Physics 2233 : Chapter 15 Examples : Wave Motion

Conventions: A (amplitude), T (period), ω (angular speed), f (frequency), k (wave number), λ (wavelength) usually all taken to be **positive**

$f = 1/T \qquad \omega = 2\pi/T = 2\pi f \qquad k = 2\pi/\lambda$

Wave function: sinusoidal wave traveling to the right (+X direction):

- $y(x,t) = A\sin(kx - \omega t)$
- $v_y(x,t) = \partial y/\partial t = -A\omega\cos(kx - \omega t)$
- $a_y(x,t) = \partial v_y/\partial t = -A\omega^2\sin(kx - \omega t) = -\omega^2 y(x,t)$.
- wave speed: $v = \lambda f = \omega/k = \lambda/T$ to the right.
- (If traveling **left** replace $-$ with $+$ in the cos and sin terms.)

Transverse waves on a wire/string: $v = \sqrt{F/\mu}$ (F is tension in wire, $\mu = M/L$ of wire)

Longitudinal waves through gas or liquid: $v = \sqrt{B/\rho}$ (E is bulk modulus of medium)

Longitudinal waves through solid: $v = \sqrt{E/\rho}$ (E is elastic modulus of medium)

Transverse (shear) waves through solid: $v = \sqrt{G/\rho}$ (G is shear modulus of medium)

Wave power (transverse waves on wire/string): $P_{avg} = 2\pi^2\mu v f^2 A^2$

Wave superposition: $y(x,t) = y_1(x,t) + y_2(x,t)$

Standing waves on a string with fixed ends: $y(x,t) = 2A\sin(kx)\cos(\omega t)$. $f_n = n(\frac{v}{2L}) = nf_1$ and $\lambda_n = \frac{2L}{n} = v/f_n$ where $n = 1, 2, 3, ...$ For such a string: $f_1 = \frac{1}{2L}\sqrt{F/\mu}$.

Range of sound audible to humans: $20\ Hz$ to $20,000\ Hz$

Wave power (pressure waves; 3D): $P_{avg} = 2\pi^2\rho S v f^2 A^2$

Intensity (power/area) $I = P_{avg}/S = 2\pi^2\rho v f^2 A^2$

Problem 2 : Provided the amplitude is sufficiently great, the human ear can respond to longitudinal waves over a range of frequencies from about 20.0 Hz to about 20.0 kHz. (a) If you were to mark the beginning of each complete wave pattern with a red dot for the long-wavelength sound, how far apart would the red dots be? (b) If you were to mark the beginning of each complete wave pattern with a blue dot for the short-wavelength sound, how far apart would the blue dots be? (c) In reality would adjacent red dots be far enough apart for you to easily measure their separation with a meter-stick? (d) In reality would adjacent blue dots be far enough apart for you to easily measure their separation with a meter-stick? (e) Suppose you repeated part A in water, where sound travels at 1480 m/s. How far apart would the red dots be ?

The distance between adjacent dots is the wavelength λ. The wavelength, velocity and frequency are related by $v = f\lambda$ or $\lambda = v/f$. The smallest frequency then will have the longest wavelength, and the highest frequency will have the shortest wavelength.

(a) For the red dots (long wavelength, therefore small frequency) we have $f = 20\ Hz$ so $\lambda = (344\ m/s)/(20\ Hz) = 17.2\ m$.

(b) For the blue dots (short wavelength, therefore high frequency) we have $f = 20,000\ Hz$ so $\lambda = (344\ m/s)/(20,000\ Hz) = 0.0172\ m$.

(c) The distance between the red dots technically could be measured with a meter-stick, although a tape measure would be more convenient since they are so far apart.

(d) The distance between the blue dots is $0.0172\ m$ or $1.72\ cm$ which could easily be measured.

(e) In water, the speed of sound is given to be about $1480\ m/s$. (This is the speed of sound near the surface anyway. In reality, it varies considerably with depth.) Now the red dots (representing the longer wavelength and therefore smaller frequency) would have a wavelength of $\lambda = v/f = (1480\ m/s)/(20\ Hz) = 74\ m$, so the dots will be $74\ m$ apart. (And correspondingly, the blue dots at the higher frequency would be $\lambda = (1480\ m/s)/(20,000\ Hz) = 0.074\ m$ or $7.4\ cm$ apart.

Problem 3 : On December 26, 2004, a great earthquake occurred off the coast of Sumatra and triggered immense waves (tsunami) that killed some 200000 people. Satellites observing these waves from space measured 800 km from one wave crest to the next and a period between waves of 1.0 hour. (a) What was the speed of these waves in meters/sec? (b) What was the speed of these waves in km/hour?

(a) The wave speed is given by $v = f\lambda$ but $f = 1/T$ so $v = \lambda/T$ which is a more convenient form here. The wavelength was $800\ km$, which is $800,000\ m$. The period is $T = 1\ hour = 3600\ s$, so $v = (800,000\ m)/(3600\ s) = 222\ m/s$.

(b) In km/hour $v = \lambda/T$ and we can just use the numbers in the units given so $v = (800\ km)/(1\ hr) = 800\ km/hr$ (which is about $500\ miles/hr$).

Problem 6 : A certain transverse wave is described by: $y(x,t) = (6.50\ mm)\sin[2\pi(\frac{x}{28.0\ cm} - \frac{t}{0.0360\ s})]$ Determine the wave's (a) amplitude, (b) wavelength, (c) frequency, (d) speed of propagation, and (e) direction of propagation.

This form directly matches one of the standard forms for the wave function: $y(x,y) = A\sin[2\pi(\frac{x}{\lambda} - \frac{t}{T})]$ so we can directly pick off the matching parameters. Here (a) $A = 6.50\ mm$, (b) $\lambda = 28\ cm = 0.28\ m$, and (c) $T = 0.0360\ s$ so $f = 1/T = 27.8\ Hz$.

(d) The wave speed is $v = \lambda f$ so $v = (0.28\ m)(27.8\ Hz) = 7.78\ m/s$.

(e) We can look at the sign of the t term to determine the direction. A negative sign means the wave is propagating in the +X direction.

Problem 8 : A water wave traveling in a straight line on a lake is described by the equation: $y(x,t) = (3.75\ cm)\sin[(0.450\ cm^{-1})x + (5.40\ s^{-1})t]$ where y is the displacement perpendicular to the undisturbed surface of the lake. (a) How much time does it take for one complete wave pattern to go past a fisherman in a boat at anchor? (b) What horizontal distance does the wave crest travel in that time? (c) What is the wave number? (d) What is the number of waves per second that pass the fisherman? (e) How fast does a wave crest travel past the fisherman? (f) What is the maximum speed of his cork floater as the wave causes it to bob up and down in the water?

This wave function is in one of the standard forms: $y(x,t) = A\sin(kx - wt)$ for waves traveling in the +X direction, or $y(x,t) = A\sin(kx + wt)$ for waves traveling in the -X direction. So this wave is apparently moving to the left (well, assuming +X is to the right) but otherwise that doesn't change the wavelength or wave speed or anything else, so we can still pick off the matching terms directly: $A = 3.75\ cm$, $k = 0.450\ cm^{-1}$, and $\omega = 5.40\ s^{-1}$.

(a) The time it takes for one complete wave pattern is the period, but $T = 1/f$ or $T = 2\pi/\omega$ so here $T = (2)(\pi)/(5.40\ s^{-1}) = 1.16\ s$.

(b) The wave travels one wavelength, and the wavelength is related to the wave number by $\lambda = 2\pi/k$ so here the wavelength (the distance from one crest to the next) is $\lambda = (2)(\pi)/(0.450\ cm^{-1}) = 13.96\ cm = 0.1396\ m$.

(c) The wave-number we already picked off the equation to be $k = 0.45\ cm^{-1}$. Converting this to meters, we note that k has units here of 'per cm', so $k = \frac{0.45}{1\ cm}$. To convert this to meters we would multiply by a factor of $\frac{100\ cm}{1\ m}$ to arrive at 45 *per meter* or $45\ m^{-1}$. We could also calculate the wave-number from $k = 2\pi/\lambda$ since we already have λ in standard metric units of meters so $k = (2)(\pi)/(0.1396\ m) = 45\ m^{-1}$.

(d) The number of waves per second is just the frequency f. $f = 1/T$ is a convenient form to use since we already calculated the period T, so $f = 1/(1.16\ s) = 0.862\ waves/sec$.

(e) Here we need to find the speed of these waves. $v = \lambda f$ is a convenient form since we have both of those, so $v = (0.1396\ m)(0.862\ s^{-1}) = 0.120\ m/s$ or $12\ cm/s$.

(f) For the vertical motion of a point on the surface, we found that the maximum y velocity was $v_{max} = \omega A$ so here $v_{max} = (5.40\ s^{-1})(3.75\ cm) = 20.25\ cm/s$ or $0.2025\ m/s$.

Problem 12 : The equation $y(x,t) = A\sin[2\pi f(\frac{x}{v} - t)]$ may be written as $y(x,t) = A\sin[\frac{2\pi}{\lambda}(x - vt)]$. (a) Use the last expression for y(x,t) to find an expression for the transverse velocity v_y of a particle in the string on which the wave travels. (b) Find the maximum speed of a particle of the string. (c) Under what circumstances is this equal to the propagation speed v?

(a) The transverse velocity $v_y = dy/dt$ (those should be partial derivatives but I can't figure out how to get that symbol to appear here at the moment). Differentiating the second equation, we get $v_y = A(\frac{2\pi v}{\lambda})\cos[\frac{2\pi}{\lambda}(x - vt)]$.

(b) The cosine function varies between 1 and -1, so the maximum value of the transverse velocity will be just $v_{max} = 2\pi v A/\lambda$

(c) This maximum velocity will be equal to the propagation speed when $2\pi v A/\lambda = v$ which will occur when $2\pi A/\lambda = 1$ or when $A = \lambda/(2\pi)$. (So for a given wavelength, if the amplitude of the wave is roughly 1/6 of that, the transverse speeds can get as high as the wave propagation speed.

Problem 14 : With what tension must a rope with length 2.50 m and mass 0.120 kg be stretched for transverse waves of frequency 40.0 Hz to have a wavelength of 0.750 m?

The velocity of transverse waves on the rope is related to the tension and the mass/length by $v = \sqrt{F/\mu}$. But the velocity is also tied to the frequency and wavelength by $v = \lambda f$. We're given $\lambda = 0.750\ m$ and $f = 40.0\ Hz$ so that fixes the wave speed to be $v = (0.750\ m)(40\ Hz) = 30\ m/s$.

The mass per unit length here is $\mu = M/L = (0.120\ kg)/(2.5\ m) = 0.048\ kg/m$.

We can now find the required tension since $v = \sqrt{F/\mu}$. We know v and μ so let's rearrange this equation to solve for F, the tension: $F = \mu v^2 = (0.048)(30)^2 = 43.2\ N$.

Problem 16 : A 1.50 m string of weight 1.25 N is tied to the ceiling at its upper end, and the lower end supports a weight W. When you pluck the string slightly, the waves traveling up the string obey the equation: $y(x,t) = (8.50\ mm)\sin[(172\ m^{-1})x - (2730\ s^{-1})t]$. (a) How much time does it take a pulse to travel the full length of the string? (b) What is the weight W? (c) How many wavelengths are on the string at any instant of time? (d) What is the equation for waves traveling DOWN the string?

For transverse waves on a string, $v = \sqrt{F/\mu}$. The wave function was given in one of the standard forms, so we can pick off the parameters: $k = 172\ m^{-1}$ and $\omega = 2730\ s^{-1}$. The wave speed, then, is $v = \omega/k = (2730)/(172) = 15.9\ m/s$.

(a) So we have a wave moving at 15.9 m/s. In a time t, it will cover a distance of $d = vt$ so $t = d/v$ or here $t = (1.50\ m)/(15.9\ m/s) = 0.0943\ s$. It takes each wave pulse less than a tenth of a second to propagate down the length of the rope.

(b) We were able to determine the wave speed, but this speed is related to the tension and the mass/length of the string by $v = \sqrt{F/\mu}$ or squaring both sides and rearranging terms, $F = v^2\mu$. (This will be a more convenient form, since we know the wave speed to be 15.9 m/s and we can use the information provided to find the μ for the string.) We were given the WEIGHT of the string, not its mass but the weight is just the mass times g, so the mass of the string must be $(1.25\ N)/(9.8\ m/s^2) = 0.12755\ kg$. Now that we know the mass and length of the string, we can calculate its μ to be $\mu = M/L = (0.12755\ kg)/(1.50\ m) = 0.0850\ kg/m$. The tension in the string therefore must be $F = v^2\mu = (15.9)^2 \times (0.0850) = 21.5\ N$. This tension is being provided by the hanging weight W so this in fact must be the weight of that object. ($W = mg$, so we could compute its mass to be $(21.5\ N)/(9.8\ m/s^2) = 2.19\ kg$, but the problem just asked for the weight, not the mass.)

(c) How many wavelengths are on the string at any given time? Well the string is 1.50 m long, and each wave has a wavelength of $\lambda = 2\pi/k = (2)(\pi)/(172\ m^{-1}) = 0.0365\ m$ so the total number of waves present is $(1.50\ m)/(0.0365\ m/wave) = 41.1\ waves$.

(d) The wave function for waves moving in the opposite direction is identical to the original equation, just changing the negative sign in front of the t term into a positive sign, so waves moving down the string would have a wave function of $y(x,t) = (8.50\ mm)\sin[(172\ m^{-1})x + (2730\ s^{-1})t]$.

Problem 18 : One end of a nylon rope is tied to a stationary support at the top of a vertical mine shaft of depth 80.0 m. The rope is stretched taut by a box of mineral samples with mass 20.0 kg attached at the lower end. The mass of the rope is 2.00 kg. The geologist at the bottom of the mine signals to his colleague at the top by jerking the rope sideways. (Do not neglect the weight of the rope.). What is the wave speed (a) at the bottom of the rope, (b) at the middle of the rope, (c) at the top of the rope?

The wave speed on the rope is given by $v = \sqrt{F/\mu}$, so depends on the tension in the rope at various points. Since we want to include the mass of the rope in our calculations, the tension will be changing. The mass per unit length μ is fixed though and is $\mu = M/L = (2.00\ kg)/(80\ m) = 0.0250\ kg/m$.

(a) At the very bottom of the rope, looking at $\Sigma F = 0$ we have the weight of the box of samples pulling down and the tension in the rope at that point pulling up, so the tension must have a value of $(20\ kg)(9.8\ m/s^2) = 196\ N$. The velocity of a transverse signal down at the bottom of the rope, then, is $v = \sqrt{F/\mu} = \sqrt{(196)/(0.025)} = 88.54\ m/s$.

(b) At the middle of the rope, we have the mass of the box (20 kg) plus half the mass of the rope (or an additional 1 kg) pulling down, and the tension in the rope at that point pulling up. So the force down at this point (and therefore the tension in the rope at that point) is $(21\ kg)(9.8\ m/s^2) = 205.8\ N$. The velocity of a transverse signal at the middle of the rope, then, is $v = \sqrt{F/\mu} = \sqrt{(205.8)/(0.025)} = 90.73\ m/s$.

(c) At the top of the rope, we have the mass of the box (20 kg) plus the full mass of the rope (or an additional 2 kg) pulling down, and the tension in the rope at that point pulling up. So the force down at this point (and therefore the tension in the rope at that point) is $(22\ kg)(9.8\ m/s^2) = 215.6\ N$. The velocity of a transverse signal up at the top of the rope, then, is $v = \sqrt{F/\mu} = \sqrt{(215.6)/(0.025)} = 92.87\ m/s$.

(So the wave travels faster and faster as it moves up the rope, being about 5 percent faster at the top, compared to the bottom.)

Problem 18 revisited : energy and power :

Let's look at the previous problem again but now in the context of energy and power. Restating the information given in the earlier version:

One end of a nylon rope is tied to a stationary support at the top of a vertical mine shaft of depth 80.0 m. The rope is stretched taut by a box of mineral samples with mass 20.0 kg attached at the lower end. The mass of the rope is 2.00 kg. The geologist at the bottom of the mine signals to his colleague at the top by jerking the rope sideways.

This time, let's ignore the change in wave speed as we move up the rope and just use the average value of about 91 m/s.

Suppose the geologist down at the bottom wiggles the rope back and forth with an amplitude of 4 cm and a frequency of 2 Hz. How much power is the geologist expending to create these waves travelling along the rope?

For transverse sinusoidal waves travelling along a wire/rope/string type geometry, the power being carried by the waves is given by: $P = 2\pi^2 \mu v f^2 A^2$. Here then:

- $\mu = M/L = (2\ kg)/(80\ m) = 0.025\ kg/m$
- $v = 91\ m/s$
- $f = 2\ Hz = 2\ s^{-1}$
- $A = 4\ cm = 0.04\ m$

The wave motion is thus carrying a power of $P = 2\pi^2 \mu v f^2 A^2 = (2)(\pi^2)(0.025)(91)(2)^2(0.04)^2 = 0.316\ W = 0.316\ J/s$.

That's not much power. One joule is just 4.184 cal, so this is just 1.32 $calories/sec$.

Every second the geologist spends wiggling the rope represents burning up just a little over a calorie of energy. It takes 1000 cal to make one food calorie (1 $Calorie$ = 1000 $calories$, probably one of the stupidest choices for units in history since they're both pronounced exactly the same). A typical candy bar is a few hundred Calories, so a few hundred thousand 'little-c' calories. It would take days to work off a candy bar at this rate...

Effect of changing wave speed

The wave speed is changing as the wave moves up along the rope, but energy needs to be conserved, so the wave must change in some way to account for this. Looking at the power equation, we have $P = 2\pi^2 \mu v f^2 A^2$, so what can change? v is getting higher as we move up the rope. We can't have fewer or more waves leaving the top of the rope than we create at the bottom (they aren't disappearing or being created out of nothing), so the frequency can't change. The only variable we have left is the amplitude. As v increases moving up the rope, the amplitude of the wave must be slightly decreasing.

Problem 20 : A piano wire with mass 3.00 g and length 80.0 cm is stretched with a tension of 25.0 N. A wave with frequency 120.0 Hz and amplitude 1.60 mm travels along the wire. (a) Calculate the average power carried by the wave. (b) What happens to the average power if the wave amplitude is halved?

Let's convert everything to standard metric units first. The mass is $M = 0.003\ kg$ and the length is $L = 0.80\ m$. The amplitude of the waves is $A = 0.0016\ m$.

(a) The average power carried by a transverse wave on a wire is given by $P_{avg} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$. We'll need the mass per unit length μ and the angular frequency ω so let's compute those first. $\mu = M/L = (.003\ kg)/(0.80\ m) = 3.75 \times 10^{-3}\ kg/m$. The angular frequency is $\omega = 2\pi f = 2\pi 120 = 754\ s^{-1}$. The average power carried by the wave then is $P_{avg} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2 = (0.5)\sqrt{(3.75 \times 10^{-3})(25.0)}(754)^2(0.0016)^2 = 0.223\ W$.

(b) The power is proportional to the square of the amplitude, so if we multiply the amplitude by 1/2, we will end up multiplying the power by $(1/2)^2$ or 1/4. Thus doing so would result in an average power transmission down this wire of only $0.223/4\ W$ or $0.056\ W$.

Problem 22 : You are investigating the report of a UFO landing in an isolated portion of New Mexico, and encounter a strange object that is radiating sound waves uniformly in all directions. Assume that the sound comes from a point source and that you can ignore reflections. You are slowly walking toward the source. When you are 7.5 m from it, you measure its intensity to be $0.11\ W/m^2$. An intensity of $1.00\ W/m^2$ is often used as the threshold of pain. How close to the source can you get before the sound intensity reaches this level?

The intensity drops off as the square of the distance from the source. If we let r_1 and I_1 be the radius and intensity at the initial point (when you are 7.5 meters away) and r_2 and I_2 be the radius and intensity at the desired position, then $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$. Rearranging this to solve for r_2: $r_2 = r_1\sqrt{I_1/I_2}$. Here, then: $r_2 = (7.5\ m)\sqrt{(0.11)/(1.00)} = 2.49\ m$.

Any closer and the intensity of the sound will exceed the pain threshold.

Problem 22 revisited : energy and power :

Let's look at the previous problem again in terms of energy and power. Initially we are standing $7.5\ m$ away and have a measured sound intensity of $0.11\ W/m^2$. We'll see later in chapter 16 that this represents a sound intensity of about 110 *decibels* ($110\ dB$) which is extremely loud - just below the pain threshhold of about $120\ dB$.

(a) How much power must the source be emitting?

$I = (power)/(area)$ so the source power would be $P = (I)(area)$. If we assume the sound is radiating out uniformly in all directions, then at this distance all that power is being spread over a spherical surface of radius $7.5\ m$, so $P = (I)(4\pi r^2) = (0.11\ W/m^2)(4)(\pi)(7.5\ m)^2 = 77.75\ W$. where here the area represents the surface of a

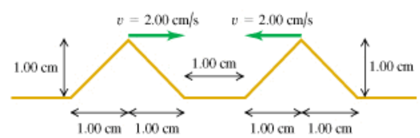
(b) If the sound is a pure $1000\ Hz$ sinusoidal signal propagating through air, how much are the individual air molecules moving back and forth as the wave passes through them?

For 3D waves like this, the power/area (intensity) is related to the various other parameters we have via: $I = 2\pi^2 \rho v f^2 A^2$, and here:

- $I = 0.11\ W/m^2$
- $\rho = 1.2\ kg/m^2$ (density of air at standard temperature and pressure)
- $v = 343\ m/s$ (speed of sound in air at STP)
- $f = 1000\ Hz = 1000\ s^{-1}$
- A is unknown

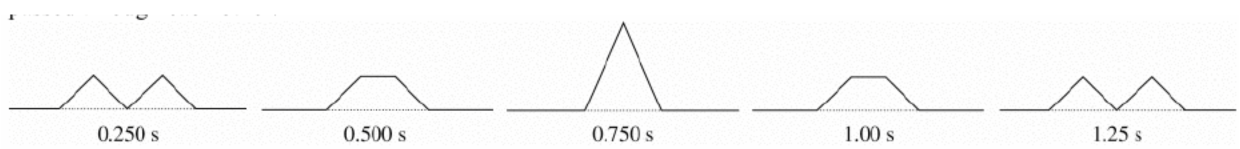
$I = 2\pi^2 \rho v f^2 A^2$, so: $0.11 = (2)(\pi)^2(1.2)(343)(1000)^2(A)^2$ which yields $A = 1.1 \times 10^{-5}\ m$ or $A = 0.011\ mm$: the molecules are only moving back and forth by about a hundredth of a millimeter, and that tiny motion hitting the eardrum is something we perceive as an almost painfully loud sound.

Problem 28 : Two triangular wave pulses are traveling toward each other on a stretched string as shown in the figure. Each pulse is identical to the other and travels at 2.00 cm/s. The leading edges of the pulses are 1.00 cm apart at t = 0. Sketch the shape of the string at $t = 0.250\ s$, $t = 0.500\ s$, $t = 0.750\ s$, $t = 1.000\ s$, and $t = 1.250\ s$.



The wave pulses will move independently, and the overall shape of the string at any time is just the algebraic sum of those two shapes. The best way to do these is on graph paper where you can sketch each wave separately in light dotted lines perhaps, than add the amplitudes point by point along the line to arrive at the amplitude of the string at that point.

For example, at $t = 0.25\ s$, the left pulse has traveled a distance of $(2.00\ cm/s)(0.25\ s) = 0.5\ cm$ to the right, while the right pulse has traveled a distance of $(2.00\ cm/s)(0.25\ s) = 0.5\ cm$ to the left. But they were only 1 cm apart to begin with, so they have closed the distance between them and are now just touching.



Problem 33 : Standing waves on a wire are described by $y(x,t) = A_{sw}\sin(kx)\cos(\omega t)$, with $A_{sw} = 2.50\ mm$, $\omega = 942\ rad/s$, and $k = 0.750\pi\ rad/m$. The left end of the wire is at $x = 0$. At what distances from the left end are (a) the nodes of the standing waves and (b) the anti-nodes of the standing wave?

(a) Nodes occur where the amplitude of the wave is always zero. That will be where $\sin(kx) = 0$ or where $kx = n\pi$ (i.e. any integer multiple of π). Rearranging this to solve for x: $x = n\pi/k = (n)(\pi)/(0.75\pi) = (1.333\ m)n$ where $n = 0, 1, 2, ...$

(b) Anti-nodes occur where the standing wave has its highest amplitude peaks. These will occur where $\sin(kx) = \pm 1$. The sine is equal to 1 or -1 when its argument is $\pi/2$ plus any integer multiple of π, which we can write as $(n + \frac{1}{2})\pi$. So at these points, $kx = (n + \frac{1}{2})\pi$ or $x = (n + \frac{1}{2})\pi/k$ but $k = 0.750\pi$ so finally $x = (n + \frac{1}{2})/0.750$ or $x = (1.333\ m)(n + \frac{1}{2})$, for $n = 0, 1, 2, ...$

Problem 34 : Adjacent anti-nodes of a standing wave on a string are 15.0 cm apart. A particle at an anti-node oscillates in simple harmonic motion with amplitude 0.850 cm and period 0.0750 s. The string lies along the +x-axis and is fixed at x=0. (a) How far apart are the adjacent nodes? (b) What are the wavelength, amplitude, and speed of the two traveling waves that form this pattern? (c) Find the maximum and minimum transverse wave speeds of a point at an anti-node. (d) What is the shortest distance along the string between a node and an anti-node?

(a) Adjacent nodes (and adjacent anti-nodes) are always $\lambda/2$ apart (i.e. every half wavelength). So if the anti-nodes are 15 cm apart, so are the nodes.

(b) The distance between the nodes (or anti-nodes) in a standing wave is exactly half the wavelength of the underlying waves that are producing this standing wave. So the wavelength of those traveling waves must be $2 \times 15\ cm$ or $\lambda = 30\ cm$. The angular frequency of the standing wave is the same as that of the underlying traveling waves, so the period is the same as well: 0.0750 s. The speed of each of the traveling waves can now be found, since $v = \lambda/T$ using one of the forms we have for v. Thus $v = (30\ cm)/(0.075\ s) = 400\ cm/s$ or $4.0\ m/s$. The amplitude of the standing wave is exactly twice the amplitude of the underlying traveling waves, so the traveling waves must have an amplitude of $(0.850\ cm)/2 = 0.425\ cm$.

(c) The wave function for the standing wave is: $y(x,t) = A_{sw}\sin(kx)\cos(\omega t)$, with the numerical values we have for this problem. The transverse speed (how fast a particle on the wire is moving up and down perpendicular to the wire) will be the partial derivative of this with respect to time, so $v_y = -A_{sw}\omega\sin(kx)\sin(\omega t)$. At an anti-node, $\sin(kx)$ is equal to 1 or -1. We're just looking for the maximum and minimum SPEED here, so we can ignore the signs. At an anti-node, then,: $|v_y| = A_{sw}\omega|\sin(\omega t)|$ The smallest value that $|v_y|$ can have will be when the sine term is zero, at which point the transverse wave speed is zero. The largest value that $|v_y|$ can have will occur when the sine term is 1 (or -1), at which point the maximum value of $|v_y|$ will be $A_{sw}\omega$.

For this particular wave, $\omega = 2\pi/T = (2)(\pi)/(0.075\ s) = 83.8\ rad/sec$, so the maximum value of $|v_y|$ will be $(0.850\ cm)(83.8\ s^{-1}) = 71.2\ cm/s$ or $0.712\ m/s$.

(d) The nodes and anti-nodes are separated by a quarter of a wavelength, so the distance between adjacent nodes and anti-nodes is $\lambda/4 = (30\ cm)/4 = 7.5\ cm$.

Problem 38 : A rope of length 1.50 m is stretched between two supports with a tension that makes the transverse waves have a speed of 48.0 m/s. What are the wavelength and frequency of: (a) the fundamental, (b) the second overtone, (c) the fourth harmonic?

For a string fixed at both ends, $\lambda_n = 2L/n$ and $f_n = n\frac{v}{2L}$ for $n = 1, 2, 3....$ Note from this the shortcuts that $\lambda_n = \lambda_1/n$ and $f_n = nf_1$.

(a) The fundamental represents the $n = 1$ case, so $\lambda_1 = 2L/1 = 3.00\ m$. $f_1 = (1)\frac{v}{2L} = (1)(48.0\ m/s)/(2 \times 1.5\ m) = 16.0\ Hz$.

(b) Overtones start their counting offset by one: the FIRST overtone is the same thing as the SECOND harmonic. So the second overtone means the third harmonic (n=3). So $\lambda_3 = \lambda_1/3 = 1.00\ m$ and $f_3 = 3f_1 = 48\ Hz$.

(c) The fourth harmonic represents the $n = 4$ case, so $\lambda_4 = \lambda_1/4 = 0.75\ m$ and $f_4 = 4f_1 = 64\ Hz$.

Problem 44 : One string of a certain musical instrument is 75.0 cm long and has a mass of 8.75 g. It is being played in a room where the speed of sound is 344 m/s. (a) To what tension must you adjust the string so that, when vibrating in its second overtone, it produces sound of wavelength 33.5 cm? (b) What frequency sound does this string produce in its fundamental mode of vibration?

This can be confusing because we're bouncing between what's going on on the wire, and what's going on in the air.

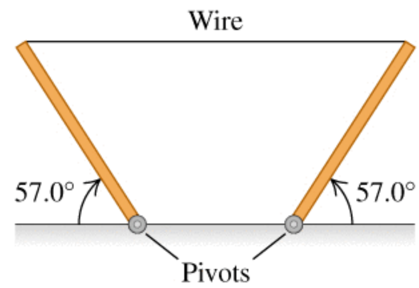
(a) Here, we have a sound wave (in air) with a wavelength of 33.5 cm or 0.335 m. What frequency does this represent? $f = v/\lambda = (344\ m/s)/(0.335\ m) = 1026.9\ Hz$. So we need the WIRE now to be producing this frequency, and more than that, it should be its second overtone (i.e. third harmonic) frequency! $f_n = n\frac{v}{2L}$ so $f_3 = 3\frac{v}{2L}$ The desired frequency is 1026.9 Hz and the string length is 0.75 m so $(1026.9) = (3)(v)/(2 \times 0.75)$ from which we can find that the velocity of waves on the WIRE must be 513.5 m/s.

The velocity of waves on the wire, though, is related to the tension in the wire and its mass per unit length via $v = \sqrt{F/\mu}$ so the required tension will be $F = \mu v^2$. We know v from the preceding paragraph, and can find μ: $\mu = M/L = (0.00875\ kg)/(0.75\ m) = 0.01167\ kg/m$. Finally then, $F = (0.01167)(513.5)^2 = 3,076.3\ N$ (This is probably higher than real string tensions would be. Typical string tensions in instruments like pianos and guitars are under 1000 N.)

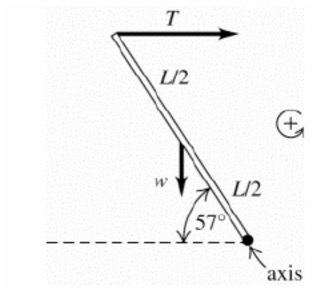
(b) The fundamentals are related by $f_n = nf_1$ so $f_3 = 3f_1$ or $f_1 = f_3/3 = (1026.9\ Hz)/3 = 342.3\ Hz$.

NOTE : the original version of this example, based on a homework problem from a previous textbook, used a wavelength (in air) of 3.35 cm, which represents a frequency of 10269 Hz, which is very high. Typically, frequencies this high would be created by very short strings under very high tension, so making the string 75 cm long resulted in a tension that was totally unrealistic. I changed the problem so that a more realistic frequency and tension would be involved.

Problem 56 : A 5.00 m, 0.732 kg wire is used to support two uniform 235 N posts of equal length. Assume that the wire is essentially horizontal and that the speed of sound is 344 m/s. A strong wind is blowing, causing the wire to vibrate in its 7th overtone. What are the frequency and wavelength of the sound this wire produces?



Focusing on the left pivot, we have two torques present: the weight of the left post, which is creating a counterclockwise (therefore positive) torque and the tension in the wire which is creating a clockwise (therefore negative) torque about that pivot. Setting up the $\Sigma\tau = 0$ equation lets us solve for the tension in the wire, which is 76.3 N. (This problem will NOT be on the test...)



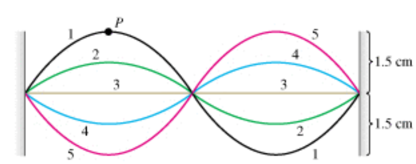
The wire has a mass per unit length of $\mu = M/L = (0.732\ kg)/(5.00\ m) = 0.146\ kg/m$. Waves on this wire then will propagate with a speed of $v = \sqrt{F/\mu} = \sqrt{(76.3/0.146)} = 22.86\ m/s$.

The fundamental frequency for this wire, then, is $f_1 = \frac{v}{2L} = (22.86\ m/s)/(2 \times 5.00\ m) = 2.286\ Hz$. The 7th overtone means the 8th fundamental, so $f_8 = 8f_1 = 18.3\ Hz$.

The frequency of the sound produced will be the same 18.3 Hz. The wavelength of this wave IN THE AIR can be found from $\lambda = v/f$ so $\lambda = (344\ m/s)/(18.3\ Hz) = 18.8\ m$.

(The wavelength of the wave IN THE WIRE can be found from $\lambda_n = 2L/n$ so $\lambda_8 = (2)(5.00\ m)/8 = 1.25\ m$.)

Problem 68 : A vibrating string 50.0 cm long is under a tension of 1.00 N. The results from five successive stroboscopic pictures are shown in the figure. The strobe rate is set at 5000 flashes per minute, and observations reveal that the maximum displacement occurred at flashes 1 and 5 with no other maxima in between. (a) Find the period, frequency, and wavelength for the traveling waves on this string. (b) In what normal mode (harmonic) is the string vibrating? (c) What is the speed of the traveling wave on the string? (d) How fast is point P moving when the string is in position 1? In position 3? (e) What is the mass of this string?



(a) Between flashes 1 and 5, the standing wave has gone through exactly one half of a complete period. (This is clearer if you can see the figure in color. The initial position of the wave looks like a sine function: i.e. it's the top curve on the left side but the bottom curve on the right side. At flash 5, the left half of the wave is now at the bottom of the figure and the right half is at the top. It has to go through this process again before it gets back to its original position (i.e. one complete period), so the time between flashes 1 and 5 represents exactly $T/2$ seconds. We have 5000 flashes per minute, or 5000/60=83.333 flashes per second. Each flash thus represents a time interval of 1/83.333 or 0.0120 s. Starting our clock at flash number 1, flash number 5 then is exactly four of these time intervals later, or $(4)(0.0120\ s) = 0.048\ s$. We just argued that this represents just HALF of a complete period, so from all this, we can say that $T = 0.096\ s$.

The frequency $f = 1/T$ so $f = 10.42\ Hz$.

We see one complete wave in the picture, so $\lambda = 0.500\ m$.

(b) The fundamental standing wave just has nodes at the end points and no nodes in between. The second harmonic (aka the first overtone) is the pattern with nodes at the ends plus one node in the middle.

(c) $v = \lambda f$ so $v = (0.500\ m)(10.42\ Hz) = 5.20\ m/s$.

(d) In position 1, point P is at its maximum displacement, so its transverse speed is zero. In position 3, the point is passing through the equilibrium position, which is the point where it has its maximum transverse speed. But this speed is $|v_{max}| = \omega A = 2\pi f A$. From the figure (you might have to blow it up) the amplitude $A = 1.5\ cm = 0.015\ m$ so $|v_{max}| = (2)(\pi)(10.42)(0.015) = 0.98\ m/s$

(e) The wave speed along the wire is given by $v = \sqrt{F/\mu}$. We know the wave speed to be 5.20 m/s and we know the tension in the wire is 1.00 N, so we can use this information to find what the mass per unit length is for this wire. Once we have that, the total mass of the wire will be μ times its length. Rearranging the equation to solve for μ: $\mu = F/v^2 = (1.00)/(5.20)^2 = 0.037\ kg/m$. The wire is 0.500 m long, so its mass must be $(0.037\ kg/m)(0.500\ m) = 0.0185\ kg$ or 18.5 g.

Problem 75 : A uniform cylindrical steel wire 55.0 cm long and 1.14 mm in diameter is fixed at both ends. To what tension must it be adjusted so that, when vibrating in its first overtone, it produces a note with frequency of 311 Hz? (Assume the string doesn't stretch, and that the density of steel is 7800 kg/m^3.)

The wave speed in a wire is given by $v = \sqrt{F/\mu}$. The frequencies of the standing waves in the wave are given by: $f_n = n\frac{v}{2L}$ or rearranging to solve for v: $v = 2Lf_n/n$. We desire a frequency of 311 Hz and for this sound to be the first overtone (i.e. the second harmonic so $n = 2$) so we need a wave speed of $v = (2)(0.55\ m)(311\ Hz)/(2) = 171.05\ m/s$.

Rearranging $v = \sqrt{F/\mu}$ to solve for the tension: $F = \mu v^2$. We have the required velocity already, but need to use the information given to figure out the mass per unit length.

The wire is being modeled as a cylinder, so it's total mass will be the density of the material times the volume of the wire, of $M = \rho(\pi r^2)L$. The mass per length is just M/L though, so $\mu = \rho(\pi r^2)$. For this wire, $r = 0.57 \times 10^{-3}\ m$ (note we had to divide the given value of the DIAMETER by two here and convert from mm to m) and $\rho = 7800\ kg/m$ so $\mu = 7.96 \times 10^{-3}\ kg/m$.

Finally, $F = \mu v^2 = (7.96 \times 10^{-3})(171.05)^2 = 233\ N$