Physics 2233 : Chapter 33 Examples : Lenses and Optical Instruments

NOTE: these examples are mostly from our previous book, which used different symbols for the object and image distances. I've tried to track them all down, but just in case I missed any:

Symbol Conventions		
Variable	Current book	Old Book
Object Distance	d_o	s
Image Distance	d_i	s'
Focal distance	f	f
Object size	h_o	y
Image size	h_i	y'
Corresponding Equations		
Lens Equation	$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$	$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$
Magnification	$m = h_i/h_o = -d_i/d_o$	$m = y'/y = -s'/s$
inverted image	$m < 0 \Rightarrow h_i < 0$	$m < 0 \Rightarrow y' < 0$

Thin Spherical Lenses

See problems 24 and 34 for sample ray diagrams. See book page 963 for 'sign rules'.

Lensmaker's equation : n is the index of refraction of the lens

in air : $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$

in other medium of index of refraction n_o : $\frac{1}{f} = \frac{n-n_o}{n_o}(\frac{1}{R_1} + \frac{1}{R_2})$

Converging Lens (thicker in the middle)

Diverging Lens (thinner in the middle)

Human Eye

'normal eye' : near-point of 25 cm, far-point at infinity

'nearsighted' : far-point closer in (corrected with diverging lens)

'farsighted' : near-point farther out (corrected with converging lens)

Magnifiers (converging lens) $M = \frac{\theta'}{\theta}$

Eye relaxed (i.e. focused at ∞) : $M = N/f$ where N is the near-point distance

Eye focused at near point: $M = \frac{N}{f} + 1$

Example 23 : An insect 3.75 *mm* tall is placed 22.50 *cm* to the left of a thin planoconvex lens. The left surface of this lens is flat, the right surface has a radius of curvature of magnitude 13.0 *cm*, and the index of refraction of the lens material is 1.70. (a) Calculate the location and size of the image this lens forms of the insect. Is it real or virtual? Upright or inverted? (b) Repeat part (a) if the lens is reversed.

(Planoconvex: one side is flat, the other side bulges out in the middle.)

(a) Our labeling scheme is that the '1' surface is the one that the light rays hit first, so here R_1 is infinity. (A flat surface is basically a spherical surface in the limit of infinitely large radius of curvature.)

On the right side, the center of curvature is out to the left of the lens. This book uses a convention such that the radii for a double-convex lens (bulging out on both sides) are both positive, so here $R_2 = +13.0\ cm$.

The object is on the left of this lens (same side as the rays incoming to the lens) so $d_o = +22.50\ cm$.

We can use the Lensmaker's equation to compute the focal distance for this lens: $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2}) = (1.70 - 1.00)(0 + \frac{1}{+13\ cm}) = (0.70)/(+13\ cm)$ from which $f = (13.0\ cm)/0.7 = 18.6\ cm$. This is positive so this is a **converging** lens.

We can now find the image distance: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or $\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$. Putting the right side over a common denominator: $\frac{1}{d_i} = \frac{d_o - f}{d_o f}$ or rearranging: $d_i = \frac{d_o f}{d_o - f} = \frac{(22.5)(18.6)}{22.5 - 18.6} = 107\ cm$. (That's positive, so the image is on the same side as the outgoing rays, making it a real image.)

The magnification factor will be $m = -d_i/d_o = -(106)/(22.5) = -4.76$. (Negative, so the image will be inverted.)

Since $m = h_i/h_o$ also, the size of the image will be $h_i = mh_o = (-4.76)(3.75\ mm) = -17.8\ mm$.

Summary: the image will be 17.8 *mm* tall, inverted, and real.

(b) **We now turn the lens around.**

Now the curved side is facing the bug so our '1' side is the curved one and the '2' side is the flat one. The center of curvature of the left side of the lens is to the right which is the 'right' side for a converging lens (the reference lens that this book uses to define signs for the radii) so $R_1 = +13\ cm$. The flat side will have an infinite radius of curvature.

We can use the Lensmaker's equation again to compute the focal distance for this lens: $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2}) = (1.70 - 1.00)(\frac{1}{+13\ cm} + 0) = (0.70)/(+13\ cm)$ from which $f = (13.0\ cm)/0.7 = 18.6\ cm$. This is exactly the **same value** we had before, and the object distance is the same, so all of the other calculations above will proceed without change. The image of the bug, again, will be 17.8 *mm* tall, inverted, and real.

This is an important generic result when we have 'thin lenses' (whether converging or diverging). It implies that we can flip the lens the other way around and it will have no effect. (There will be some effects but the differences are in the higher order terms we ignored when we used the thin lens / small angle approximation to derive the lens-related equations.)

Example 24 : A lens forms an image of an object. The object is 16.0 cm from the lens. The image is 12.0 cm from the lens on the same side as the object. (a) What is the focal length of the lens? Is the lens converging or diverging? (b) If the object is 8.5 mm tall, how tall is the image? Is it upright or inverted? (c) Draw a principal ray diagram.

A converging lens will have $f > 0$ so that rays coming from infinitely far away from one side will get focused on the other side. Conversely, a diverging lens will have $f < 0$. So from the sign of f, we can determine the type of lens.

(a) Here we have the image and object distances. The object is 16.0 cm from the lens, so $d_o = 16.0\ cm$. (If only a single lens is involved, the object will always on the same side as the light rays coming in towards the lens, so d_o is always positive, no matter what the configuration - again, as long as we only have one lens involved.) For the image though, we have the image on the same side as the object in this case. Lenses don't turn light around (like a mirror does) so that means the image is **not** on the same side as the outgoing rays, so d_i is negative here: $d_i = -12.0\ cm$.

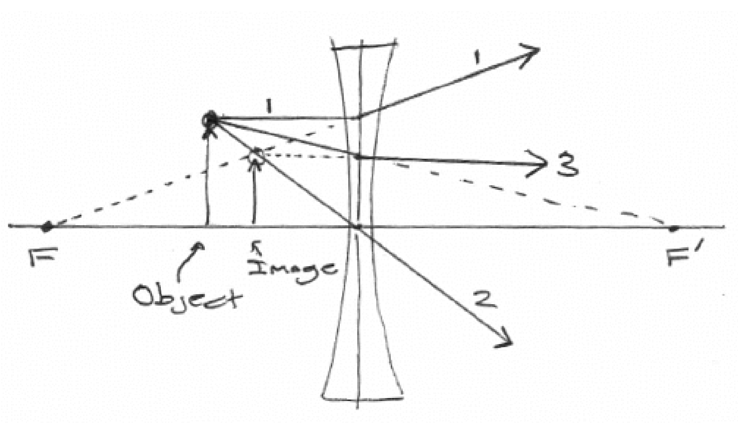
For a lens: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$. Putting the left hand side over a common denominator: $\frac{d_o + d_i}{d_o d_i} = \frac{1}{f}$ or finally $f = \frac{d_o d_i}{d_o + d_i}$. For this particular case: $f = \frac{(16\ cm)(-12\ cm)}{16\ cm + (-12\ cm)} = -48.0\ cm$.

Since $f < 0$, this must be a **diverging** lens.

(b) The magnification factor $m = -d_i/d_o = -(-12)/(16) = +0.750$ so the object is shrunk by this factor. $m = h_i/h_o$ so $h_i = mh_o = (0.750)(8.5\ mm) = 6.38\ mm$. Since m was positive, the image will be upright.

(c) Principal ray diagram:

Ray 1 : a ray parallel to the axis should be refracted by the lens so that it goes through (or appears to have come from) the point F. From the arrowhead, we start drawing a line parallel to the axis. It hits the lens at some point. We now draw a line that goes through this point and the point F. The part of this line that is on the outgoing side will be where the light ray actually goes. The dotted part leading back to F is where this ray appears to be coming from.



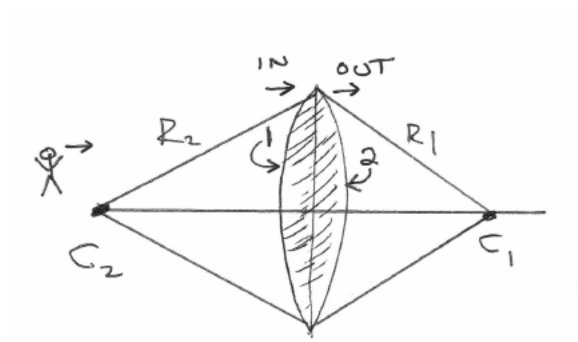
Ray 2 : A ray from the point through the vertex of the lens just keeps going in a straight line.

Ray 3 : A ray that heads directly towards F' (or appears to come from F') should become a ray parallel to the axis. So here we draw a line connecting the top of the arrow to F' and where it intersects the lens, it turns into a line parallel to the axis.

Where these three lines intersect or appear to intersect, that's where the top of the arrow is in the image. Based on the sketch, the image is upright, shrunk a bit, and virtual. (And the ray diagram results appear consistent with the numerical values that were given or calculated.)

Example 30 : Six lenses in air are shown in the figures below. Each lens is made of a material with index of refraction $n > 1$. Considering each lens individually, imagine that light enters the lens from the left. Show that the three lenses in the first figure (a) have **positive** focal lengths (and hence are **converging** lenses). Show that the three lenses in the second figure (b) have **negative** focal lengths (and hence are **diverging** lenses).

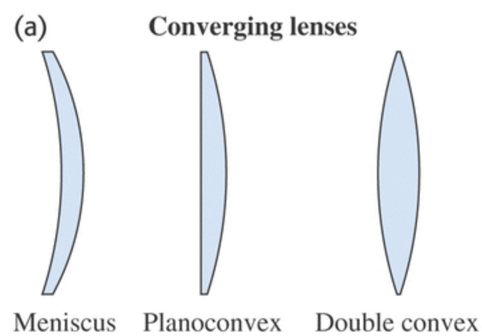
In this problem, the object is on the left side, so in all these lenses, the left will be the '1' side and the right will be the '2' side, when it comes to figuring out the signs of R_1 and R_2. This book uses the convention that both radii are positive when the lens is a double-convex type (bulging outward on both sides). If a center of curvature for a given face is on the 'wrong' side, that radius will be negative.



Converging lenses ($f > 0$) : Starting from the Lensmaker's equation, $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$ if f is to be positive, then (since $n > 1$) we can see that $\frac{1}{R_1} + \frac{1}{R_2} > 0$ (We can't really go much further here, since these radii may be negative. $1/4$ is smaller than $1/3$ but $-1/4$ is larger than $-1/3$ (since 'larger' means to the right on the number line...) Rather than derive special cases for one or both of the radii being negative, we'll just proceed with what we have.)

Diverging lenses ($f < 0$) : for these lenses (again since $n > 1$), we see that $\frac{1}{R_1} + \frac{1}{R_2} < 0$

Ultimately, we need to compare the inverses of the radii of curvature, being careful to account for the signs of R.



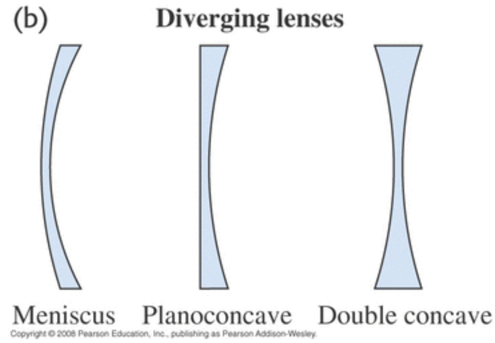
Meniscus: Both C_1 and C_2 are to the left of this lens, so $R_1 < 0$ and $R_2 > 0$. Side 1 is 'flatter' so the **magnitude** of R_1 will be larger. $|R_1|$ will be larger than $|R_2|$ which means that $|1/R_1|$ will be smaller than $|1/R_2|$ thus $\frac{1}{R_1} + \frac{1}{R_2}$ will be positive, making $f > 0$ and this must be a converging lens.

Planoconvex: This is just a special case of the lens we just did, where R_1 has increased to be infinitely large. Now $1/R_1 = 0$ but R_2 is still some positive number, so $\frac{1}{R_1} + \frac{1}{R_2}$ will still be positive, making $f > 0$ and this is also a converging lens.

Double convex: This is the 'reference' lens as far as our sign conventions go and both radii here are positive, so automatically $\frac{1}{R_1} + \frac{1}{R_2}$ will be positive, making $f > 0$ and again this is a converging lens.

(continued next page)

Meniscus: In this case, the centers of curvature for the two sides are both to the right of the lens. Side 1 is 'flatter' so must have a larger radius of curvature: $|R_1|$ will be larger than $|R_2|$ which means that $|1/R_1|$ will be smaller than $|1/R_2|$.
Comparing this to our reference lens, R_1 will be positive and R_2 will be negative.
In our expression $\frac{1}{R_1} + \frac{1}{R_2}$ the first term is positive and the second term is negative but has a larger magnitude, so the net result will be negative, making $f < 0$ and thus this is a diverging lens.



Plano-concave: This is a special case of the lens we just did, where the left side being flat implies that R_1 is infinity, so $1/R_1 = 0$. R_2 is negative, so $1/R_2$ is also negative, making $\frac{1}{R_1} + \frac{1}{R_2}$ negative, so ultimately the focal length of this lens will be negative. This will be a diverging lens.

Double concave: The center of curvature for the left face is to the left of the lens and the center for the right face is to the right. Both of these are on the 'wrong' side from our reference lens as far as signs are concerned, so both R_1 and R_2 will be negative. Thus $\frac{1}{R_1} + \frac{1}{R_2}$ is negative, and this lens has a negative focal length, making it a diverging lens.

Example 32 : A converging lens with a focal length of 12.0 cm forms a virtual image 8.00 mm tall, 17.0 cm to the right of the lens. Determine the position and size of the object itself. Is the image upright or inverted? Are the object and image on the same side or opposite sides of the lens? Draw a principal ray diagram for this situation. (Note: way too much set-up thought needed to work this one; nothing this weird will be on the test without being described better...)

Note that this is a single lens and we have a virtual image; that means the image must be **not** on the outgoing ray side, which means that $d_i = -17\ cm$. They also said this image is on the right side of the lens, so our figure here is **turned around** from the way we usually draw them: the outgoing rays here are heading to the left, so the incoming rays must be on the right-hand side.

So here we have $d_i = -17.0\ cm$, and $f = +12\ cm$ (positive since this is a converging lens).

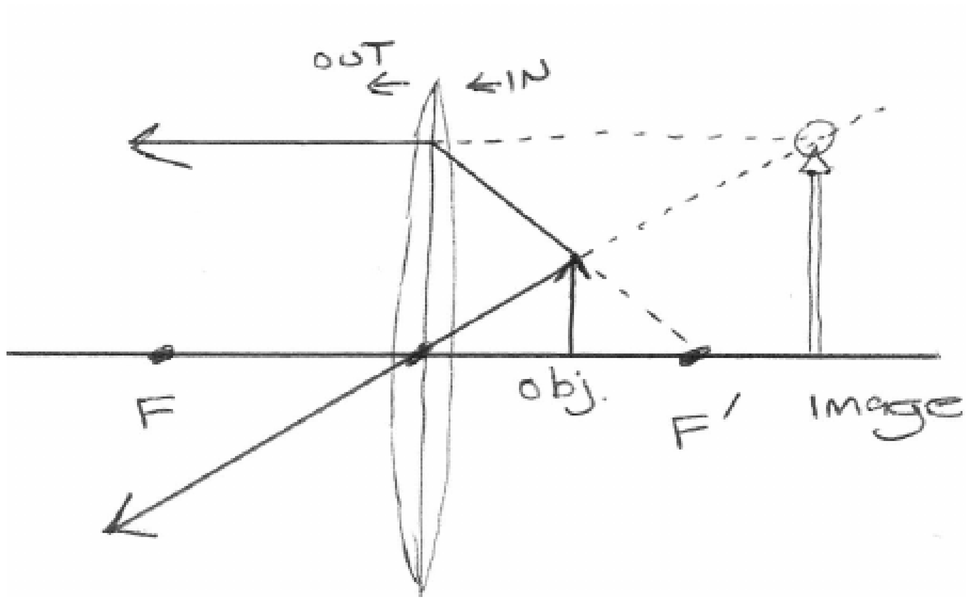
We can use the object-image relationship to find the object distance d_o now: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$. Rearranging the terms to solve for d_o (since that's what we're looking for here ultimately): $\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{d_i - f}{d_i f}$ or finally $d_o = \frac{d_i f}{d_i - f}$. For this problem then: $d_o = \frac{(-17.0\ cm)(12\ cm)}{(-17\ cm)-(12\ cm)} = +7.03\ cm$. Positive d_o implies the object is on the same side as the incoming rays (the rays from the object as they 'come in' to the lens), so it must be on the right side of the lens. (We already knew that from the first paragraph where we argued that this figure is turned around backwards from the way we usually draw it.)

The magnification of the image is $m = -d_i/d_o = -(-17\ cm)/(7.03\ cm) = +2.42$. This is positive so the image is upright. The magnification also relates the sizes of the image and object: $m = h_i/h_o$ so $h_o = h_i/m = (8\ mm)/2.42 = 3.31\ mm$.

m was positive, so the image and object have the same orientation.

Using the numbers we got above, we see that the object is positioned inside of the focal length of this lens. A ray that comes from (or appears to come from) the focus on the right-hand side should refract into a ray parallel to the axis as it heads off to the left. (So here we draw a line from F' to the head of the arrow and then on to the lens, at which point it turns into a line parallel to the axis. The image of the arrowhead must lie somewhere in the direction defined by that line.

Another ray we can draw is one between the arrowhead and the vertex of the lens. Extending that line in each direction we can see where this ray intersects the one we just drew, and that's where the image of the arrowhead must be. (It's usually a good idea to draw everything as dotted lines, or light colored lines initially just to lay out the geometry. Then we can make the lines more solid to represent the real path that light takes along these rays, and project these lines back if needed to see where they intersect.



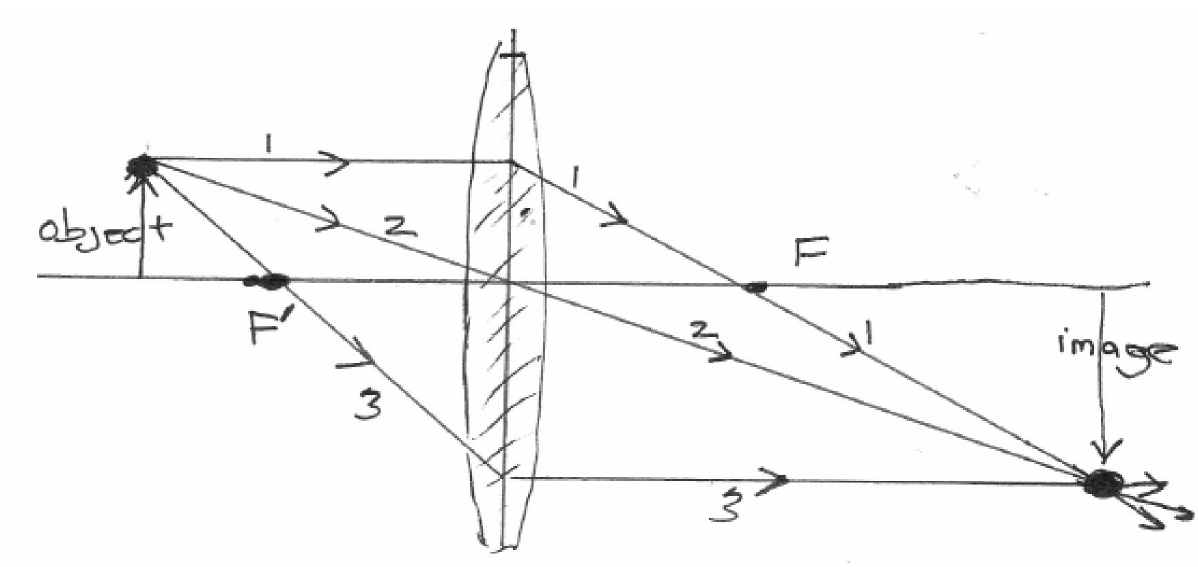
Example 34 : An object is 16.0 cm to the left of a lens. The lens forms an image 36.0 cm to the right of the lens. (a) What is the focal length of the lens? Is the lens converging or diverging? (b) If the object is 8.0 mm tall, how tall is the image? Is it upright or inverted? (c) Draw a principal ray diagram.

For a single lens, the object is (always) on the same side as the rays incoming towards the lens, so $d_o = 16\ cm$. The image is over on the same side as the outgoing rays, so $d_i = +36.0\ cm$.

(a) The focal length can be found easily since $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$. Putting the left hand side over a common denominator: $\frac{d_o + d_i}{d_o d_i} = \frac{1}{f}$ or finally $f = \frac{d_o d_i}{d_o + d_i}$. For this particular case: $f = \frac{(16\ cm)(36\ cm)}{16\ cm\ +\ 36\ cm} = 11.1\ cm$. This is positive, so the lens is converging.

(b) The magnification factor is $m = -d_i/d_o = -(36)/(16) = -2.25$ so $h_i = mh_o = (-2.25)(8.0\ mm) = -18.0\ mm$. The image is 18 mm tall and inverted.

We draw all three of the standard principal rays for lenses here, using the point at the top of the arrowhead as our object point for which we want to find the corresponding image point. We have a converging lens, so if the object is on the left side, F will be on the right side and F' will be on the left side of the lens.
Ray 1 : a ray from the object parallel to the axis will go through focal point F. We start drawing a line parallel to the axis and when the line hits the lens, change its direction so it passes through F (and extend the line).
Ray 2 : a ray from the object through the vertex will just continue without bending.
Ray 3 : a ray through F' (or appearing to come from F') will refract at the lens into a line parallel to the axis. So we put a ruler down between the arrowhead and F' and draw a line between those points, extending it until it hits the lens. At that point, the line refracts into a perfectly horizontal line parallel to the axis. All three of these lines nominally should intersect at the location of the image of our point.



Example 40a : An object placed 20 cm to the left of a lens produces a **real** image 10 cm from the lens. Determine the focal length and type (converging or diverging) and the magnification of the lens.

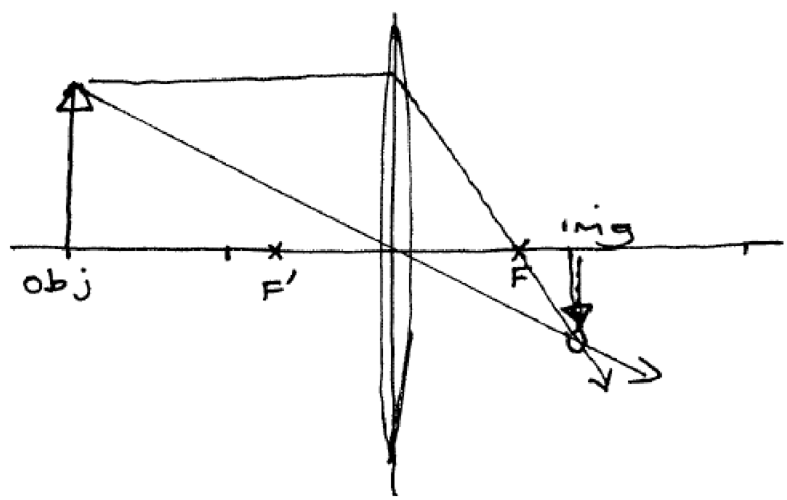
Let's think about the geometry here. We have the object on the left, the lens in the middle, and a real image being formed somewhere but is that image on the left or right of the lens? The object is on the left, so that's the 'incoming' side of the lens, with the right side being where the 'outgoing' rays are. The problem states that the image here is real, which means it must be on the 'outgoing' ray side, which means for this lens the image must be on the right-hand side of the lens.

We can determine the signs now: $d_o = +20$ and $d_i = +10$.

Magnification : We can already determine that, since $m = -d_i/d_o = -(10)/(20) = -0.5$ so the image will be smaller and inverted.

Focal length : $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ which we can rearrange into: $f = \frac{d_o d_i}{d_o + d_i}$. With the image and object distances we have here: $f = \frac{(20)(10)}{20+10} = 200/30 = +6.67\ cm$. This is positive, so this is a **converging** lens.

Ray diagram (NOTE: we couldn't draw this until we knew what the focal length was.)
A ray from a point on the object that heads out parallel to the lens axis will refract and pass through the focal point on the other side of the lens.
A ray through the vertex of the lens just keeps going in a straight line.
The image of the starting point will be located at the intersection of the rays.



Example 40b : An object placed 20 *cm* to the left of a lens produces a **virtual** image 10 *cm* from the lens. Determine the focal length and type (converging or diverging) and the magnification of the lens.

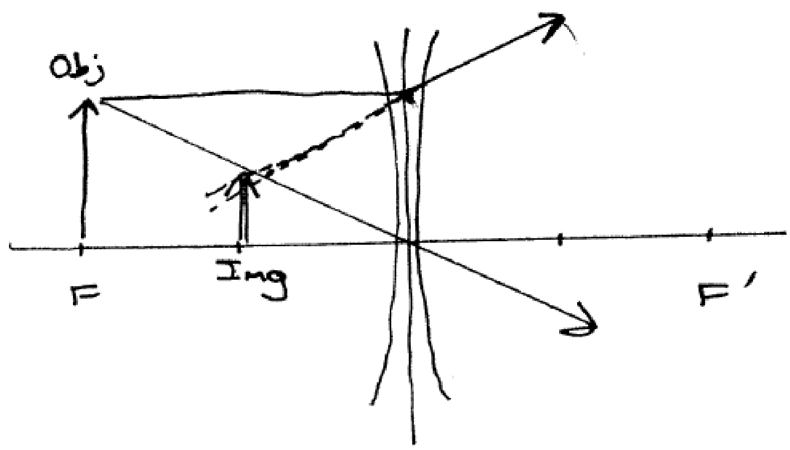
Let's think about the geometry here. We have the object on the left, the lens in the middle, and a virtual image being formed somewhere but is that image on the left or right of the lens? The object is on the left, so that's the 'incoming' side of the lens, with the right side being where the 'outgoing' rays are. The problem states that the image is virtual, so it must **not** be on the 'outgoing' ray side, which means the image in this problem must be on the left side of the lens.

We can determine the signs now: $d_o = +20$ and $d_i = -10$.

Magnification : We can already determine that, since $m = -d_i/d_o = -(-10)/(20) = +0.5$ so the image will be smaller and upright.

Focal length : $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ which we can rearrange into: $f = \frac{d_o d_i}{d_o + d_i}$. With the image and object distances we have here: $f = \frac{(20)(-10)}{20+(-10)} = -200/10 = -20.0\ cm$. This is negative, so this is a **diverging** lens.

Ray diagram (NOTE: we couldn't draw this until we knew what the focal length was.)
A ray parallel to the axis refracts to appear to have come from F.
A ray through the vertex continues in a straight line.
Where these intersect (or appear to be coming from in the case of a diverging lens) that's where the image of the point is.



Example 40c : An object placed 20 *cm* to the left of a lens produces a **virtual** image 30 *cm* from the lens. Determine the focal length and type (converging or diverging) and the magnification of the lens. (Note that compared to the previous problem, the virtual image is further out than before.)

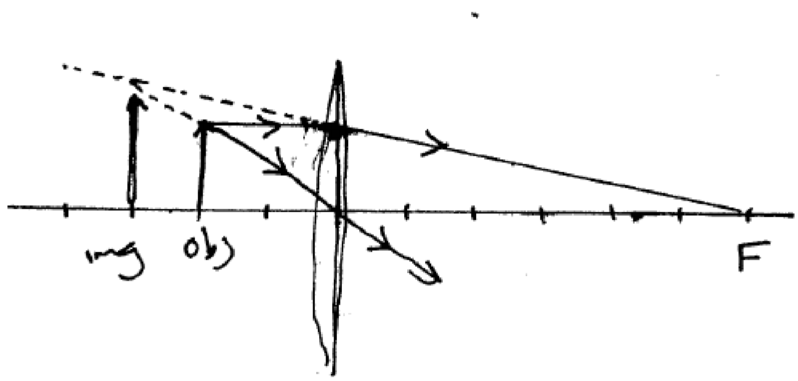
Let's think about the geometry here. We have the object on the left, the lens in the middle, and a virtual image being formed somewhere but is that image on the left or right of the lens? The object is on the left, so that's the 'incoming' side of the lens, with the right side being where the 'outgoing' rays are. The problem states that the image here is virtual, so it must **not** be on the 'outgoing' ray side, which means the image is on the left side of the lens.

We can determine the signs now: $d_o = +20$ and $d_i = -30$.

Magnification : We can already determine that, since $m = -d_i/d_o = -(-30)/(20) = +1.5$ so the image will be larger and upright.

Focal length : $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ which we can rearrange into: $f = \frac{d_o d_i}{d_o + d_i}$. With the image and object distances we have here: $f = \frac{(20)(-30)}{20+(-30)} = (-600)/(-10) = +60.0\ cm$. This is positive, so this is a **converging** lens.

Ray diagram (NOTE: we couldn't draw this until we knew what the focal length was.)
Converging lens, so we draw a ray parallel to the axis; it refracts at the lens and heads towards F on the outgoing side.
Ray through the vertex continues in a straight line.
Where these rays appear to be coming from is where the image of the point is located.



Example 42 : A lens in air has a focal length of 20 cm. If we put this lens under water, describe the image that it will form if an object is placed 10 cm to the left of this lens. Assume the lens is made of glass with $n = 1.5$, the index of refraction of air is essentially $n = 1$ and the index of refraction of the water is $n = 1.33$.

We can determine the focal length of the lens under water from knowledge of it's focal length in air.

From the (generic version of the) Lensmaker's equation, the focal length in air (subscript 'a') will be:

$\frac{1}{f_a} = \frac{n_g - n_a}{n_a}(\frac{1}{R_1} + \frac{1}{R_2})$

where n_g is the index of refraction of the glass lens and n_a is the index of refraction of the air (essentially 1).

The focal length under water will be:

$\frac{1}{f_w} = \frac{n_g - n_w}{n_w}(\frac{1}{R_1} + \frac{1}{R_2})$

where n_g is the index of refraction of the lens and n_w is the index of refraction of the water (1.33).

We don't know the radii of the two sides of the lens, but don't need them:

Dividing the first equation by the second:

$\frac{f_w}{f_a} = \frac{n_g - n_a}{n_a} \times \frac{n_w}{n_g - n_w} = \frac{1.5-1}{1} \times \frac{1.33}{1.5-1.33} = (0.5)(7.8235..) = 3.912.$

The focal length under water then is 3.912 times the focal length in air, or about $f_w = (20\ cm)(3.912) = 78.2\ cm$.

IN WATER : the focal length of the lens is 78.2 cm so the image will form at: $d_i = (d_o f)/(d_o - f) = (10)(78.2)/(10 - 78.2) = 782/(-68.2) = -11.5\ cm$ which will be a virtual image. $m = -d_i/d_o = -(-11.5)/(10) = +1.15$ so the image will be upright and only slightly larger than the object.

What would the image have looked like in air?

IN AIR : if we place an object 10 cm from the lens, it will form an image at $d_i = (d_o f)/(d_o - f) = (10)(20)/(10 - 20) = -20\ cm$ so we have a virtual image. $m = -d_i/d_o = -(-20)/(10) = +2$ so the image will be upright and magnified by a factor of 2.

Note that when used under water, the magnification is much weaker. This is often expressed as lenses being 'less effective' under water. Underwater cameras are often enclosed in water-tight enclosures so that the lens remains surrounded by air in order to avoid having to deal with this changing focal length effect.

Example 45a : In a particular camera, the lens is located 2 cm from the film (or image sensor in the case of a digital camera). This camera perfectly focuses an object that is located 1 m from the lens.

(a) What is the focal length of this lens?

This is a real image (same side as the 'outgoing' rays), so the image distance $d_i = +2\ cm$. The object is on the same side as the 'incoming' rays, so $d_o = +100\ cm$.

Focal length: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ which we can rearrange into: $f = \frac{d_o d_i}{d_o + d_i}$. With the image and object distances we have here: $f = \frac{(100)(2)}{100+2} = 200/102 = 1.96078..\ cm$. This is positive, so this is a **converging** lens.

(b) If we point this camera at an object $2\ m$ **away, where will the image form?**

Image Distance: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ which we can rearrange into: $d_i = \frac{d_o f}{d_o - f}$. Here, the focal length is still 1.96 cm but the object distance has increased to $d_o = +200\ cm$ so:

$d_i = \frac{(200)(1.96)}{200-1.96} = 1.979\ cm$.

The film/sensor is located 2 cm from the lens though, which means that the image is now forming slightly in front of the film, missing it by $2 - 1.979 = 0.021\ cm$ or about a fifth of a millimeter. The rays that exactly converge on this focal point will continue on past there until they reach the film/sensor, resulting is a (very) slightly blurry image.

(c) If we point this camera at an object infinitely far away, where will the image form?

When d_o becomes infinity, the image forms at the focal length $d_i = f$ so in this case the image will form 1.96 cm from the lens. The film/sensor is 2 cm from the lens, so this image will be about 0.04 cm or two fifths of a millimeter in front of the film. As in (b), the rays continue until they hit the film, and will now be slightly more out of focus (blurred).

(d) If we point the camera at something $20\ cm$ **in front of the lens, where will the image form?**

$d_i = \frac{d_o f}{d_o - f} = \frac{(20)(1.96)}{20-1.96} = 2.17\ cm$ which is almost 2 millimeters behind the film or sensor. This image will be even more blurry than the ones in the previous parts.

The closer the object comes to the lens, the further behind the sensor the image will form, creating increasingly out-of-focus pictures.

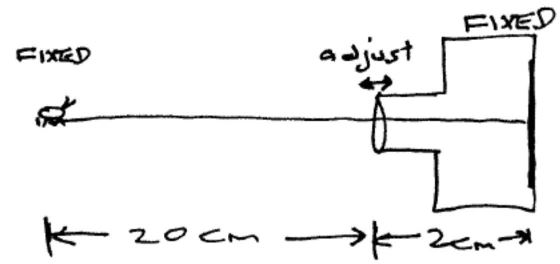
(e) Where do we need to move the lens so that an object exactly 20 cm in front of the lens will **focus perfectly on the sensor?**

The lens of some cameras can move in and out, altering the distance from the lens to the film or sensor, allowing the user to adjust things so that the image distance exactly lands on the sensor.

Here, we have the same physical lens, so $f = 1.96\ cm$ still and we have $d_o = 20\ cm$ so where will this form an image (then we'll shift the lens so that it's exactly that distance from the film).

$d_i = \frac{d_o f}{d_o - f} = \frac{(20)(1.96)}{20-1.96} = 2.174..\ cm$ so we'll need to move the lens so that instead of being 2 cm away from the sensor, it's now 2.17 cm from the sensor: the lens needed to be moved slightly farther out (and to maintain the exact 20 cm distance to the object, the whole camera would need to be moved backwards slightly).

Example 45b : Suppose we re-do the last part of the previous problem assuming that the **object** and **camera** are both **fixed in place**. In part (e) of the previous problem, we assumed that the distance from the lens to the object was still 20 cm but since we had to increase the lens-film distance, that meant that in effect we had to move the whole camera slightly farther from the object. It wasn't much - just a couple of millimeters, but suppose the camera body and the thing we're photographing are fixed. Where do we need to put the lens now?



Initially, the object was located 20 cm in front of the lens, and the sensor was 2 cm behind the lens. Those two points then are separated by 22 cm and we can't change this since the camera is locked in place. That does give us an additional relationship we can use though: $d_o + d_i = 22\ cm$.

We still have our lens equation: $d_i = \frac{d_o f}{d_o - f}$ but we can replace all occurrences of d_o with $22 - d_i$ and we know $f = 1.96$ so:

$d_i = \frac{(22-d_i)(1.96)}{22-d_i-1.96}$ or $d_i = \frac{(22-d_i)(1.96)}{20.04-d_i}$.

Multiplying both sides by the denominator on the right (and letting $d_i = x$ to simplify solving the equation a bit):

$(20.04 - x)(x) = (22 - x)(1.96)$

If we expand out and recollect all these terms, this becomes the quadratic equation:

$x^2 - 22x + 43.12 = 0$ which has solutions: $d_i = 19.82\ cm$ or $d_i = 2.175\ cm$.

It appears that there are two locations we can place the lens. Why the ambiguity?

Notice that the two distances add up to the original 22 cm separation distance. Since we can 'turn around' a lens and get the exact same focal length according to the Lensmaker's equation, an object at A that focuses at B means that an object at B would focus at A. We see this in the lens equation as well if we just swap the locations of d_i and d_o. If we put an object where the image was, we'll get an image where the object was.

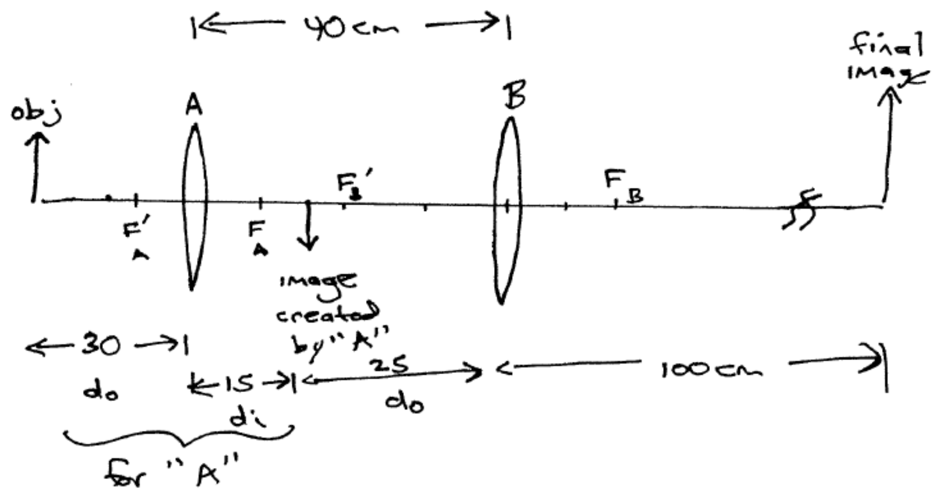
The solution we're looking for here would be the $d_i = 2.175\ cm$ one, where we just make a tiny adjustment to the lens location to bring the object into focus.

(If we keep enough significant figures, this result is **slightly** different than the position we had in part (e) in the previous problem. The closer the object is to the lens, the more different they will be.)

(Note that means the bug is now $22 - 2.175 = 19.825\ cm$ from the lens, instead of 20 cm which means the magnification factor is $m = -d_i/d_o = -(2.175)/(19.825) = -0.109$.)

Example 50a : Multiple Lenses

Suppose we have two lenses separated by 40 *cm* set up as in the figure. They're both converging lenses and the first has $f = +10$ *cm* and the second has $f = +20$ *cm*. An object is placed 30 *cm* to the left of the first lens. Where will the image formed by these two lenses working together be?



Rays from the object will create an image; that image then acts as the object for the second lens.

Where will the intermediate image created by the left lens be?

$d_i = \frac{d_o f}{d_o - f} = \frac{(30)(10)}{30-10} = 300/20 = +15$ *cm* so this will be a real image, 15 *cm* to the right of the first lens.

That image now becomes the object for the second lens. Where is it located according to the second lens? The lenses are 40 *cm* apart, and the first lens has created an image 15 *cm* to the right of the first lens, so this 'object' is $40 - 15 = 25$ *cm* to the left of the second lens. That's on the same side as the rays 'incoming' to the second lens, which makes it a positive object distance: $d_o = +25$ *cm*. We can now find where the second lens will create an image:

$d_i = \frac{d_o f}{d_o - f} = \frac{(25)(20)}{25-20} = 500/5 = +100$ *cm* so this will be a real image, 100 *cm* to the right of the second lens.

Magnification : the intermediate image, created by the first lens, will have a magnification of $m = -d_i/d_o = -(15)/(30) = -0.5$

The magnification of the second lens will be: $m = -d_i/d_o = -(100)/(25) = -4.0$.

Starting from the original object then: the first lens inverts the object and reduces it in size by a factor of 1/2; that reduced and inverted image then gets 'processed' by the second lens, which multiplies it by 4 and inverts it again. The net result of these two: $(-0.5)(-4.0) = +2.0$ is an image that is twice the size of the original object, and upright.

Utility of Multiple Lenses

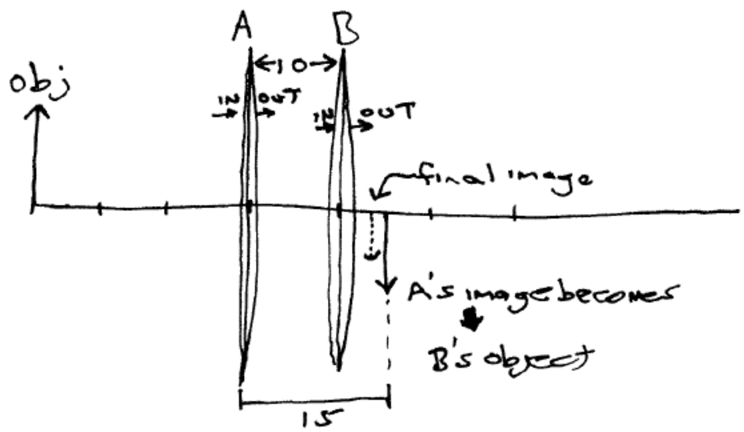
Why use two lenses? Can we not achieve the same result with a single lens? Can't we find some lens that by itself creates a real, upright, magnified image? Unfortunately not, which is easy to show:

Here we're looking for a lens that creates a real image, so both the object and image distances must be positive. Unfortunately, $m = -d_i/d_o$ and both distances here are positive so the magnification factor will be negative: the image will be **inverted**. We wanted an upright image, so we're out of luck: there's no single lens that will do what we want.

In the case of a projector, say, that means that we can use two lenses to turn an upright object into a magnified upright image. If we only have one lens, the image will be upside down. Usually we can deal with this by just inverting the object to begin with, but that's not always convenient.

Example 50b : Multiple Lenses

Suppose we have the same two lenses from the previous problem but this time they are separated by just 10 *cm*. They're still both converging lenses and the first has $f = +10\ cm$ and the second has $f = +20\ cm$. An object is placed 30 *cm* to the left of the first lens. Where will the image formed by these two lenses working together be?



Note that the image formed by the first lens is now beyond the second lens. This is the 'object' for the second lens, but now the object is **not** on the same side as the 'incoming' rays for this lens. That's alright - it just means that the object distance is negative (see the **sign rules** on page 963 of the book).

The image from the first lens is forming 15 *cm* to the right of the first lens. The second lens is located 10 *cm* to the right of the first one, which means the 'object' for the second lens is 5 *cm* from the second lens and on the wrong side: $d_o = -5\ cm$ now.

$d_i = \frac{d_o f}{d_o - f} = \frac{(-5)(20)}{-5-20} = (-100)/(-25) = +4.0\ cm$

The image forms 4 *cm* to the right of the second lens, making it still a real image (it's on the 'outgoing' ray side of that lens).

How large will this final image be? The magnification factor for the second lens is $m = -d_i/d_o = -(4)/(-5) = +0.8$; so the overall magnification for the two lenses working together will be -0.5 from the first lens and $+0.8$ from the second or overall: $(-0.5)(+0.8) = -0.4$.

The final image will be real, inverted, and reduced to 0.4 of its original size.

Equivalent Lens

Can we find a single lens that does exactly the same thing? I.e., it should produce an image in the same place as the double lenses did, and it should also be the same magnification: everything identical.

Let's try a single lens located where the first one was. The final image was 4 *cm* to the right of the second lens, which means it's 14 *cm* to the right of the first lens: $d_i = +14\ cm$. The original object was 30 *cm* to the left of the first lens so $d_o = +30$, but that makes the magnification factor $m = -d_i/d_o = -14/30 = -0.467$ which is **not** exactly what we got from the two lenses working together.

Unfortunately, if we want to replace these two lenses with a single one, we'll need to find a new place to put it.

We want the original magnification, so $m = -d_i/d_o = -0.4$ so $\boxed{d_i = 0.4d_o}$ but we also want the object and image to be in the same place, which means $d_o + d_i = 30 + 14 = 44\ cm$. That's two equations with two unknowns we can solve directly, resulting in $d_o = 31.43\ cm$ and $d_i = 12.57\ cm$.

What focal length would such a lens have to have to produce this?

$f = \frac{d_o d_i}{d_o + d_i} = \frac{(31.43)(12.57)}{31.43+12.57} = 8.98\ cm$

Net result: this single lens needs to have a smaller focal length, and we need to place it so that it's slightly farther away from the object (31.43 cm instead of the original 30 cm).

Important : note how this problem is **different** from Example 45b. In that problem, we had a fixed object and a fixed image location and were looking for where to position the **existing** camera lens so that a sharp, in focus image would form. That sounds similar, but in that case, the focal length of the lens was fixed: we were stuck to using the same lens we started with and were only allowed to play with its position. In that problem we were able to find a spot, but the result was a slightly different magnification of the object.

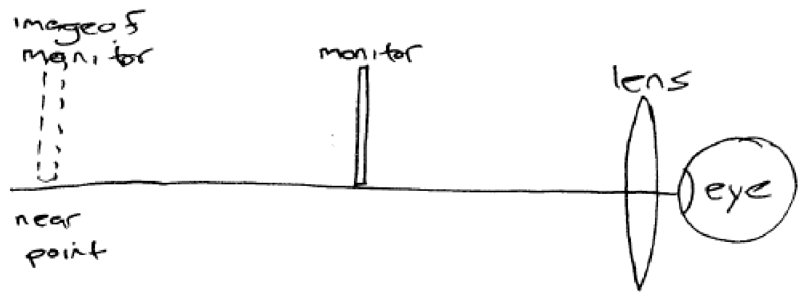
In **this** problem, we're throwing away the two lenses we started with and looking for a new single lens that would have the same effect, and this time we were successful: the single lens produced the identical image (location and size) as the two lenses we started with.

It worked out this time, but recall that in the previous problem (50a), it didn't work out there because there is no single lens that can produce an upright, real image.

Basically you have to work through each of these types of problems separately.

Example 60a : Correcting for far-farsightedness

A person is far-sighted, which means their 'near-point' is farther out than normal. Suppose their near-point is located at 50 cm. They want to be able to comfortably read a computer monitor (or phone or tablet) that is 25 cm in front of them. What corrective lens is needed? (Ignore the small distance between the lens and the eye and just measure all distances relative to the lens location, but see example 60d where we do a more correct analysis.)



The lens here needs to 'move' the object (the monitor) so that it appears to be at their near-point, where they can comfortably focus. The lens will be sitting right in front of the person's eyes, so essentially we have a situation here where we have a real object at 25 cm that needs to be converted into an image at 50 cm. What signs should we have here? We have the object, then a lens, then the person's eye. The object is on the side of the rays incoming to the lens, so we have $d_o = +25\ cm$. The rays continue through the lens into the eye, which means that the image is **not** on the outgoing-ray side, meaning the image will be virtual: $d_i = -50\ cm$.

What focal length lens will do this?

We can rearrange the lens equation, $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ into the form: $f = \frac{d_o d_i}{d_o + d_i} = \frac{(25)(-50)}{25+(-50)} = (-1250)/(-25) = +50\ cm$.

This is positive, so apparently we need a converging lens (one that's thicker in the middle than near the edges) with that focal length.

The eye doctor would use the lens power instead of the focal length: $P = 1/f$ (with f measured in meters) so here $f = +0.50\ m$ and $P = 1/f = +2.0\ D$ (recall in that lens power is measured in inverse meters, which are called Diopters).

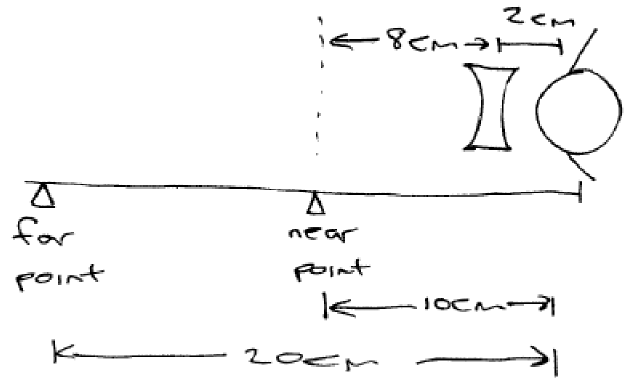
Will the monitor screen 'look' larger or smaller?

The magnification factor here is $m = -d_i/d_o = -(-50)/(25) = +2.0$ which means that the here we have the monitor appearing to be twice as far away, but twice as large: the image of the monitor (what we see when we look through the eyeglasses) takes up the **same field-of-view** (same angle) as the actual monitor (without the glasses) would have taken up.

The screen looks the same size, it's just in focus now!

Example 60b : Correcting for near-sightedness

A person is very near-sighted, with a near-point of 10 *cm* and a far-point of 20 *cm*. What lens should we construct so that objects far away (like road signs or movie screens) can be seen clearly (i.e. not blurry). How will this lens affect their near-point?
Note: in this case, do not ignore the lens-eye distance; assume the lens is about 2 cm in front of the eye.



Essentially we're trying to turn an object at infinity into an image located at their far-point. The object is on the same side as the light rays coming into the lens, so the object distance will be positive infinity.

In the lens equation, the distances are all measured relative to the lens location. We want the image to form 20 *cm* from the person's eye, which means 18 *cm* in front of the lens. So ultimately: we're trying to make an image that is 18 *cm* from the lens. The image is clearly not on the same side as the outgoing rays, so this is a virtual image and $d_i = -18\ cm$.

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ so in this case: $\frac{1}{\infty} + \frac{1}{-18} = \frac{1}{f}$ or $f = -18\ cm$.

The focal length is negative, so this is a diverging lens (thinner in the center than at the edges). Lens power: $P = 1/f$ with f in meters, so $P = 1/(-0.18\ m) = -5.56\ D$. (Note that the lens power for correcting near-sightedness is negative.)

How will this lens affect their near-point?

We're trying to find where the person's new near-point will be when they're wearing these glasses. Their eye has a near point of 10 *cm*, which is 8 *cm* in front of the lens. Where would an object have to be so that it forms an image there? (Careful here: we're wanting the image to form beyond the lens, not on the side as the outgoing rays, so $d_i = -8\ cm$.)

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or rearranging:

$d_o = \frac{d_i f}{d_i - f} = \frac{(-8)(-18)}{(-8)-(-18)} = +144/10 = 14.4\ cm$.

An object that is 14.4 *cm* in front of the lens (16.4 *cm* in front of their eye) will now appear at their near-point. They used to be able to focus on objects as close as 10 *cm* from their eye, now they can only focus on objects as close as 16.4 *cm* in front of their eye, so these types of corrective lenses have that downside.

Another downside of near-sightedness is that you usually need to get glasses prescribed specifically for you. Far-sighted people that want to be able to look at something close up can buy 'reading glasses' cheaply at pharmacies. Those involve converging lenses, with positive 'lens powers'. You'll see plenty of glasses on a display listing lens power of +1 D or +2 D and so on, but I've never seen glasses with a negative lens power on display.

Example 60c : Correcting for near-sightedness

A person is very near-sighted, with a near-point of 10 *cm* and a far-point of 20 *cm*. What **contact lens** should we construct so that objects far away can be seen clearly (i.e. not blurry). How will this lens affect their near-point?

Note: in the previous problem, we accounted for the fact that the eyeglass lens was 2 *cm* away from the person's eye but in the case of a contact lens we can ignore than: the lens is in direct contact with the person's eye, so we don't have to make any of those adjustments.

Essentially we're trying to turn an object at infinity into an image located at their far-point. The object is on the same side as the light rays coming into the lens, so the object distance will be positive infinity.

In the lens equation, the distances are all measured relative to the lens location. We want the image to form 20 *cm* from the person's eye, which means 20 *cm* in front of the lens (different from the previous problem since the lens is in contact with the eye now). So ultimately: we're trying to make an image that is 20 *cm* from the lens. The image is clearly not on the same side as the outgoing rays, so this is a virtual image and $d_i = -20\ cm$.

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ so in this case: $\frac{1}{\infty} + \frac{1}{-20} = \frac{1}{f}$ or $f = -20\ cm$ (different from the eyeglass solution previously).

The focal length is negative, so this is a diverging lens (thinner in the center than at the edges).

How will this lens affect their near-point?

We're trying to find where the person's new near-point will be when they're wearing these glasses. Their eye has a near point of 10 *cm*, which is 10 *cm* in front of the lens (again, we no longer have to account for the lens-eye distance with contact lenses). Where would an object have to be so that it forms an image there? (Careful here: we're wanting the image to form beyond the lens, not on the side as the outgoing rays, so $d_i = -10\ cm$.)

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or rearranging:

$d_o = \frac{d_i f}{d_i - f} = \frac{(-10)(-20)}{(-10)-(-20)} = +200/10 = 20.0\ cm.$

So an object that is 20 *cm* in front of the lens (20 *cm* in front of their eye since this is a contact lens) will now appear at their near-point. They used to be able to focus on objects as close as 10 *cm* from their eye, now they can only focus on objects as close as 20 *cm* in front of their eye. This contact lens had an even worse impact on viewing close-up objects than the eyeglasses in the previous example had.

Example 60d : Correcting for far-sightedness redux

Let's go back to problem 60a for a minute. In 60b, we accounted for the extra 2 cm distance between the eyeglass lens and the eye. In 60a, the problem stated to ignore that, but let's redo that problem taking it into account.

Going back to the figure from 60a, we have the object 25 cm from the eye so it will be $25-2 = 23$ cm from the lens: $d_o = +23$ cm. We want the image to form at the near-point of this person's eye, which is 50 cm from their eye, or 48 cm from the lens: $d_i = -48$ cm.

What is the focal length we need now?

$f = \frac{d_o d_i}{d_o + d_i} = \frac{(23)(-48)}{23+(-48)} = (-1104)/(-25) = +44.2$ cm instead of the $+50$ cm we found in the original version of the problem.

We probably should account for that distance between the eye and the lens for these types of corrective lenses also.

Example 70 : Apparent Magnification

Let's use the numbers from **Example 34** and assume that a person is standing 1 meter to the right of the lens. What is the angular size of the object and the image? What angular magnification does this represent? (Compute it for real - do not use the magnifier rules; we'll see why at the end.)

In that problem, we had an object that was 8 mm tall, located 16 cm to the left of the lens. It formed an image 36 cm to the right of the lens, and we found the image to be -18 mm tall. We found that this lens had a focal length of $f = 11.1$ cm.

Now, we're standing 1 meter (100 cm) over on the right of this lens (in line with the axis of the lens).

Angular size of the object : we are located 116 cm from the object (one meter from us to the lens, and another 16 cm from the lens to the object), and the object has a height of 8 mm, so the angular size, in radians, can be found from $s = r\theta$ (arc-length equal to the distance times the angle subtended, in radians). Here we have $r = 116$ cm and the 'arc-length' subtended is (close enough) the size of the object, so $\theta = s/r = (0.8\ cm)/(116\ cm) = 0.006897\ rad$.

Angular size of the image : the image is 36 cm to the right of the lens, and we are standing 100 cm to the right of the lens, so we are $100 - 36 = 64$ cm from the image. The angular size now is: $\theta = s/r = (1.8\ cm)/(64\ cm) = 0.028\ rad$.

The angular magnification then is $(0.028\ rad)/(0.006897\ rad) = 4.1\times$.

In problem 34, we found that the magnification factor for this lens was $m = 2.25$ but remember that factor just tells us the relationship between the physical size of the object and image. The image is in a different position than the object, so we perceive an entirely different **apparent** or **angular** magnification (represented using a capital M, rather than a lower case m).

Relationship to magnifier rules

Take care here. M is still defined as θ'/θ but in the section on magnifiers, we assumed that the image was virtual and the distance from the lens to the eye could be ignored. Neither of those was true for this problem.

More generic (and complex) versions of these equations exist but it's usually simpler to just apply the process we went through in this problem for those cases.

Example 71 : Magnifying Glass (1)

What is the focal length of a magnifying glass of 3.8× magnification for a relaxed normal eye? What will the magnification be when examining something close up (with the eye focused at the near-point?)

For a 'normal' eye, the near-point is at $N = 25\ cm$.

For a relaxed eye (far-point at infinity), the magnification is $M = N/f$ so $f = N/M = (25\ cm)/(3.8) = 6.58\ cm$

When the eye is focusing on something at it's near-point, $M = \frac{N}{f} + 1$ so here $M = \frac{25}{6.58} + 1 = 4.8\times$.

Example 72 : Magnifying Glass (2)

Sherlock Holmes is using an 8.8 cm focal length lens as his magnifying glass. To obtain maximum magnification, where must the object be placed (assume a normal eye) and what will be the magnification?

The maximum magnification occurs when the image is at the near-point of the eye; any closer and we won't be able to see it clearly. The image will be 'not on the outgoing ray side', so it's a virtual image: $d_i = -25\ cm$.

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or rearranging: $d_o = \frac{d_i f}{d_i - f} = \frac{(-25)(8.8)}{(-25)-(8.8)} = 6.51\ cm$.

The image is at the near-point, so $M = \frac{N}{f} + 1 = \frac{25}{8.8} + 1 = 3.84\times$

Example 73 : Magnifying Glass (3)

A small insect is placed 5.85 cm from a +6.00 cm focal length lens. Calculate the position of the image and the angular magnification?

Image distance:

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or rearranging: $d_i = \frac{d_o f}{d_o - f} = \frac{(5.85)(6.0)}{(5.85)-(6.0)} = -234\ cm$

Magnification:

The image of the bug is over 2 meters away, far beyond the person's near point so we'll use $M = N/f$ (instead of $M = 1 + (N/f)$ when the person is focused at their near-point).

Here then: $M = N/f = (25\ cm)/(6.0\ cm) = 4.17\times$

Example: Image Formation with Two Lenses

Binoculars and some small telescopes are constructed using two lenses, one at each end of the device.

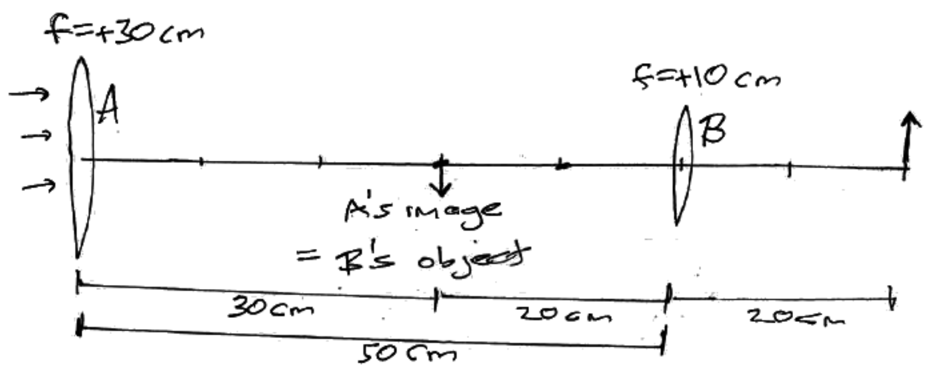
Consider an optical instrument that has an $f = +30\ cm$ converging lens at one end (the end that gets pointed at an object), and an $f = +10$ converging lens at the other end (the end you look through).

Case 1 : converging lenses separated by $50\ cm$

If the lenses are separated by $50\ cm$, where would an object very far away form an image?

The first lens will create an image at $d_i = \frac{d_o f}{d_o - f}$. If $d_o >> f$, the denominator is essentially just d_o, so the image will form at $d_i = f = +30\ cm$, making it a **real** image. The magnification for this lens will be $m_1 = h_i/h_o = -d_i/d_o$ and since both d_o and d_i are positive, the image will be **inverted**.

The image formed by the first lens now becomes the object for the second lens. Rays (photon paths) from the object actually do pass through the image, so in effect the second lens 'sees' the image formed by the first lens as if it were an actual object located at that point.



The lenses are separated by $50\ cm$ here, so this 'object' is located $20\ cm$ to the left of the second lens. The lens will turn this into an image where? $d_i = \frac{d_o f}{d_o - f} = \frac{(20)(10)}{20-10} = \frac{200}{10} = +20\ cm$ so the image will form $20\ cm$ to the right of the second lens - same side as the rays outgoing from that lens, so again this is a **real** image.

The second lens will introduce another magnification factor: $m_2 = -d_i/d_o = -(20)/(20) = -1$ which means that the final image will be **real** and **upright**.

That's useful, since if we use this instrument the final image will be rightside up, but unfortunately the image is forming floating out to the right of the second lens, so our **eye** would need to be even further to the right to be able to see it. Basically, you'd need to hold the binoculars or telescope way out in front of you.

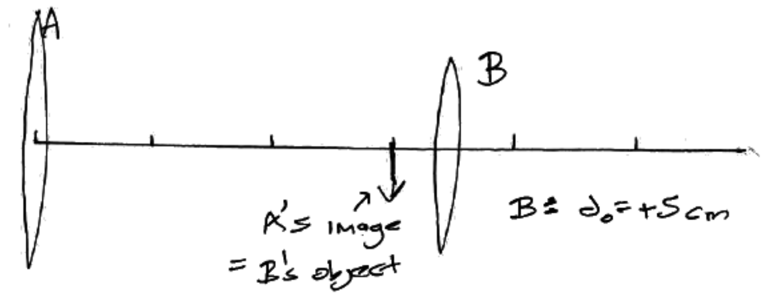
We'd like to be able to put our eye right up to the second lens, which means we **need** the final image to form over on the left of the second lens: i.e., we need a virtual image: we need the final d_i calculated for the second lens to be **negative**. How can we make that happen?

The image distance formed by the second lens will be $d_i = \frac{d_o f}{d_o - f}$ and we want that to be negative. One way to achieve this is to make d_o smaller than f : that is, slide the second lens to the left a bit. Let's try that next.

Example: Image Formation with Two Lenses (continued)

Case 2 : converging lenses separated by $35\ cm$

If we put the second lens just $35\ cm$ from the first one, the image from the first lens is still forming $30\ cm$ to the right of the first lens, meaning it's now just $5\ cm$ from the second lens. Where will the final image form now?



$d_i = \frac{d_o f}{d_o - f} = \frac{(5)(10)}{5-10} = \frac{50}{-5} = -10\ cm$. That's to the left of the lens, a virtual image, so it's where we need it to be.

What about the magnification now? The second lens will introduce an additional magnification factor of $m = -d_i/d_o = -(-10)/5 = +2$. The first lens created an upside-down image, which now becomes the object for the second lens, which doubles its size, but leaves it upside down.

So we were only partially successful here. We have a virtual image, as needed, but it's upside down.

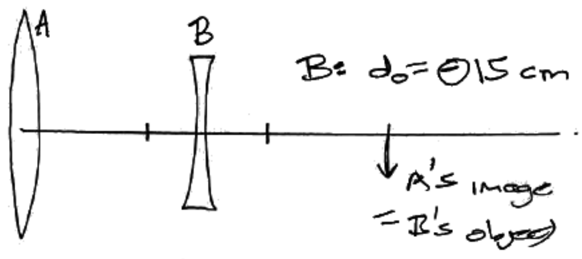
How can we create a virtual but upright final image?

Example: Image Formation with Two Lenses (continued)

Case 3A : converging and diverging lenses : image formation

One solution involves using both an initial converging lens but then a diverging lens for the second one. We can try various separation distances again, but here's one solution that works.

Suppose we have our original $f = +30\ cm$ converging lens on the front of the spyglass or binoculars, but then an $f = -10\ cm$ diverging lens on the other end (the end we look through), and suppose these lenses are now separated by 15 cm.



The first lens forms (or at least tries to form) an image 30 cm to the right of the first lens, but before it does so, the rays (photons) run into the second lens.

In effect, the 'object' for the second lens is now on the wrong side of that lens: it's over to the right, instead of being on the left where it should be. That's okay though: we can still use our lens equation if we consider an object on the wrong side of the lens (that is, on the outgoing ray side instead of the incoming ray side) as a **negative object distance**, specifically here: $d_o = -15\ cm$ for the second lens.

Where does the second lens lens form it's image?

$d_i = \frac{d_o f}{d_o - f} = \frac{(-15)(-10)}{-15-(-10)} = \frac{150}{-5} = -30\ cm.$

The final image appears 30 cm to the **left** of the second lens (meaning it's actually floating out in front of the first lens). It's still a virtual image: we can still stick our eye right up against the second lens and see this image.

What about it's orientation now?

The second lens will introduce a magnification factor of $m = -d_i/d_o = -(-30)/(-15) = -2$, which means it takes the first lens's image (which was upside down) and flips it over again, making it now upright. Success!

Example: Image Formation with Two Lenses (continued)

Case 3B : converging and diverging lenses : overall magnification

Let's finish this off by determining the apparent (i.e. the angular) magnification of the object. Essentially we're trying to decide if the final image 'looks' larger than the object, in an angular (i.e. field-of-view) sense.

Suppose we use this instrument to look at an object that is 2 m tall and 30 m away.

The first lens will form an image at $d_i = \frac{d_o f}{d_o - f} = \frac{(3000)(30)}{3000-30} = +30.3\ cm$. This image will be magnified by $m = -d_i/d_o = -(30.3)/3000 = -0.0101$ so the actual height of this image will be $h_i = mh_o = (-0.0101)(2\ m) = -0.0202\ m = -2.02\ cm$.

The lenses are separated by 15 cm, so this image is 15.3 cm to the right of the second lens (i.e. on the outgoing ray side), making $d_o = -15.3\ cm$ when we apply our lens equation to the second lens.

Where will the second lens form it's image?

$d_i = \frac{d_o f}{d_o - f} = \frac{(-15.3)(-10)}{-15.3-(-10)} = \frac{153}{-5.3} = -28.9\ cm$ (i.e. 28.9 cm to the left of the second lens: still virtual). This lens introduces a magnification factor of $m = -d_i/d_o = -(-28.9\ cm)/(-15.3\ cm) = -1.89$.

The 2.02 cm upside down image that the first lens created now becomes a $h_i = mh_o = (-1.89)(-2.02\ cm) = +3.81\ cm$ tall final image. (Positive, so it's right-side up this time.)

How about the angular magnification now?

The original object has an angular size of $\theta_{obj} = (size)/(distance) = (2\ m)/(30\ m) = 0.067\ rad$.

The final image was 3.81 cm tall and is located 28.9 cm from our eye, so it has an angular size of $\theta_{img} = (size)/(distance) = (3.81\ cm)/(28.9\ cm) = 0.132\ rad$.

The **apparent magnification** then is $M = \theta_{img}/\theta_{obj} = 0.132/0.067 = 1.98$.

The image takes up twice as much space in our field-of-view as the original object did if we just looked at it without using this instrument.

Note: actual binoculars and spyglasses can usually achieve much higher (apparent, angular) magnifications and they do this by optimizing the separation distance between the two lenses. Try changing that distance from the 15 cm we used here to maybe 14 cm or 16 cm and see how that effects the final overall magnification.