Physics 2233 : Chapter 16 Examples : Sound

Generic relationships: $k = 2\pi/\lambda \quad \omega = 2\pi/T \quad f = 1/T \quad v = \lambda/T = \lambda f = \omega/k$

Sound is a longitudinal (pressure) wave

Range of sound audible to humans: 20 Hz to $20,000$ Hz

Speed of sound in air: $v \approx (331 + 0.60T)$ for T in $deg\ C$ (about 343 m/s at STP, i.e. $20^o\ C$)

Displacement: $D(x,t) = A\sin(kx - \omega t)$ which implies Pressure: $\Delta P = -\Delta P_{max}\cos(kx - \omega t)$

where: $\Delta P_{max} = BAk = \rho v^2 Ak = 2\pi\rho v A f$

Intensity (power/area): $I = 2\pi^2\rho v A^2 f^2 \qquad I = (\Delta P_{max})^2/(2v\rho)$

Intensity in Decibels: $\beta = 10\log_{10}(I/I_o)$ where $I_o = 1\times10^{-12}\ W/m^2$

Stringed Instruments (fixed at each end):

$\lambda_n = \frac{2L}{n} = v/f_n$ and $f_n = n(\frac{v}{2L})$ for $n = 1, 2, 3, ...$ where $v = \sqrt{F_T/\mu}$

Wind Instruments:

Open pipe: $\lambda_n = \frac{2L}{n}$ and $f_n = n(\frac{v}{2L})$ (for $n = 1, 2, 3...$)

Closed pipe: $\lambda_n = \frac{4L}{n}$ and $f_n = n(\frac{v}{4L})$ (for $n = 1, 3, 5, ...$)

Interference (beats) : $f_{avg} = \frac{1}{2}(f_1 + f_2) \qquad f_{beat} = |f_2 - f_1|$

Doppler Effect (using book's conventions) :

$f' = f\cdot(v \pm v_{obs})/(v \mp v_{src})$ where v = sound speed, v_{obs} = observer speed and v_{src} = source speed. Upper sign if moving toward other; lower sign if moving apart (treat each separately).

Intensity of Various Sounds		
Source	Sound Level (dB)	Intensity W/m^2
jet plane at 30m	140	100
pain threshhold	120	1
loud rock concert	120	1
siren at 30m	100	1×10^{-2}
truck traffic	90	1×10^{-3}
busy street	80	1×10^{-4}
noisy restaurant	70	1×10^{-5}
talk at 50 cm	60	3×10^{-6}
quiet radio	40	1×10^{-8}
whisper	30	1×10^{-9}
rustling leaves	10	1×10^{-11}
threshhold of hearing	0	1×10^{-12}

Material Properties at $20^o\ C$ and 1 atm		
Material	Sound speed m/s	Density kg/m^3
Air	343	1.20
Helium	1005	0.18
Pure Water	1440	1000
Sea Water	1560	1030
Iron, steel	5000	7800
Glass	4500	2800
Aluminum	5100	2700
Hardwood	4000	1000
Concrete	3000	1400

These were old homework problems from the previous version of the textbook that relate to the material from this chapter. (The previous book covered some of this material differently and developed some different equations since many quantities are related and the same equations can be written many different ways. Hopefully I've tracked down and replaced everything using equations and terms used in our current textbook.)

Example 3 : Consider a sound wave in air that has displacement amplitude $2.00 \times 10^{-2} mm$. Calculate the pressure amplitude for frequencies of (a) 150 Hz, (b) 1500 Hz, and (c) 15000 Hz. In each case, compare the result to the pain threshold, which is 30 Pa.

The pressure is related to the amplitude of the waves by $p_{max} = BkA$ where B is the bulk modulus of the material. For air, at 20^o at sea level, this is $1.42 \times 10^5\ Pa$. We need the wave number k in this equation though. An equation that directly links wave number and frequency is: $v = \omega/k$ so $k = \omega/v$. But $\omega = 2\pi f$ so $k = 2\pi f/v$. We know this sound wave is propagating through the air, so we'll take the wave speed to be 344 m/s (although technically it varies with temperature, and they didn't give us a temperature here).

Finally the amplitude was given in millimeters. There are 1000 mm in one meter, so this displacement amplitude represents $A = 2 \times 10^{-5}\ m$.

(a) At $f = 150 Hz$ we have $k = (2)(\pi)(150)/(344) = 2.74\ rad/m$. For the given displacement amplitude of the wave, then, the maximum pressure this sound generates is $p_{max} = BkA = (1.42 \times 10^5)(2.54)(2 \times 10^{-5}) = 7.78\ Pa$. (All the individual terms were in proper metric units, so the final answer will come out in the proper metric units for pressure, which is pascals (i.e. newtons per square meter). This is well below the pain threshold.

(b) At $f = 1500\ Hz$ we are multiplying the value of f by 10 compared to part (a). Since $k = 2\pi f/v$, that means the wave-number will also be 10 times larger. Since $p_{max} = BkA$, that means the maximum pressure will also be ten times larger so here $p_{max} = 77.8\ Pa$, which is over the pain threshold of 30 Pa.

(c) At $f = 15000\ Hz$ we are multiplying the value of f by 10 compared to part (b), so by the same arguments given there, we'll end up multiplying the maximum pressure by yet another factor of ten, resulting in 778 Pa, or about 26 times the pain threshold.

One thing to note here. $15,000\ Hz$ is within the range of human hearing. Even a minuscule displacement amplitude of 0.02 mm would produce a disabling pressure. So the actual sounds we hear in everyday life must have displacement amplitudes much smaller than this, showing how sensitive the ear is.

Example 4 : A loud factory machine produces sound having a displacement amplitude of $1.00\mu m$, but the frequency of this sound can be adjusted. In order to prevent ear damage to the workers (which will certainly occur by a pressure of 30.0 Pa), the maximum pressure amplitude of the sound waves is limited to 10.0 Pa. Under the conditions of this factory, the bulk modulus of air is $1.42 \times 10^5 Pa$. What is the highest frequency sound to which this machine can be adjusted without exceeding the prescribed limit? Is this frequency audible to the workers?

Similar to the previous problem, we have $p_{max} = BkA$, $k = 2\pi/\lambda = 2\pi f/v$ so $p_{max} = 2\pi f BA/v$ or rearranging, $f = vp_{max}/(2\pi BA)$. The machine is producing sound with $A = 1.00\mu m$ which is $A = 1 \times 10^{-6}\ m$. The bulk modulus of air is $1.42 \times 10^5\ Pa$, and the speed of sound is 344 m/s. So for the given desired maximum pressure of 10 Pa, we can calculate the desired frequency to be $f = \frac{(344)(10)}{(2)(\pi)(1.42\times10^5)(1\times10^{-6})} = 3856\ Hz$.

The range of human hearing generally runs from 20 Hz to $20,000\ Hz$, so this frequency will be audible.

Note: The maximum pressure is proportional to the frequency in $p_{max} = 2\pi f BA/v$ so any higher frequency will produce pressures above the limit of 10 Pa. (And lower frequencies will produce less painful pressures...)

Example 5 : (a) In a liquid with a density of 1300 kg/m^3, longitudinal waves with a frequency of 400 Hz are found to have a wavelength of 8.00 m. Calculate the bulk modulus of this liquid.
(b) A metal bar with a length of 1.50 m has a density of 6400 kg/m^3. Longitudinal sound waves travel from one end of the bar to the other in a time interval of $3.90 \times 10^{-4}\ s$. What is Young's modulus for this metal?

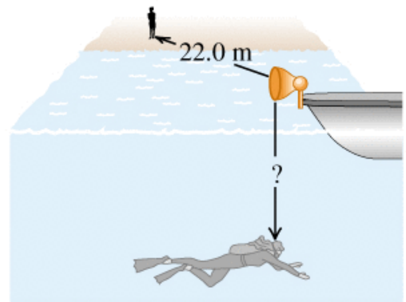
This problem basically shows a method of calculating physical properties such as the bulk modulus of a liquid, or the Young's modulus of a metal using sound.

(a) In a liquid, the wave speed is given by $v = \sqrt{B/\rho}$. We know the density of this liquid, so if we can find the wave speed, we can solve for B: $B = \rho v^2$. We do know the wavelength and frequency of these waves, so we can find the wave velocity to be $v = \lambda f = (8\ m)(400\ s^{-1}) = 3200\ m/s$. Thus $B = \rho v^2 = (1300)(3200)^2 = 1.33 \times 10^{10}\ Pa$. (All the individual terms were in proper units, so B will come out in proper metric units as well, and for a bulk modulus, those units are pascals.)

(Note: the given density sounds like a lot, but consider pure water. It has a density of 1 gram per cubic centimeter. Converting to standard metric units, this is 1000 kg/m^3, so this liquid is only 30 percent denser than water. One cubic meter of water (a cube only about three feet on each side, would have a mass of 1000 kg which is a weight of 2200 pounds - over a ton. Imagine the force an entire swimming pool full of water on the roof of a building is producing.)

(b) Here we are given information that lets us calculate the wave speed. The 'sound' took 3.90×10^{-4} seconds to travel 1.5 m so the wave speed must be $v = d/t = (1.50\ m)/(3.90 \times 10^{-4}\ s) = 3846\ m/s$. The wave speed in this type of material is given by $v = \sqrt{Y/\rho}$ though so we can rearrange this to find $Y = \rho v^2$. For this particular metal, then: $Y = (6400)(3846)^2 = 9.47 \times 10^{10}\ Pa$. (Again, if we use proper metric units for all the individual terms, the final answer will come out in the proper metric units for a Young's modulus, which is pascals or newtons/meter.)

Example 7 : A submerged scuba diver hears the sound of a boat horn directly above her on the surface of the lake. At the same time, a friend on dry land 22.0 m from the boat also hears the horn . The horn is 1.20 m above the surface of the water. What is the distance (labeled by the question mark in the figure) from the horn to the diver? (Both air and water are at 20^oC.)



For the person on the shore, the sound travels entirely through the air. For the person in the water, the sound travels the first 1.2 m in air, then the rest of the time it's under the water. Since for the first 1.2 m, the sound is traveling through the air for both listeners, let's discard that time interval and just look at the rest of the interval. Now we have sound traveling $22 - 1.2 = 20.8\ m$ through the air to the land-based listener, and some distance d entirely under the water to get to the diver.

The time interval $t = d/v$ is the same for each listener for this part of the path. For the land-based listener, $t = (20.8\ m)/(344\ m/s) = 0.0605\ s$. For the diver, $t = d/v$ but now we know that $t = 0.00605\ s$ and we also know that $v = 1482\ m/s$ (the speed of sound in water at 20 degrees) so $(0.0605\ s) = (d)/(1482\ m/s)$ or $d = 89.6\ m$. We can avoid some potential round off error by writing this a little differently and bypass calculating the time interval itself. The time intervals are the same so $d_a/v_a = d_w/v_w$ where 'a' represents the 20.8 m the sound is traveling in the air to the land-based listener during this interval and 'b' represents the unknown distance the sound is traveling in the water to the diver. Then $d_w = d_a \times \frac{v_w}{v_a} = (20.8\ m)\frac{1482}{344} = 89.61\ m$.

The problem didn't ask for how deep the diver was, though. They want to know how far the diver is from the horn. The direct distance from the horn to the diver then is the 1.20 m from the horn to the surface of the water, plus the 89.6 m that the diver is under the water, for a total distance of 90.8 m.

Example 11 : An 80.0-m-long brass rod is struck at one end. A person at the other end hears two sounds as a result of two longitudinal waves, one traveling in the metal rod and the other traveling in the air. What is the time interval between the two sounds? Take the speed of sound in air to be 344 m/s. (Use 8600 kg/m^3 for the density of brass and 9.00×10^{10} Pa for the Young's modulus of brass.)

The distance the sound travels is related to the velocity and time by $d = vt$, or $t = d/v$. In air, the sound wave travels 80.0 m at a speed of 344 m/s so $t = (80.0\ m)/(344\ m/s) = 0.2326\ s$.

In the brass rod, the sound will travel the same distance but at a considerably different speed. The speed of sound in the metal rod will be given by $v = \sqrt{Y/\rho} = \sqrt{(9 \times 10^{10})/(8600)} = 3235\ m/s$ (nearly ten times the speed of sound in air). The time in the brass rod then is $t = d/v = (80.0\ m)/(3235\ m/s) = 0.0247\ s$.

The time interval between these two sounds then is $(0.2326\ s) - (0.0247\ s)$ or $0.2079\ s$.

Example 14 : Use information from the table to answer the following questions about sound in air. At 20^oC the bulk modulus for air is $1.42 \times 10^5 Pa$ and its density is $1.20 kg/m^2$. At this temperature, what are the pressure amplitude (in Pa and atm) and the displacement amplitude: (a) for the softest sound a person can normally hear at 1000 Hz and (b) for the sound from a riveter at the same frequency. (c) How much energy per second does each wave deliver to a square 5.00 mm on a side?

Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, I (W/m^2)
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

We have equations that directly relate intensity to the pressure amplitude: $I = \frac{p_{max}^2}{2\sqrt{\rho B}}$ and to the displacement amplitude $I = \frac{1}{2}\sqrt{(\rho B)}\omega^2 A^2$. Rearranging these equations to solve for the pressure and displacement amplitudes, we have:
$p_{max} = \sqrt{2I\sqrt{\rho B}}$ and $A = \frac{1}{\omega}\sqrt{\frac{2I}{\sqrt{\rho B}}}$.
Throughout, we have a frequency of 1000 Hz which represents an angular frequency of $\omega = 2\pi f = 6283 rad/s$.

(a) The softest sound a person can hear has an intensity of $1 \times 10^{-12}\ W/m^2$. We have all the other constants we need, so we can calculate the pressure amplitude to be $p_{max} = 2.9 \times 10^{-5}\ Pa$. One atmosphere is a pressure of $1.013 \times 10^5\ Pa$ so this represents $p_{max} = 2.8 \times 10^{-10}\ atm$. The displacement amplitude for this sound would be $1.1 \times 10^{-11}\ m$.

(b) For the riveter, we can take a shortcut and notice that A is proportional to $\sqrt{I}$ so the new amplitude will be $(1.1 \times 10^{-11}\ m)\sqrt{\frac{3.2\times10^{-3}}{1.0\times10^{-12}}}$ or $6.2 \times 10^{-7}\ m$.

The maximum pressure p_{max} is also proportional to $\sqrt{I}$ so the value here is related to the value in (a) by $p_{max} = (2.9 \times 10^{-5}\ Pa)\sqrt{\frac{3.2\times10^{-3}}{1.0\times10^{-12}}}$ or 1.6 Pa which is equal to $1.6 \times 10^{-5}\ atm$.

(c) The intensity is the amount of power per area, so if we multiply the intensity by the area of interest, we find the amount of power passing through that area. The 5.00 mm size is roughly an ear hole, although here we're looking at a tiny square that is this length along each side. The area of this square shape is $A = L^2 = (5 \times 10^{-3}\ m)^2$ or $2.5 \times 10^{-5}\ m^2$. For the softer sound, then, this represents a power of $(I)(area) = (1 \times 10^{-12}\ W/m^2)(2.5 \times 10^{-5}\ m^2) = 2.5 \times 10^{-17}\ W$. For the riveter, this represents a power of $(I)(area) = (3.2 \times 10^{-3}\ W/m^2)(2.5 \times 10^{-5}\ m^2) = 8.0 \times 10^{-8}\ W$.

Example 20 : (a) When four quadruplets cry simultaneously, how many decibels greater is the sound intensity level than when a single one cries? (b) To increase the sound intensity level again by the same number of decibels as in part (a), how many more crying babies are required?

(a) In terms of decibels, the intensity is $\beta = (10\ dB)\log(I/I_o)$. Comparing two intensity levels, we derived that: $\Delta\beta = \beta_2 - \beta_1 = (10\ dB)\log(I_2/I_1)$. In this case, we are told that I_2 (the intensity with four crying babies) is exactly four times the initial intensity, so $\Delta\beta = (10\ dB)\log(4I/I) = (10\ dB)\log(4) = 6.02\ dB$. A 6 dB increase in the sound level means the underlying intensity went up by a factor of 4.

(b) In (a), we found that we have to multiply the number of babies by four to produce a 6 dB increase in the sound intensity. To get yet another 6 dB increase, we have to multiply by another factor of 4 giving us 16 babies altogether (an additional 12).

Example 21 : A baby's mouth is a distance of 30 cm from her father's ear and a distance of 1.50 m from her mother's ear. What is the difference between the sound intensity levels heard by the father and by the mother?

As we change the distance from the source, the intensity falls off as the square of the distance: $I = P/r^2$. Comparing two intensities then: $I_2/I_1 = r_1^2/r_2^2$. The difference in two intensities in terms of decibels, though, is given by $\Delta\beta = \beta_2 - \beta_1 = (10\ dB)\log(I_2/I_1)$, so here $\Delta\beta = (10\ dB)\log(r_1^2/r_2^2)$, or in the more traditional form: $\Delta\beta = (20\ dB)\log(r_1/r_2)$.

The intensity is going to be higher at the person who is closer, so let's label the father as '2' and the mother as '1' (that way $\Delta\beta = \beta_2 - \beta_1$ will be a positive number). So here $r_1 = 1.50\ m$ and $r_2 = 0.30\ m$ so $r_1/r_2 = 5$ and $\Delta\beta = (20\ dB)\log(5) = 13.98\ dB$.

A useful rule of thumb comes from this equation. Each time we double our distance from a source, the sound intensity drops by a factor of 6.02 dB (usually just rounded off to 6 dB). Each time we cut the distance from a source in half, the sound intensity goes up by 6.02 dB.

Example 24 : The fundamental frequency of a pipe that is open at both ends is 594 Hz. (a) How long is the pipe? (b) If one end is now closed, find the wavelength of the new fundamental. (c) If one end is now closed, find the frequency of the new fundamental.

(a) For an open pipe, the fundamental frequency is $f_1 = \frac{v}{2L}$. Here we are given f_1 and we know the speed of sound to be 344 m/s so the only unknown is the length of the pipe. Rearranging the equation, we have $L = \frac{v}{2f_1} = (344)/(2 \times 594) = 0.290\ m$.

(b) If we close one end, we must have a node at that end, and an anti-node at the open end (with no other nodes or anti-nodes between, since this is the fundamental). The distance between a node and an anti-node represents one quarter of a wavelength. So here this quarter of a wavelength must be L meters long: $\lambda/4 = 0.290\ m$ or $\lambda = 1.16\ m$.

(c) If we close one end, the fundamental frequency of this 'closed pipe' is given by $f_1 = \frac{v}{4L}$. But we can write this as $f_1 = \frac{1}{2} \times \frac{v}{2L}$. That last fraction is just the fundamental frequency for the OPEN pipe that we were given though, so for the close pipe, $f_1 = \frac{1}{2}(594) = 297\ Hz$.

Example 27 : The human vocal tract is a pipe that extends about 17 cm from the lips to the vocal folds (also called 'vocal cords') near the middle of your throat. The vocal folds behave rather like the reed of a clarinet, and the vocal tract acts like a closed pipe. Estimate the first three standing-wave frequencies of the vocal tract. Use $v = 344m/s$. (The answers are only an estimate, since the position of lips and tongue affects the motion of air in the vocal tract.)

For a closed pipe, the fundamental frequencies are given by $f_n = n\frac{v}{4L}$ for $n = 1, 3, 5, ...$ The lowest frequency then would be $f_1 = (344\ m/s)/(4 \times 0.17\ m) = 505.88\ Hz$. The next two would be $f_3 = 3f_1 = 1517.6\ Hz$ and $f_5 = 5f_1 = 2529\ Hz$.

(So here we have a series of particular frequencies that will cause standing waves to form. For these frequencies, then, we have nodes putting pressure on the vocal tract at fixed locations. I imagine then it might be more difficult for a singer to maintain one of these frequencies, than to maintain other frequencies. Any singers want to comment? Are there certain frequencies that are harder to maintain for a long time? On the other hand, the throat is not a perfect cylinder, so this effect may be blurred out..)

Example 28 : The auditory canal of the ear is filled with air. One end is open, and the other end is closed by the eardrum. A particular person's auditory canal is 2.40 cm long and can be modeled as a pipe. (a) What is the fundamental frequency and wavelength of this person's auditory canal? Is this sound audible? (b) Find the frequency of the highest audible harmonic of this person's canal. Which harmonic is this?

(a) We will model the ear as a closed pipe of length $L = 0.024\ m$. For a closed pipe, $\lambda_1 = 4L$ so in the ear canal, $\lambda_1 = (4)(0.024\ m) = 0.096\ m$. The fundamental frequency will be $f_1 = \frac{v}{4L} = \frac{344}{(4)(0.024)} = 3583.33\ Hz$. This is in the audible range of 20 to $20,000\ Hz$.

(b) For closed pipes, the fundamental frequencies are $f_1 = \frac{v}{4L}$ and $f_n = nf_1$ where $n = 1, 3, 5, ...$ If we set f_n to $20,000\ Hz$ then nominally $n = f_n/f_1 = (20000)/(3583.33) = 5.58$. Well n can't be a fraction, so it looks like $n = 5$ will be the highest (odd) value that n can have and still produce a frequency that is inside the human hearing range. (That means that there are only three frequencies: $f_1 = 3583.33\ Hz$, $f_3 = 3f_1 = 10,750\ Hz$, and $f_5 = 5f_1 = 17,917\ Hz$ which will produce standing waves in the ear canal. (Something special should probably happen at these frequencies. Perhaps they would be more painful since we would have standing waves in the ear canal, producing pressure maxima at fixed locations. A real ear canal isn't a perfect cylinder of the given length, so these variations probably eliminate whatever effect would be produced here.)

Example 31 : You blow across the open mouth of an empty test tube and produce the fundamental standing wave of the air column inside the test tube. The speed of sound in air is 344 m/s and the test tube acts as a closed pipe. (a) If the length of the air column in the test tube is 14.0 cm, what is the frequency of this standing wave? (b) What is the frequency of the fundamental standing wave in the air column if the test tube is half-filled with water?

With a closed pipe, the fundamental wavelength will be $\lambda_1 = 4L = (4)(0.14\ m) = 0.56\ m$. This implies a frequency of $f_1 = v/\lambda = (344\ m/s)/(0.56\ m) = 614.3\ Hz$. Half filling the test tube with water cuts the length of the air column in half to the wavelength of the first fundamental will be cut in half, meaning that the frequency will be multiplied by 2, producing a sound of frequency $(2)(613.4\ Hz) = 1228\ Hz$.

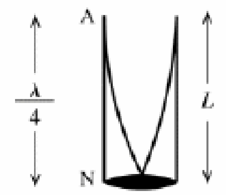


Figure 16.31

Example 38 : Two guitarists attempt to play the same note of wavelength 6.50 cm at the same time, but one of the instruments is slightly out of tune and plays a note of wavelength 6.52 cm instead. What is the frequency of the beat these musicians hear when they play together?

The beat frequency created by two nearby frequencies is $f_b = |f_1 - f_2|$ and we could convert each of the given wavelengths into frequencies and just compute it directly. Since we often have sounds in terms of wavelengths instead of frequencies, it is convenient to derive a general equation in terms of the wavelengths directly.

Since $v = f\lambda$, we have $f = v/\lambda$. So here, $f_b = |f_1 - f_2|$ becomes $f_b = |\frac{v}{\lambda_1} - \frac{v}{\lambda_2}|$ or $f_b = v|\frac{1}{\lambda_1} - \frac{1}{\lambda_2}|$. But we can write this as $f_b = v|\frac{\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}|$ or since the wavelengths are positive, finally: $f_b = v\frac{|\lambda_2 - \lambda_1|}{\lambda_1 \lambda_2}$.

Here, $\lambda_1 = 0.0650\ m$ and $\lambda_2 = 0.0652\ m$ so $f_b = (344)\frac{(0.0002)}{(0.0650)(0.0652)} = 16.2\ Hz$.

Example 39 : Two organ pipes, open at one end but closed at the other, are each 1.14 m long. One is now lengthened by 2.00 cm. Find the frequency of the beat they produce when playing together in their fundamental.

The fundamental frequency of a closed pipe is $f_1 = v/(4L)$. The beat frequency we usually write as: $f_b = |f_1 - f_2|$ where f_1 and f_2 label the two frequencies involved. We've already used the subscript 1 to represent the fundamental though, so let's rewrite this to avoid confusion as: $f_{beat} = |f_a - f_b|$ where f_a and f_b are now the two frequencies under consideration.

Since the closed pipe has fundamentals related to the length as $f_1 = v/(4L)$ we can rewrite the beat frequency in terms of the length of the pipes directly (since that's what we're changing): $f_{beat} = \frac{v}{4}|\frac{1}{L_a} - \frac{1}{L_b}|$ or further as $f_{beat} = v\frac{|L_a - L_b|}{4L_a L_b}$.

In this case, $L_a = 1.14\ m$ and $L_b = 1.16\ m$ so $f_{beat} = v\frac{|L_a - L_b|}{4L_a L_b}$ or $f_{beat} = (344)\frac{(0.02)}{(4)(1.14)(1.16)} = 1.3\ beats/s$.

Example 41 : On the planet Arrakis a male ornithoid is flying toward his mate at a speed of $25.0 m/s$ while singing at a frequency of 1200 Hz. If the stationary female hears the sound at a frequency of 1240 Hz, what is the speed of sound in the atmosphere of Arrakis?

(NOTE: updated to reflect this book's version of the Doppler equation)

The Doppler equation relates the frequencies of the source and listener to their speeds (assuming the air is stationary and not moving also): $f' = f \cdot (v \pm v_{obs})/(v \mp v_{src})$ where v = sound speed, v_{obs} = observer speed and v_{src} = source speed. Upper sign if moving toward other; lower sign if moving apart (treat each separately).

Here, the listener is not moving so $v_{obs} = 0$. The source is moving TOWARDS the listener, so we'll use the upper sign on that term, leading to:

$f' = f \cdot (v + 0)/(v - v_{src})$

The source is moving at a speed of $v_{src} = 25\ m/s$, the source frequency is $f = 1200\ Hz$ which the listener hears as $f' = 1240\ Hz$ so:

$1240 = 1200 \frac{v}{v-25}$

Mutiplying both sides by $(v - 25)$ and collecting terms, we can rearrange this into $v = \frac{(25)(1240)}{1240-1200} = 775\ m/s$

Example 42 : A police car with its 300 Hz siren is moving away from a warehouse at a speed of $20.0 m/s$. What frequency does the driver of the police car hear reflected from the warehouse? (Use 344 m/s for the speed of sound in air.)

There are two steps here. First, the siren is moving away from the wall, so AT THE WALL, there will be a frequency of some (lower) value impinging on the wall. That same frequency will then bounce off the wall and head off towards the police car. Now we have THAT frequency heading towards the police car, which is moving away (producing a yet lower frequency when that sound reaches the moving car).

Here, we will need to apply the Doppler equation twice: first to figure out what frequency is hitting (and then radiating from) the wall, then what frequency that turns into when it arrives at the ears of the policeman.

What frequency arrives at (and reflects from) the wall?

$f' = f \cdot (v \pm v_{obs})/(v \mp v_{src})$ where v = sound speed, v_{obs} = observer speed and v_{src} = source speed. Upper sign if moving toward other; lower sign if moving apart (treat each separately).

Here the 'listener' is the (stationary) wall, so $v_{obs} = 0$. The source is emitting a frequency of $f = 300$ and is moving away from the listener, so we'll use the lower sign in the doppler equation on that term:

$f' = (300\ Hz) \cdot (344 + 0)/(344 + 20) = 283.5\ Hz$.

Frequency heard at the moving police car.

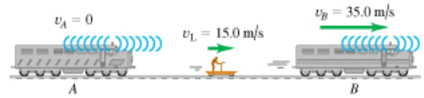
Now we have a 283.5 Hz source (coming from the stationary wall) and we need to find out what frequency is received at the moving police car.

$f' = f \cdot (v \pm v_{obs})/(v \mp v_{src})$ and now $f = 283.5\ Hz$; the source is not moving so $v_{src} = 0$; the listener is moving away from the source so we'll use the lower sign on that term, leading to:

$f' = (283.5\ Hz) \cdot (344 - 20)/(344 + 0) = 267.0\ Hz$.

We could take this problem one step further. A person inside the police car is being hit by two frequencies: the original 300 Hz siren, and the 267 Hz sound reflected from the wall of the building. This will produce a beat frequency of $f_b = |f_1 - f_2| = |300 - 267| = 33\ Hz$.

Example 43 : Two train whistles, A and B, each have a frequency of $392 Hz$. A is stationary and B is moving toward the right (away from A) at a speed of $35.0 m/s$. A listener is between the two whistles and is moving toward the right with a speed of $15.0 m/s$. No wind is blowing. Take the speed of sound to be 344 m/s. (a) What is the frequency from train A as heard by the listener? (b) What is the frequency from train B as heard by the listener? (c) What is the beat frequency detected by the listener?



(a) Train A:

$f' = f \cdot (v \pm v_{obs})/(v \mp v_{src})$ where v = sound speed, v_{obs} = observer speed and v_{src} = source speed. Upper sign if moving toward other; lower sign if moving apart (treat each separately).

Here the source is not moving, so $v_{src} = 0$. The observer is moving away from train A, so we'll use the lower sign on that term, leading to: $f' = (392\ Hz)(344 - 15)/(344 + 0) = 375\ Hz$.

(b) Train B: This time, we have a source emitting $f = 392\ Hz$. The listener is moving towards the source of the sound, so we'll use the upper sign on the v_{obs} term. The source is moving away from the listener, so we'll need the lower sign on the v_{src} term. This leads to:

$f' = (392\ Hz)(344 + 15)/(344 + 35) = 371\ Hz$

Note that in both cases here, the frequency heard by the (moving) person in the middle was less than the frequency the whistles were putting out.

(c) The beat frequency created here will be $f_b = |f_1 - f_2| = |375 - 371| = 4\ Hz$.

Example 44 : A railroad train is traveling at a speed of 25.0 m/s in still air. The frequency of the note emitted by the locomotive whistle is 400 Hz. (Assume the speed of sound in air is 344 m/s.) (a) What is the frequency and wavelength of the sound waves in front of the locomotive? (b) What is the frequency and wavelength of the sound waves behind the locomotive?

(a) In this case, the listener is not moving so $v_{obs} = 0$. The source is moving towards the listener, so we'll use the upper sign on the v_{src} term:

$f' = f \cdot (v \pm v_{obs})/(v \mp v_{src})$ becomes: $f' = (400\ Hz)(344 + 0)/(344 - 25) = 431\ Hz$.

$f = v/\lambda$ so $\lambda = v/f = (344)/(431) = 0.798\ m$.

(b) In this case, the listener is not moving so $v_{obs} = 0$. The source is moving away from the listener, so we'll use the lower sign on the v_{src} term:

$f' = f \cdot (v \pm v_{obs})/(v \mp v_{src})$ becomes: $f' = (400\ Hz)(344 + 0)/(344 + 25) = 373\ Hz$.

$\lambda = v/f = (344)/(373) = 0.922\ m$.

Example 46 : A sound source producing 1.00 kHz waves moves toward a stationary listener at one-half the speed of sound. (a) What frequency will the listener hear? (b) Suppose instead that the source is stationary and the listener moves toward the source at one-half the speed of sound. What frequency does the listener hear? (c) Why do these differ?

(a) In this case the source is emitting a frequency of $f = 1000\ Hz$. The observer is stationary so $v_{obs} = 0$. The source is moving towards the listener, so we'll use the upper sign on the v_{src} term.

$f' = f \cdot (v \pm v_{obs})/(v \mp v_{src})$ becomes: $f' = (1000\ Hz)(344 + 0)/(344 - 177) = 2000\ Hz$.

(b) In this case, the source is emitting a frequency of $f = 1000\ Hz$. The source is stationary, so $v_{src} = 0$. The listener is moving towards the source so we'll use the upper sign on the v_{obs} term.

$f' = f \cdot (v \pm v_{obs})/(v \mp v_{src})$ becomes: $f' = (1000\ Hz)(344 + 177)/(344 + 0) = 1500\ Hz$.

(c) Let's think of what's happening from the point of view of the listener in each case. In (a), the air isn't moving and the source is moving towards the listener, producing the higher frequency. In (b), the listener is moving so he is feeling the air rush past him at half the speed of sound. So to the listener, the source is moving towards him AND the air that sound is in is also moving towards him, resulting in a different frequency.

Example 50 : In the not-too-distant future, it should be possible to detect the presence of planets moving around other stars by measuring the Doppler shift in the infrared light they emit. If a planet is going around its star at $50.0km/s$ while emitting infrared light of frequency $3.330 \times 10^{14} Hz$, what frequency light will be received from this planet when it is moving directly away from us? (Note: Infrared light is light having wavelengths longer than those of visible light.)

The Doppler equation when the wave speed is the speed of light is: $f_R = \sqrt{\frac{c-v}{c+v}} f_S$, so here: $f_R = \sqrt{\frac{3.0\times10^8 - 50000}{3.0\times10^8+50000}} (3.333 \times 10^{14}\ Hz = 3.329 \times 10^{14}\ Hz$.

For velocities that are small relative to the speed of light (which $50,000\ m/s$ certainly is), this is not a very convenient (or accurate) way to do this calculation. It relies on the calculator retaining a lot of digits.

More normally, we do these by calculating the **change** in frequency. So we let $f_R = f_S + \Delta f$. Inside the square root, we divide the numerator and denominator by c, resulting in: $f_R = \sqrt{\frac{1-v/c}{1+v/c}} f_S$. But from a Taylor's series analysis, $\frac{1}{1+x} = 1 - x + (higher\ order\ terms)$ if x is small, so $\frac{1-v/c}{1+v/c}$ is approximately equal to $(1 - v/c)^2$. Going back to our equation for f_R, then: $f_R = \sqrt{(1 + v/c)^2} f_S$ or $f_R = (1 + \frac{v}{c}) f_S = f_S + \frac{v}{c} f_S$ and finally we can pick off $\Delta f = \frac{v}{c} f_S$. This lets us calculate the CHANGE in the frequency with much greater precision.

Example 64 : Not test material, but an interesting case in how the properties of a material limit what we do do with it.

One type of steel has a density of 7800 kg/m^3 and a breaking stress of 7.0×10^8 N/m^2. A cylindrical guitar string is to be made out of a quantity of steel with a mass of 4.00 g. (a) What is the length and radius of the longest and thinnest string that can be placed under a tension of 900 N without breaking? (b) What is the highest fundamental frequency that this string could have?

(a) The 'breaking stress' tells us what tension a wire of a given cross sectional area can maintain without breaking. If a single wire can support some weight before breaking, then two identical such wires, or one wire with twice the area of the first, would be able to support twice the weight without breaking. So $(breaking\ stress) \times (area) = (maximum\ tension\ before\ breaking)$. For this particular type of steel: $(7 \times 10^8\ N/m^2)(A) = (900\ N)$ from which $A = 1.2857 \times 10^{-6}\ m^2$. But $A = \pi r^2$ so we can convert this to a radius of $r = 6.40 \times 10^{-4}\ m$ ($0.64\ mm$). Anything thinner and the force/area would exceed the breaking stress. This is the thinnest we can make this wire and have it (barely) survive a tension of 900 N.

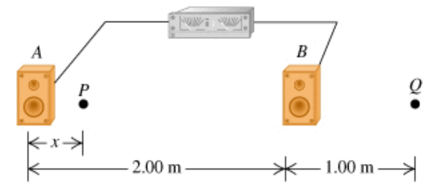
The mass of this 'cylinder' of steel will be its density times its volume: $M = \rho(\pi r^2)(L)$ or $M = \rho AL$. It is more accurate to use this second form because we calculated A, then used A to calculate r. Each time we do a calculation, we introduce more numerical and round-off error. $M = \rho AL$ so $0.004\ kg = (7800\ kg/m^3)(1.2857 \times 10^{-6}\ m)(L) = 0.399\ m$ (or 40 cm, which is a bit shorter than the length of a typical real guitar string).

(b) A guitar string is locked down at both ends, so the lowest frequency mode here will be $f_1 = \frac{v}{2L}$ but $v = \sqrt{F/\mu}$ so $f_1 = \frac{v}{2L}\sqrt{\frac{F}{m/L}}$ This simplifies to: $f_1 = \frac{1}{2}\sqrt{\frac{F}{ML}}$ or $f_1 = \frac{1}{2}\sqrt{\frac{900}{(0.004)(0.399)}} = 375.5\ Hz$.

If we lower the tension in the string, it will produce a lower frequency. If we raise the tension it would produce a higher frequency, but we did this calculation for the string being under the maximum possible tension before it breaks, so this string can't be adjusted to produce any higher frequency. If we do need to tune it to a higher frequency, we'll need to replace the string with something made of a different material.

Interference Examples : **Not on this test - we'll come back to this topic in a later chapter**

Example 33 : Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. Consider point Q along the extension of the line connecting the speakers, 1.00 m to the right of speaker B. Both speakers emit sound waves that travel directly from the speaker to point Q. (a) What is the lowest frequency for which constructive interference occurs at point Q? (b) What is the lowest frequency for which destructive interference occurs at point Q?



Constructive interference occurs when the difference in the distances of each source from the listener is exactly an integer number of wavelengths. The interference is destructive when this difference of path lengths is half a wavelength plus an integer number of wavelengths. The lowest frequency implies the largest wavelength. Here the listener is 3 m from speaker A and 1 m from speaker B, so the difference in the paths is 2 m.

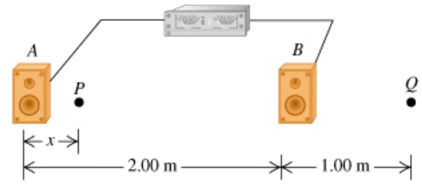
(a) In the case of constructive interference, we need to fit some integer number of wavelengths in the path difference, so $2_m = n\lambda$ so $\lambda = (2_m)/n$. The frequency is $f = v/\lambda$ so this gives us frequencies of $f = (344)/(2/n)$ or $177n$. The lowest frequency then will be just 177 Hz.

(b) In the case of destructive interference, we need the path difference to represent a half a wavelength plus some integer number of waves or $(2\ m) = (n + \frac{1}{2})\lambda$ or $\lambda = 2/(n + \frac{1}{2})$ for $n = 0, 1, 2, ...$ The corresponding frequencies then will be $f = v/\lambda = (344)/(2/(n + \frac{1}{2})) = (n + \frac{1}{2})(344/2)$ or $f = (n + \frac{1}{2})177\ Hz$. Since our series runs from $n = 0, 1, 2, ...$, the lowest frequency then will be (at $n = 0$) 177/2 or 86 Hz.

The book argues this slightly differently using a different series for n. They argue that for destructive interference, the path difference has be half a wavelength plus multiples of a full wavelength, so the path difference of two meters represents the series: $\frac{1}{2}\lambda$, $(\frac{1}{2} + 1)\lambda = \frac{3}{2}\lambda$, $(\frac{1}{2} + 2)\lambda = \frac{5}{2}\lambda$, and so on. They then write this as $n\frac{\lambda}{2}$ for $n = 1, 3, 5, ...$ These have to equal the path difference of 2 m so we can write $n\frac{\lambda}{2} = 2$ for $n = 1, 3, 5, ...$ or $\lambda = 4/n$ for $n = 1, 3, 5, ...$

The corresponding frequencies then are: $f = v/\lambda = \frac{v}{(4/n)} = nv/4 = (n)(344)/(4) = 86n$, again for $n = 1, 3, 5, ...$ Again, the lowest frequency comes out to be (at $n = 1$ here): 86 Hz.

Example 34 : Two loudspeakers, A and B, are driven by the same amplifier and emit sinusoidal waves in phase. Speaker B is 2.00 m to the right of speaker A. The frequency of the sound waves produced by the loudspeakers is 206 Hz. Consider point P between the speakers and along the line connecting them, a distance x to the right of speaker A. Both speakers emit sound waves that travel directly from the speaker to point P. (a) For what values of x will destructive interference occur at point P? (b) For what values of x will constructive interference occur at point P? (c) Interference effects like those in parts A and B are almost never a factor in listening to home stereo equipment. Why not?



Constructive interference occurs when the difference in the distances of each source from the listener is exactly an integer number of wavelengths. The interference is destructive when this difference of path lengths if half a wavelength plus an integer number of wavelengths. The listener is located BETWEEN the two speakers, so the values of x are limited to the range from 0 to 2.0 m. Sound travels from A to the listener, and from B to the listener. The path length from A to the listener is x. The path length from B to the listener is $L - x$ (where L is the distance between the two speakers, here 2.0 m). (The path length is always some positive number of meters: the DISTANCE from the source to the listener). The difference in the path length (we'll call that Δl) then is $\Delta l = (L-x)-x = L-2x$ or rearranging $x = (L-\Delta l)/2$ or for this specific situation, $x = (2-\Delta l)/2$ or $x = 1-0.5\Delta l$. It's the difference in path length (Δl) that has to be any multiple of the wavelength (for constructive interference) or equal to half a wavelength plus some multiple of wavelengths (for destructive interference). The multiples in each case can be either positive or negative, representing either one of the speakers being further away. Once we have Δl from those considerations, we can find the corresponding values for x.

The frequency of 206 Hz represents a wavelength of $\lambda = v/f = (344)/(206) = 1.67$ m.

(a) For destructive interference, $\Delta l = (n + \frac{1}{2})\lambda$. For $n = 0$, we have $\Delta l = \frac{1}{2}\lambda = (0.5)(1.667) = 0.833$ m. This gives $x = 1.0 - \Delta l/2 = 0.58$ m. For $n = 1$, we have $\Delta l = (3/2)\lambda = 2.50$ m so $x = -0.25$ m which is outside the line between the two speakers. Any higher positive values of n will just put this node further and further away. For $n = -1$, we have $\Delta l = (-1 + \frac{1}{2})\lambda = -0.833$ m from which $x = 1.0 - \Delta l/2 = 1.42$ m. You should check but any other values again put x outside the range of $[0, 2\ m]$ that is required for the point to be between the two speakers.

So we have exactly two points between the speakers where destructive interference will occur: $x = 0.58$ m and $x = 1.42$ m. Note that the second position is exactly 0.58 m to the LEFT of the right speaker, so we have symmetrically located positions here.

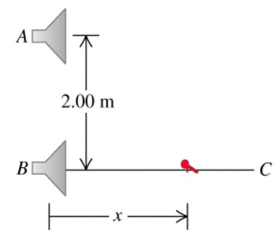
(b) For constructive interference, the path difference has to be any integer multiple of the wavelength, that is: $\Delta l = n\lambda$ where $(n = 0, \pm1, \pm2, ...)$. $n = 0$ implies $\Delta l = 0$ (i.e. there are the exact same number of waves from A to the listener as there are from B to the listener, so the listener must be the same distance from each speaker.) But $x = 1.0 - \Delta l/2$ so $\Delta l = 0$ implies that $x = 1.0$ m. (This is exactly at the midway point between the two speakers, so that makes sense. The sound from each speaker arrives at the same time - the waves will always be in phase, so they will definitely be constructively interfering.)

For $n = 1$ we have $\Delta l = (1)(1.667) = 1.667\ m$ so $x = 1.0 - (1.667)/2 = 0.167\ m$. For $n = -1$ we have $\Delta l = (-1)(1.667) = -1.667\ m$ so $x = 1.0 - (-1.667)/2 = 1.883\ m$. Other values of n put the values of x outside of the range $[0, 2\ m]$.

(Again, note that these two points are symmetrically located: the first is $0.167\ m$ to the right of the left speaker, and the second is $0.167\ m$ to the left of the right speaker.)

(c) Treating speakers as point sources is a poor approximation for these dimensions, and sound reaches these points after reflecting off the floor, ceiling, walls, and any objects in the room, so it is not likely one could perceive these interference effects.

Example 70 : (Not for the first test; we'll see this again later.) Two identical loudspeakers are located at points A and B, 2.00 m apart. The loudspeakers are driven by the same amplifier and produce sound waves with a frequency of 784 Hz. Take the speed of sound in air to be 344 m/s. A small microphone is moved out from point B along a line perpendicular to the line connecting A and B (line BC in the figure). (a) At what distances from B will there be destructive interference? (b) At what distances from B will there be constructive interference? (c) If the frequency is made low enough, there will be no positions along the line BC at which destructive interference occurs. How low must the frequency be for this to be the case?



(a) Destructive interference occurs when the path difference is equal to half a wavelength (plus any positive or negative integer number of wavelengths). The wavelength of this sound is $\lambda = v/f = (344)/(784) = 0.439\ m$.

Let's set things up symbolically at first. If the separation between the speakers is denoted by h, then the distance from B to the listener is x and the distance from A to the listener is $\sqrt{h^2+x^2}$. The path difference then is $\sqrt{h^2+x^2} - x$ (which will always be positive) and that result must be an odd-integer multiple of the wavelength. So: $\sqrt{h^2+x^2} - x = n(\lambda/2)$ where $n = 1, 3, 5, ...$ We're looking for the values that x can have. If we add x to both sides of this equation and square the resulting equation, we have $h^2 + x^2 = (x + \frac{n\lambda}{2})^2 = x^2 + x\frac{n}{\lambda} + (\frac{n\lambda}{2})^2$. We can now subtract x^2 from each side and rearrange to solve for x: $x = \frac{h^2}{n\lambda} - \frac{n\lambda}{4}$. Plugging in the specific values for h and λ here: $x = \frac{9.112}{n} - 0.1098n$. $n = 1$ gives $x = 9.00\ m$. $n = 3$ gives $x = 2.71\ m$. $n = 5$ gives $x = 1.27\ m$. $n = 7$ gives $x = 0.53\ m$. $n = 9$ gives $x = 0.026\ m$. Any larger values for n produce negative values for x (points to the left of the speakers, and again we were only interested in the $x > 0$ solutions. So those five points are the only places where destructive interference will occur.

(b) For constructive interference, the path difference must be equal to any integer number of full wavelengths (even or odd), so: So: $\sqrt{h^2+x^2} - x = n\lambda$ where $n = 1, 2, 3, ...$ ($n = 0$ means there are zero wavelengths between the two, which would imply they are on top of one another, so we know this series has to start with at least $n = 1$.) Doing the same sort of algebraic trickery above, we arrive at $x = \frac{h^2}{2n\lambda} - \frac{n\lambda}{2} = \frac{4.556}{n} - 0.2195n$.

At $n = 1$ we have $x = 4.34\ m$. At $n = 2$ we have $x = 1.84\ m$, and so on with the remaining solutions being 0.86 m and 0.26 m.

(c) It is claimed that if we make the frequency low enough, there will not be any points where destructive interference occurs. The hand-waving argument here proceeds as follows: The lowest frequency for which destructive interference will occur at $x = 0$ will be when h is exactly half a wavelength. If $\lambda/2$ is any larger than h we can't have any destructive interference at all. A more mathematical approach would be to take the final equation we derived for the locations of the points x for destructive interference: $x = \frac{h^2}{n\lambda} - \frac{n\lambda}{4}$ and couple it with the fact that x has to be positive. Then for no solutions to exist, we would want the right hand side of that equation to be less than zero. $\frac{h^2}{n\lambda} - \frac{n\lambda}{4} < 0$ implies that $\frac{h^2}{n\lambda} < \frac{n\lambda}{4}$. Rearranging this we arrive at: $\frac{2h}{n} < \lambda$. We want to find the lowest frequency, but that implies the largest possible λ. The largest λ occurs when $n = 1$ so $\lambda > 2h$ is the solution again. $f = v/\lambda$ to larger values of the wavelength imply lower values for f. So $f < \frac{v}{2h}$ of $f < 86\ Hz$ here.