PH2233 Fox : Lecture 04
Chapter 15 : Wave Motion

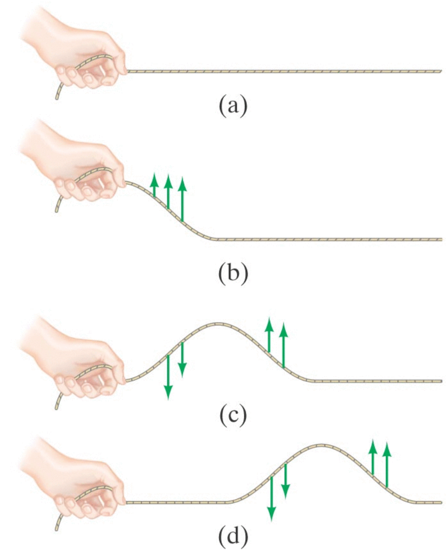
(Note: the first part of today was finishing up the last bits from Chapter 14 : see the last chapter 14 pdf.)

15.1 : characteristics of wave motion

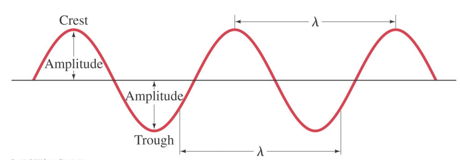
Waves are a common form of periodic motion where a disturbance propagates through a material: ripples on water, sound (which we'll see is tiny pressure fluctuations propagating through the air or some other medium), we'll even see later how light and radio waves are basically ripples of electric and magnetic fields propagating at the speed of light.

I started off demonstrating waves propagating through a 'slinky' toy and we saw that a disturbance induced at one end is seen to travel along the 'medium'.

Here, we give one end of the string (or slinky) a quick pulse to the side and see that this 'pulse' shape travels along the string.



If we keep wiggling the end of the string, some shape is seen to propagate along the material. This figure shows a snapshot of the string as the pulse passes by, just to introduce some naming conventions.

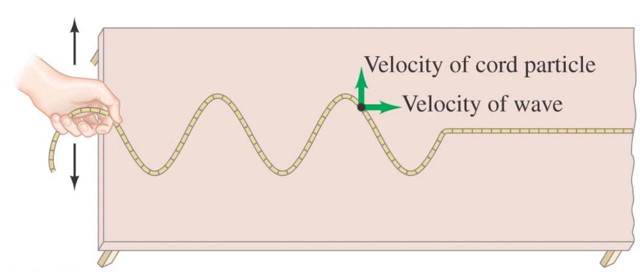


Like with periodic motion, we can define the **amplitude** as how far a point on the string moves as a result, with the maximum positive displacements labelled as **crests** (or peaks) and the maximum negative displacements labelled as **troughs**.

With a periodic wave shape like this, we can also define the wavelength λ with the shape basically repeating every length λ along the string.

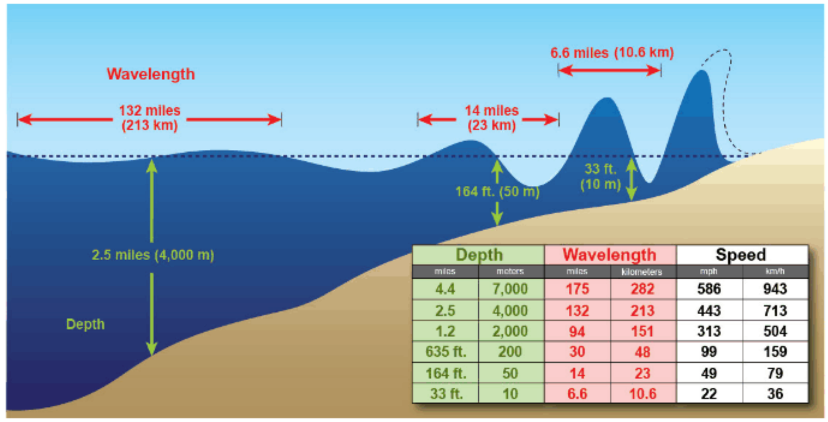
As this sinusoidal shape propagates off to the right, we see that one wavelength will pass by a given point in one period of time. We can thus define a **wave speed** of $\boxed{v = \lambda/T}$.

This figure highlights an important concept. As the wave passes through some point, that point is seen to move about it's original location (maybe up and down, or left and right), but there's no net motion of that point. There's no net **transport** of material involved here. It's the **disturbance** that's propagating, not the material itself. In this figure, the disturbance (the wave) is travelling to the right with some speed. As it passes by some point, that point is moving up and down (in this figure) at some entirely different velocity.



Quick Examples

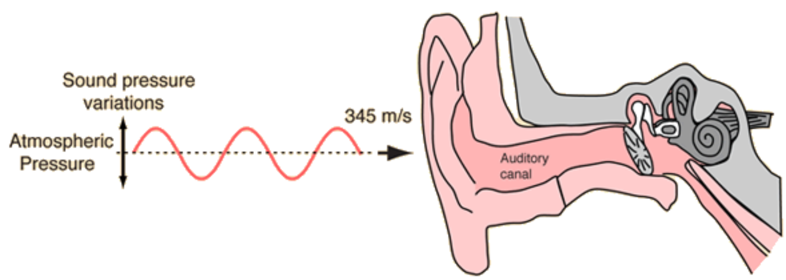
Tsunami Waves : Back in 2004 an earthquake caused a significant vertical displacement of the seafloor, which in turn generates a very large and destructive tsunami. Satellites with radar observed the wavelength of the disturbance to be about 800 km and the period was about 1 $hour$.
We can use that information to determine the **propagating speed** of this disturbance. $v = \lambda/T = (800\ km)/(1\ hr) = (800,000\ m)/(3600\ sec) = 222\ m/s$ (just under 500 $miles/hour$) which is about 2/3 the speed of sound!



It's important to note that this was how the disturbance propagated while in deep water, and the actual vertical displacement of water was less than a meter, which means that a boat out in the deep ocean would have moved up and down that far as the wave passed it but this process would have taken an hour so wouldn't even have been noticed.

(It's when the disturbance moves into shallow water that the amplitude builds up to destructive levels. Fortunately the wave speed gets dramatically lower in shallow water at least.)

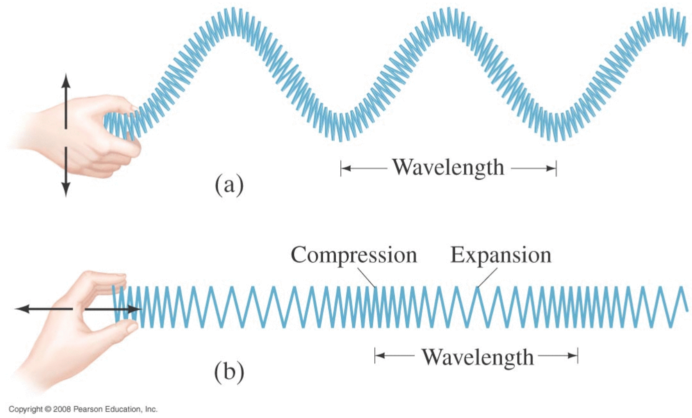
Human Hearing : The ear nominally responds to sound frequencies from 20 Hz to 20,000 Hz (that range typically shrinks with age - I really can't hear anything above about 4000 Hz ...). The speed of sound in air is about 343 m/s, so what wavelengths does this range represent?
$v = \lambda/T = \lambda f$ so $\lambda = v/f = (343\ m/s)/f$.
A $f = 20\ Hz$ sound would have $\lambda = (343\ m/s)/(20\ /s) \approx 17\ m$ -
A $f = 20,000\ Hz$ sound would have $\lambda = (343\ m/s)/(20,000\ /s) \approx 0.017\ m$ or just under 2 cm.
If you could 'see' the pressure disturbances in the air, the peaks would be that far apart from one another.



15.2 : types of waves

Two common types of waves are shown in this figure.

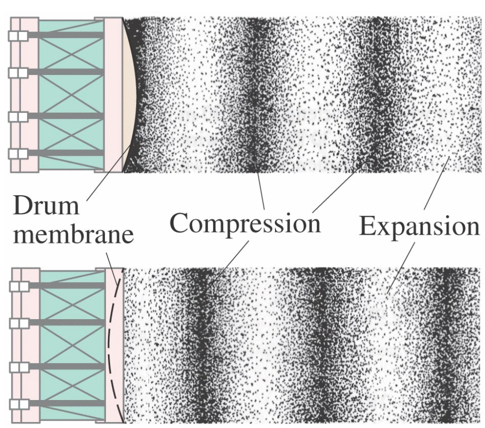
- **Transverse** (also called shear waves or S-waves)
- **Longitudinal** (also called pressure waves or P-waves)

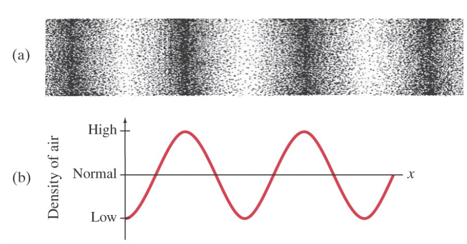


Transverse Waves (upper figure): If we introduce a disturbance by moving one end back and forth, points in the slinky move perpendicular to the medium. These are called **transverse** waves. These waves are present in stringed musical instruments (piano, guitar, etc), but also can be present in earthquakes. The molecules in the rock are being moved sideways relative to the 'next' set of molecules in the medium and this 'side to side' motion is commonly called a **shear wave** or simply an **S-wave** in that context. (It's also fairly common for materials to have less strength in the 'shear' direction: I usually use the example of a toothpick in this context. If you hold each end of a toothpick and try to break it half by pulling outward on each end, it's almost impossible. But if you push the two ends laterally in opposite directions, the toothpick easily snaps in half.) (Note that these types of waves won't exist in liquids or gasses since if the molecules in the medium move this way, they don't exert any force on the 'next' set of molecules: molecules in gasses and liquids can 'slide' laterally easily.)

Longitudinal Waves (lower figure) : In the lower figure, molecules in the medium are moving back and forth in the same direction as the wave itself, which means they're alternately being pushed into and away from each other, creating compressions and expansions: i.e. zones of higher and lower pressure, so these are often called **pressure waves** or simply **P-waves**. This type is present in earthquakes too and are how sound propagates through gases, liquids, and solids. They're the type of waves present in wind instruments (trumpet, flute, tuba, organ, etc).

Here's an illustration of the sound P-waves being generated by a drumhead oscillating back and forth.

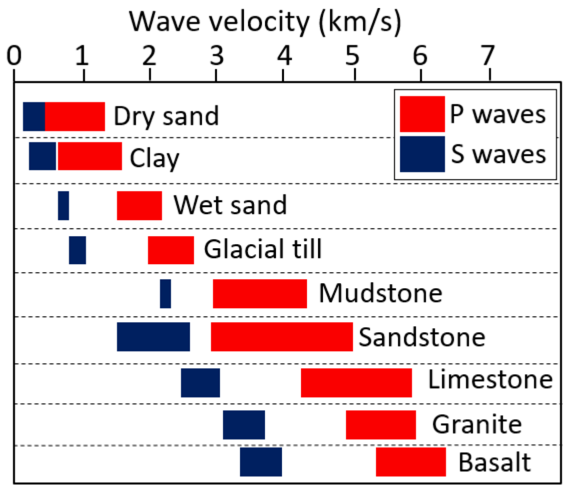




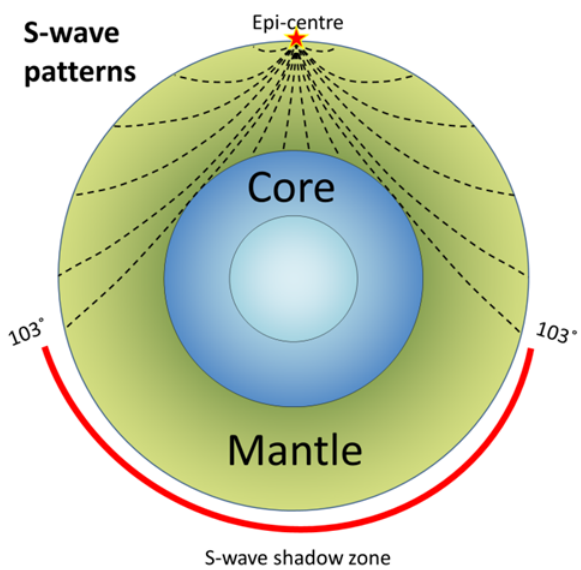
Why is this difference important? Each of these wave types, since they represent **different mechanics** at the molecular level, can propagate at quite **different speeds**.

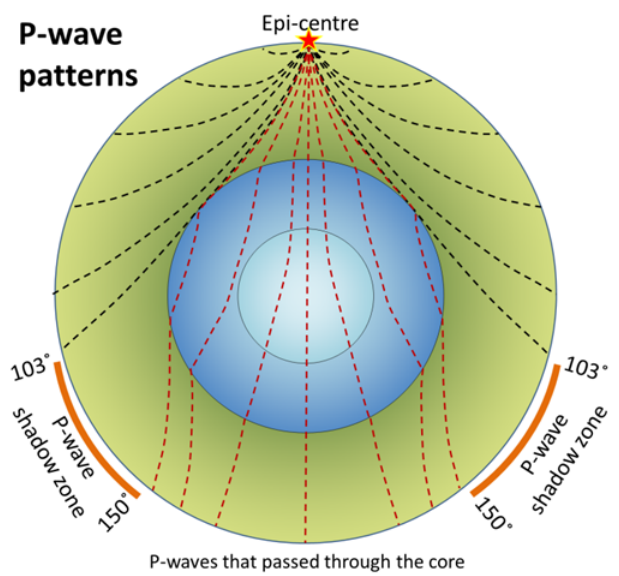
The chart on the right shows wave speeds in various different rock types (in km/s). The speed for P-waves tends to be much higher than S-waves in pretty much any real-world material.

Geologists and oil companies prospecting on land sometimes set off explosions or use vibration trucks to send sound energy into the ground, which can generate both of these types of waves and cause a more complicated post-processing step.



These different wave types can be useful also. S-waves won't propagate through the Earth's liquid inner core or molten outer core, but P-waves will, so earthquakes (and formerly nuclear bomb testing) provide information about the structure of the core.

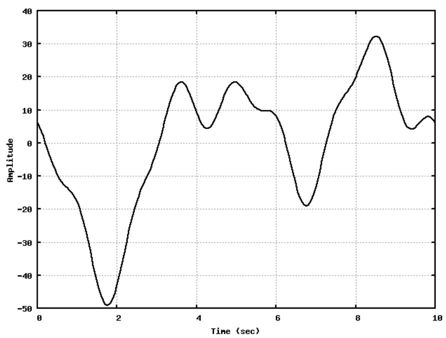




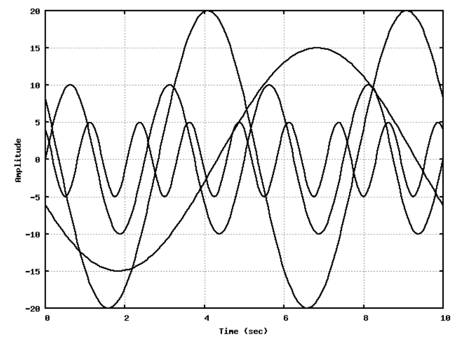
15.4 : mathematical representation of a traveling wave

Let's briefly jump ahead and look at how waves are usually modelled. We've already started drawing them as sine waves and there's a reason for that. Basically **any** real, physical disturbance and the resulting wave shape can be written as a combination of sine (or cosine, or both) waves.

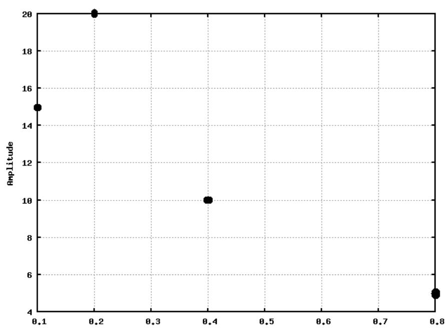
The time series in the upper left figure looks pretty random but it turns out it was built from just four sine waves of different wavelengths and phase shifts. Those four waves are shown in the upper right, and their specific amplitudes and phases are shown in the bottom left and right panels respectively.



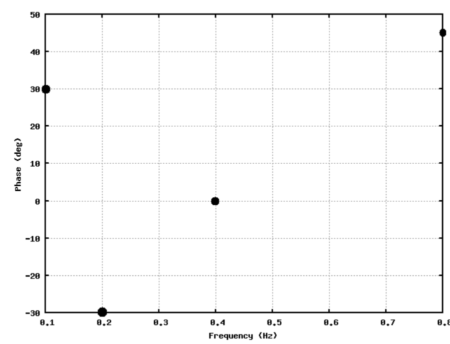
Time Series



The four sines that created it



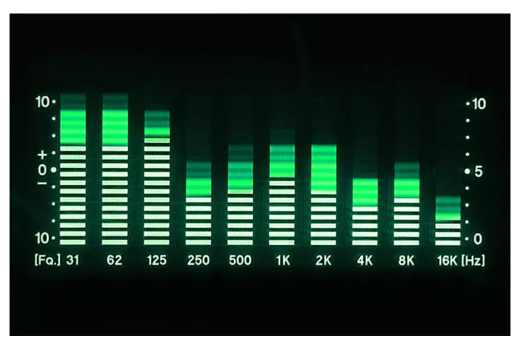
Amplitudes



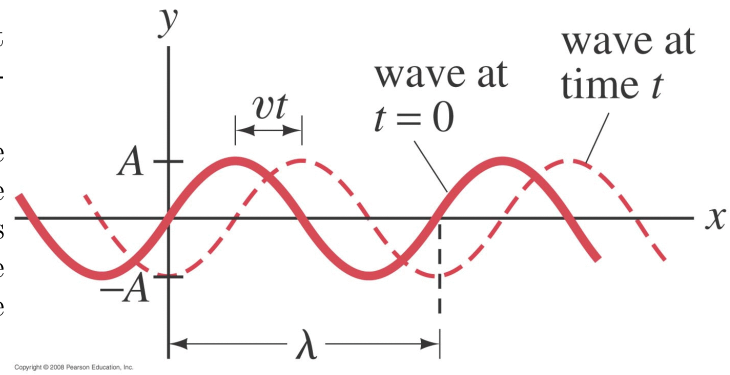
Phase Shifts

The lower figures represent the 'spectrum' of the time series and the process of deconstructing a time series into the sines (and/or cosines) that make it up is called the 'Fourier transform'. **Any real, physical time series can be converted into underlying pure sines and cosines like this, which is why we'll start off looking at waves that can be presented as a single sine wave.**

You've likely seen these in the context of music. Music, voice, earthquakes, etc are made of a continuum of amplitudes and phases that are often averaged over a few frequency ranges and displayed like the audio spectrum shown on the right. Your music player might have a display like that in the context of the 'equilizer' settings, where you can boost or suppress various ranges of frequencies.



Now that we've justified things a bit, let's look at how we can mathematically represent a propagating sine wave.
The bold line represents what the wave looks like at $t = 0$ and the wave here is bodily moving to the right at a wavespeed of v. The dotted line shows what the disturbance looks like at some later time t, when the wave has moved to the right a distance of $d = vt$.



Suppose we move along with the wave at the same speed. We don't see the wave changing at all now, and could describe it (in our coordinate system that's moving along with the wave) as: $\boxed{D(x') = A\sin(2\pi\frac{x'}{\lambda})}$.

Going back to our fixed coordinate system now: $x = x' + vt$ so $x' = x - vt$ so we can write D in our fixed coordinate system as: $D(x,t) = A\sin(2\pi\frac{x-vt}{\lambda})$ or:

$$D(x,t) = A\sin(\frac{2\pi x}{\lambda} - \frac{2\pi vt}{\lambda})$$

Let's simplify this a bit. $v = \lambda/T$ so this becomes:

$$D(x,t) = A\sin(\frac{2\pi x}{\lambda} - \frac{2\pi t}{T})$$

This still looks a little kludgy, but $\boxed{2\pi/T = \omega}$, the **angular frequency**.

Let's define an analogous symbol for the wavelength: $\boxed{2\pi/\lambda = k}$, called the **wave number**. **NOTE**: don't confuse this with the spring constant that also uses the symbol k.

Now we can write this travelling wave (moving in the +X direction) in a simpler form:

$$\boxed{D(x,t) = A\sin(kx - \omega t)}$$

We can go through the same process for a wave travelling to the LEFT (the -X direction) and find:

$$\boxed{D(x,t) = A\sin(kx + \omega t)}$$

NOTE: $v = \frac{\lambda}{T} = \frac{2\pi}{k}\frac{\omega}{2\pi} = \frac{\omega}{k}$ so we can connect the wavespeed to the wavelength and period, or directly to the angular frequency and wave number.

Taking a snapshot of this wave at $t = 0$: $D(x,0) = A\sin(kx)$ which is just the periodic shape we expect.

IF we stand at a fixed location, say $x = 0$ and look at what happens, we see that $D(0,t) = A\sin(\omega t)$ and the point just moves up and down sinusoidally as expected as the wave passes by this point. So this $D(x,t)$ expression is periodic in space and time simultaneously.

Some common (sloppy) symbolism that occurs in the wave field:

$D(x,0) = D(x)$: i.e. at $t = 0$ we just have some particular spatial form and the same symbol D with just a single argument is sometimes used for this.

The first form (with the two arguments in this case) refers to the **complete travelling wave**, giving it's amplitude at any position, at any time. The second form (with a single argument) is just the spatial part: a snapshot of what the wave looks like at $t = 0$.

Suppose we have any arbitrary initial shape $D(x)$ (might be anything - an exponential, GAUSSIAN, a square wave, etc). Then what would this thing look like propagating? Do the same thing we did at the start. We're essentially replacing the x in the (non-moving) version with $x - vt$ to give us the moving version. I.e.: $D(x,t) = D(x - vt)$ for this shape propagating to the right, or $D(x + vt)$ if it's moving to the left.

Gaussian shape example : $D(x) = e^{-x^2}$, so the propagating version would be $D(x,t) = e^{-(x-vt)^2}$ giving us the displacement of any point along the string (or whatever the medium is) at any time.

That $D(x,t)$ form describes the entire wave for all x and all t in a single equation, all at once.

Needless to say, the expression can be pretty convoluted, but for a continuous, pure sine wave it's just $D(x,t) = A\sin(kx \pm \omega t)$ and we'll use that form to extract a lot of information about waves in general.

Simple Example

As a preview, recall the tsunami wave we started with, passing under a boat out in the deep ocean. The amplitude of the wave (in deep water) was $A = 1\ m$. The wavelength was $\lambda = 800\ km = 800,000\ m$, and the period was $T = 1\ hr = 3600\ s$.

That means $k = 2\pi/\lambda = 7.854 \times 10^{-6}\ m^{-1}$ and $\omega = 2\pi/T = 1.745 \times 10^{-3}\ s^{-1}$.

The wave speed $v = \lambda/T = \omega/k = (1.745 \times 10^{-3}\ s^{-1})/(7.854 \times 10^{-6}\ m^{-1}) = 222.2\ m/s$ (same as before of course).

What about the vertical motion of the boat as this wave passes under it? The boat (hopefully) will be floating on the water, so will go up and down as the surface of the water does the same. Let's say the boat is located at $x = 0$, with the wave passing to the left (the negative X direction, or to the West in the usual convention). Then it's displacement would be $D(0,t) = A\sin(0 + \omega t)$.

- The vertical velocity of this point will be $v_y = \frac{\partial D(x,t)}{\partial t} = A\omega\cos(\omega t)$ which means the vertical velocity of the boat will vary between $\pm A\omega = \pm(1\ m)(1.745 \times 10^{-3}\ s^{-1}) = \pm 1.745 \times 10^{-3}\ m/s$ or about $\pm 2\ mm/s$ (that's **millimeters** per second).

- The maximum vertical **acceleration** the boat will feel means taking one more time derivative: $a_y = \frac{\partial v_y}{\partial t} = -A\omega^2\sin(\omega t)$ which means the acceleration will vary between $\pm A\omega^2 = (1\ m)(1.745 \times 10^{-3}\ s^{-1})^2 = 3 \times 10^{-6}\ m/s^2$ which would be about $(0.0000003)g's$.

Both the vertical velocity and acceleration are so tiny that anyone on a boat out in deep water would not even be aware that a tsumani passed under them.