write the area as $A = XY$ where X is the length of side of the rectangle in the x-direction (left-right) and Y is the length of the side in the y-direction (north-south, or up-down on the page). We can write $\xi = -\Delta\Phi/\Delta t$ where $\Phi = BA$ here (with A being the part of the area that is inside the field, since $B = 0$ outside the region shown). $\xi = -\Delta(BA)/\Delta t = -\Delta(BXY)/\Delta t = -BY\Delta X/\Delta t$. This is now in the form we need. The X (left-right) length of the loop is DECREASING by 2 cm/s, so $\Delta X/\Delta t = -0.02\ m/s$.

Finally, $\xi = -(1.25\ T)(0.40\ m)(-0.02\ m/s) = +0.01\ V$.

(Even with this very large magnetic field, moving the loop slowly is inducing only a very small EMF in the loop.)

Electromagnetic Induction (5)

How fast (in m/s and mph) would a 5.00 cm copper bar have to move at right angles to a 0.650 T magnetic field to generate 1.50 V (the same as a AA battery) across its ends? Does this seem like a practical way to generate electricity?

This is a classic 'motional EMF' problem (see section 29-3). In general $\xi = \int(\vec{v} \times \vec{B}) \cdot d\vec{l}$. Here everything is constant and $\vec{v}$ is perpendicular to $\vec{B}$. The cross product produces a vector along the bar, so the dot product goes away as well. In the end, this simply reduces to $\xi = vBL$ (figure 29.12).

Here then, we want ξ to be 1.50 V and we have the length of the rod and the strength of the magnetic field so $\xi = vBL$ becomes: $(1.5) = (v)(0.65)(0.05)$ or $v = 46.15\ m/s$ which is about 103 $miles/hr$.

This is pretty fast, and 0.65 T is a large magnetic field, so it would take rooms worth of equipment to generate this 1.5 V of EMF. Doesn't sound very practical...

Electromagnetic Induction (6)

A long straight solenoid with a cross sectional area of 8.00 cm^2 is wound with 90 turns of wire per centimeter, and the windings carry a current of 0.350 A. A second winding of 12 turns encircles the solenoid at its center. The current in the solenoid is turned off such that the magnetic field of the solenoid becomes zero in 0.04 s. What is the average induced EMF in the second winding?

The current in the solenoid creates a strong magnetic field in its interior. When we turn off the circuit, the current takes a brief amount of time to drop to zero, during which time the magnetic field inside the solenoid is dropping to zero as well.

Looking at this from outer coil's point of view, it sees a changing magnetic field (thus a changing magnetic flux), but that will create an emf in the outer coil.

Note: the problem gave us no information on the direction the wires were wound on the solenoid, or the direction around the outer loop, so we'll just worry about the magnitude of the effect here. We can't say anything about the direction of the EMF induced in the outer loop.

The magnetic field inside a solenoid is uniform and of magnitude $B = \mu_o n I$ where I is the current flowing in the wires making up the solenoid, and n is the number of turns per unit length. We need everything in standard metric units, so $n = 90\ turns/cm = \frac{90\ turns}{1\ cm} \times \frac{100\ cm}{1\ m} = 9000\ turns/meter$. The magnetic field throughout the interior of the solenoid then is $B = \mu_o n I = (4\pi \times 10^{-7})(9000)(0.350) = 3.958 \times 10^{-3}\ T$. This field is parallel to the axis of the solenoid. When the current shuts off, this magnetic field drops to zero in 0.04 s. (Note: the magnetic field inside this solenoid is roughly 100 times larger than the earth's field.)

Let's move to the outer windings now. They encompass a certain magnetic flux, and that flux is changing, dropping to zero in a short time interval. Changing magnetic flux induces a current. The total flux through the $N = 12$ turns of the outer coil is $\Phi_B = NBA$ initially, and $\Phi_B = 0$ after the current is gone. That initial flux is $\Phi_B = NBA$ and we need everything in standard metric units. B is nonzero only over an area of 8 cm^2 which is $8 \times 10^{-4}\ m^2$ so $\Phi_B = (12)(3.958 \times 10^{-3})(8 \times 10^{-4}) = 3.80 \times 10^{-5}\ Wb$. The induced EMF is $\xi = -\Delta\Phi/\Delta t = -(0 - 3.8 \times 10^{-5})/(0.04) = 9.5 \times 10^{-4}\ V$.

This is basically a type of transformer. Imagine if some sinusoidally varying voltage were being applied to the inner solenoid. The wire has some resistance, so this voltage turns into a sinusoidally varying current. This causes a sinusoidally varying magnetic field in the interior of the solenoid, which means that the outer coil is seeing a changing magnetic flux. The derivative of this flux produces a cosinusoidally varying EMF in the outer loop. The geometry (area, number of turns on wire in each coil) enter in as various scale factors and we end up turning a varying voltage of one amplitude (120 V, say) into a varying voltage of a different amplitude: maybe 12 V or 18 V or something else that the parts in your computer are designed for. The 'cost' here is that these varying voltages are out of phase from the original, but that usually doesn't matter.

Electromagnetic Induction (7) : Hand-cranked 'battery'

You are shipwrecked on a deserted tropical island. You have some electrical devices that you could operate using a generator but you have no magnets. The earth's magnetic field at your location is horizontal and has magnitude $8.0 \times 10^{-5}\ T$, and you decide to try to use this field to power a generator formed by rotating a large circular coil of wire at a high rate. You need to produce a peak EMF of $9.0\ V$ and estimate that you can rotate the coil at $30\ rpm$ by turning a crank handle. You also decide that to have an acceptable coil resistance, the maximum number of turns the coil can have is 2000. (a) What area must the coil have? (b) If the coil is circular, what is the maximum translational speed of a point on the coil as it rotates?

If we orient the coil so that the plane of the coil is vertical and then rotate it about the Z axis, we'll basically have the area vector $\vec{A}$ pointing in the same direction as $\vec{B}$ and then rotating to be perpendicular to the field, then in line again, and so on. Imagine a big circular ring with one side on the ground and the other up in the air.

The direction of $\vec{A}$ will be spinning around so that the angle between $\vec{A}$ and $\vec{B}$ will be increasing as $\theta = \omega t$. We can write the magnetic flux through this area as $\Phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta = BA\cos\omega t$. That is the flux through **each loop** of the coil, and have enough wire to make 2000 loops, so the total flux will be $\Phi = 2000BA\cos\omega t$.

The induced EMF then will be $\xi = -d\Phi/dt = 2000BA\omega\sin\omega t$. The EMF will vary sinuisoidally between $2000BA\omega$ and $-2000BA\omega$. We desire this maximum EMF to be $9.0\ V$ so we need $2000BA\omega = 9$.

We have B, the strength of the (weak) magnetic field of the earth. Apparently we think we can spin this thing at $30\ rpm$ or 30 complete revolutions per minute. Converting this angular speed to radians per second: $\omega = 30\frac{rev}{min} \times \frac{2\pi\ rad}{1\ rev} \times \frac{1\ min}{60\ sec} = \pi\ rad/sec$. (This corresponds to a period for one complete rotation of $T = 2\pi/\omega = 2\ s$

(a) In our equation for the EMF, then, we have: $9 = 2000BA\omega = (2000)(8 \times 10^{-5})(A)(\pi)$ from which $A = 17.9\ m^2$.

(b) If the coil is in the shape of a circle, the area is $A = \pi r^2$ so this area represents a radius of $17.9 = \pi r^2$ or $r = 2.39\ m$. (That makes the diameter about $4.8\ m$ or around 15 feet.) A point on the edge of the circle moving the fastest would have a tangential speed of $v = r\omega = (2.39\ m)(\pi\ rad/s) = 7.5\ m/s$.

At this point, this does not seem very practical. Imagine a ring 15 feet across that we are trying to keep spinning in a circle so that it makes a complete revolution every 2 seconds. There would certainly be a lot of air resistance, and it would be quite a lot of work to generate that $9\ V$ of 'battery power.'

On the other hand, this process is used by wind turbines to generate electricity. The wind causes a large propeller to rotate which is attached to a large coil that is now rotating in a magnetic field created by powerful permanent magnets, generating considerable amounts of power. Small turbines can generate power in the kilowatt range, with large turbines (such as those used in 'wind farms') can generate power in the megawatt range.

EMF induced in a necklace near a high frequency current

Occasionally you run into warning signs around electronics that suggest removing any rings, watches, necklaces, even belts - basically anything that might represent a 'loop' where an EMF could be induced.

Suppose we're standing **5** meters from a modest **2 A** current that's oscillating at **1 GHz**. We're wearing a necklace, which we'll model as a circular loop with a radius of **5 cm**. Estimate how much power the necklace will be emitting as heat.

The varying current in the wire will create a time-varying magnetic field swirling around the wire. That means the magnetic flux through the necklace will be changing with time, but that implies a voltage is being generated around the loop. The necklace will have some very low resistance, so this 'resistor' will be emitting $P = V^2/R$ watts of heat.

The current is varying sinusoidally with time, so $I = I_o \sin(\omega t)$

This current will create a magnetic field curling around the wire with a magnitude of $B = \frac{\mu_o I}{2\pi r}$. The magnetic flux through the necklace will be $\Phi_B = \int \vec{B} \cdot \vec{A}$. Let's assume the necklace is oriented so the magnetic field is coming straight through it, so $\vec{B}$ and $\vec{A}$ are in the same direction. Technically B depends on r, but let's assume it's approximately constant here. Then $\Phi_B = BA = \frac{\mu_o I_o A}{2\pi r} \sin(\omega t)$.

The EMF generated will be $\xi = -d\Phi_B/dt = -\frac{\mu_o I_o A \omega}{2\pi r} \cos(\omega t)$.

So: we have a sinusoidally-varying voltage around the loop represented by the necklace. The amplitude of this voltage will be $V_{max} = \frac{\mu_o I_o A \omega}{2\pi r}$.

So far so good - let's put our numbers in place now. The necklace has a radius of 5 cm so the area will be $A = \pi(0.05)^2 = 7.85 \times 10^{-3}\ m^2$. The angular frequency $\omega = 2\pi f = 6.28 \times 10^9\ s^{-1}$. The current amplitude here is $I_o = 2\ amps$, and we're $r = 5\ m$ away. Putting all that together:

$V_{max} = \frac{(4\pi \times 10^7)(2)(0.00785)(6.28 \times 10^9)}{(2)(\pi)(5)} \approx 4\ volts.$

The power this 'resistor' will emit as heat will be $P = V^2/R$, so what's the resistance of the necklace? Well, if we 'unroll' the necklace, we can think of it as a wire with some length and some cross-sectional area. The resistance of this 'wire' then will be $R = \rho L/A$ (different A than above... this A is the cross sectional area of the 'wire'.) $L = 2\pi r = 0.314\ m$. The cross sectional area could be anything, but suppose the necklace is about 2 mm by 1 mm, so $A = (0.002)(0.001) = 2 \times 10^{-6}\ m^2$. ρ for gold is about 2.4×10^{-8} so $R = \rho L/A = (2.44 \times 10^{-8})(0.314)/(2 \times 10^{-6}) \approx 4 \times 10^{-3}\ \Omega$.

$P = V^2/R = (4)^2/(0.004) = 4000\ watts$. This thin gold necklace will be emitting around 4000 watts of heat just by being in the presence of this high frequency oscillating current. Hence the warning about removing jewelry and anything else that might have loops that could have voltages induced in them. (Including phones, laptops, or anything else with circuitry inside since those are full of loops of various sizes...)

EMF Induced in a Moving Conductor (A)

Consider a 1.2 m long antenna mounted vertically on a vehicle that is travelling to the east at 30 m/s. Assuming the Earth's magnetic field is directly to the north, what voltage difference will be generated between the top and bottom of the antenna?

Let's look at the 'free electrons' in the antenna. The antenna doesn't have a net charge (probably) but the outermost electrons on the metal forming the antenna are so loosely bound that they are essentially free to move around within the metal. The antenna is moving to the east, so the free electrons within it are travelling to the east through a magnetic field that is pointing to the north. They will feel a force of $F = q\vec{v} \times \vec{B}$ which is directed vertically downward. (Note: the $\vec{v}$ to the east crossed into the magnetic field $\vec{B}$ to the north will produce a vector pointing UP but when we multiply by q, which is negative, the force will be DOWN.) The net effect is that electrons are flowing downward in the antenna.

We end up with the top part of the antenna being positively charged and the bottom negatively charged. That's an electric field in the antenna that is pointing downward. $V = -\int \vec{E} \cdot d\vec{l}$ so the voltage is increasing as we move upward along the antenna.

The magnitude of this effect is $\xi = BLv$. Here, $B = 5 \times 10^{-5}\ T$ (the weak magnetic field of the Earth), $L = 1.2\ m$ (length of the antenna), and $v = 30\ m/s$ so $\xi = (5 \times 10^{-5})(1.2)(30) = 1.8 \times 10^{-3}\ volts$ which is present, but probably too small to worry about.

Transformer (A)

Consider a solenoid with 1000 loops that's connected to a 120 V AC source that's providing 1 A of current. We'll add an insulating coating, then wrap another soleoid around this one. The outer solenoid has just 40 loops. What current and voltage will be coming out of the outer solenoid?

This is a transformer - the side that's connected to the EMF source is called the **primary**, and the other side is the **secondary**.

The voltage across the ends of each loop of a coil are related to the rate of change of the magnetic flux inside that coil via $\xi = -d\Phi_B/dt$. We have N loops, so ignoring signs, we can say that the voltage across the primary will be $V_p = N_p d\Phi_B/dt$ and the voltage across the secondary will be $V_s = N_s d\Phi_B/dt$. It's the same flux for both of them, so $V_p/N_p = V_s/N_s$ or $\boxed{V_s = V_p(N_s/N_p)}$ which in this case will be $V_s = (120\ V)(40/1000) = 4.8\ V$.

Transformers are usually very efficient, so the input and output electrical power is essentially the same on each side. $P = IV$ so $I_p V_p = I_s V_s$ or $\boxed{I_s = I_p(N_p/N_s)}$ which in this case will be $I_s = (1\ A)(1000/40) = 25\ A$.

Transformer (B)

Consider the problem of transferring electrical power over long distances - hundreds or even thousands of miles. Some amount of current is flowing through these long wires. Recall from an early chapter that the resistance of wire is $R = \rho L/A$ (where L is the length of the wire, and A is it's cross-sectional area). Since the wire has some resistance, some electrical power will be lost due to that: $P_{lost} = I^2 R_{wire}$ so the higher the current, the more power will be lost.

Is there a trick where we can reduce the current but still transmit the same amount of power to the end users hundreds of miles away?

Suppose we have a $100,000\ W$ generator that's producing $100\ V$. That implies a current of $P = VI$ or $I = P/V = (100,000)/(100) = 1,000\ amps$. If our powerline wires have a very low resistance - say 1 ohm, then the power loss due to this resistance would be $P = I^2 R = (1000)^2(1) = 1,000,000\ W$ which is more than we're generating... (Obviously that won't happen, but it does show that we'll essentially lose all the power we're generating to heat.)

Suppose we send our source through a transformer or a series of transformers that convert that $100\ V$ voltage source into a $10,000\ V$ signal? From the previous example, this $100:1$ increase in the voltage will come with a $1:100$ reduction in the current, so we now have a source that is providing $10\ A$ at $10,000\ V$.

How much power will be lost due to the resistance of the wires now? $P = I^2 R = (10)^2(1) = 100\ W$ which is almost nothing compared to the $100,000\ W$ the power station is producing.

There are several practical limits that get in the way. Transformers are not 100 percent efficient, so we'll lose some power there. Also the higher the voltage we send down the wires, the more the insulating coating will degrade, and so on. Still, real world long distance power lines typically operate in the range of $30,000\ V$ to $150,000\ V$.