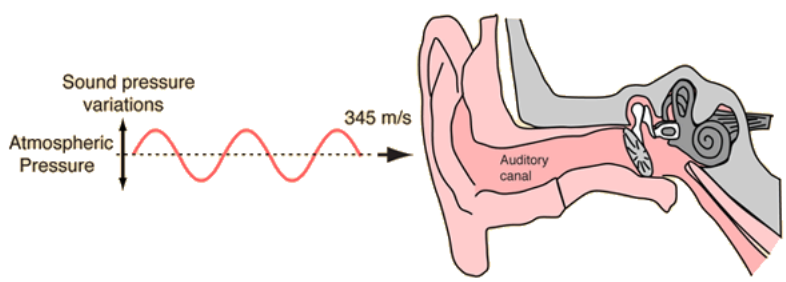
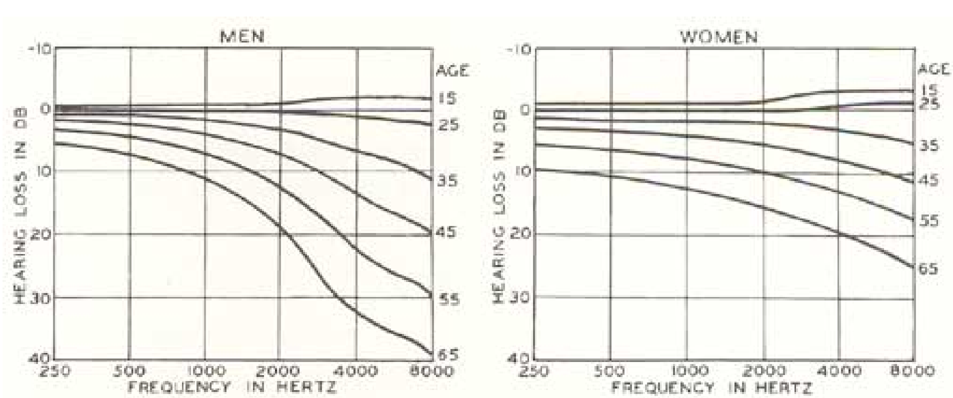
PH2233 Fox : Lecture 07
Chapter 16 : Sound

16.1 : characteristics of sound

- Sound is a longitudinal wave, commonly propagating through air (any gas really) or water (or any liquid). Pretty much any P-waves can be considered 'sound' waves though.
- Speed of sound : from the previous chapter (for gasses and liquids) $v = \sqrt{B/\rho}$ where B is the bulk modulus of the medium and ρ the volume density.
- In gasses and liquids, the density can change with temperature (especially with gasses), so the speed of sound will vary. (thermodynamics: $PV = nRT$). Approximate speed of sound in air (for 'human' range of temperatures anyway):
  $v \approx (331 + 0.60T)\ m/s$ where T is the temperature in deg C.
- **Loudness** : related to the intensity of the waves (twice the intensity 'sounds like' twice as loud, but our hearing is not very linear)
- **Pitch** : the frequency
- Range of human hearing : roughly 20 to 20,000 Hz.
- Ultrasound: f above 20,000 Hz
- Infrasound: f below 20 Hz (can have health effects)



Typical Audio Sensor in mammals



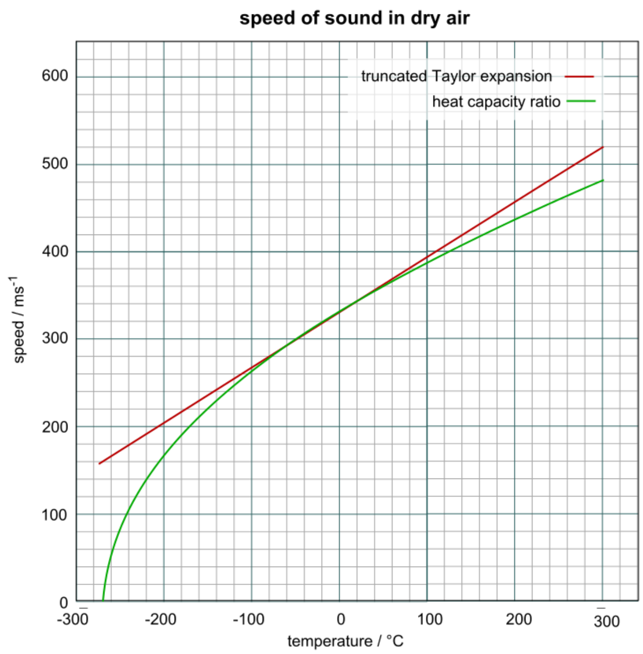
Audio Range and Sensitivity decreases with age

Speed of Sound in Air

If we look at dry air at standard pressure (1 atmosphere), the speed of sound can vary significantly.

On Earth, the lowest recorded temperature was $T = -89.2^oC$ (about -128^oF) in 2011 in Antarctica. The highest recorded temperature was $+56.67^oC$ (about 134^oF) in 1913 in California in the aptly named 'Furnace Creek' area.

Over that range, the $v \approx (331 + 0.60T)\ m/s$ approximation is quite good, so we'll be safe using it for most scenarios we'll encounter.

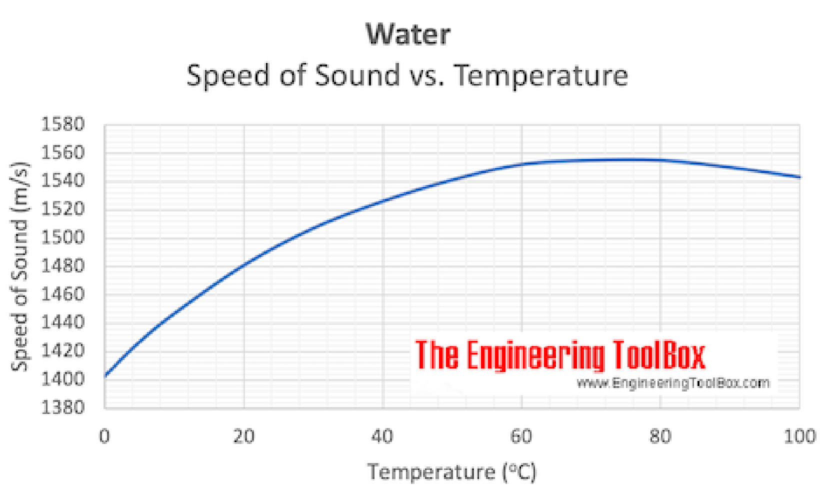


Example : The flash from a lightning strike travels at the speed of light; the sound from the thunder travels at the speed of sound. If we are exactly **one mile** away from the strike, how many seconds later will we hear the sound?

- Coldest place on earth: $v = 331 + (0.6)(-89.2) = 277\ m/s$ so $t = d/v = (1609\ m)/(277\ m/s) = 5.8\ sec.$
- At STP ($T = 20^o\ C$): $v = 331 + (0.6)(20.00) = 343\ m/s$ so $t = d/v = (1609\ m)/(365\ m/s) = 4.7\ sec.$
- Hottest place on earth: $v = 331+(0.6)(56.67) = 365\ m/s$ so $t = d/v = (1609\ m)/(365\ m/s) = 4.4\ sec.$

Speed of Sound in Water

Unlike the situation above, the speed of sound in water varies much more with temperature than the speed of sound in air over the range of temperatures we'll likely be interested in. This variation is something that sonar systems do need to account for.



Speed of Sound in Water (at standard pressure)

16.2 : mathematical representation of longitudinal waves

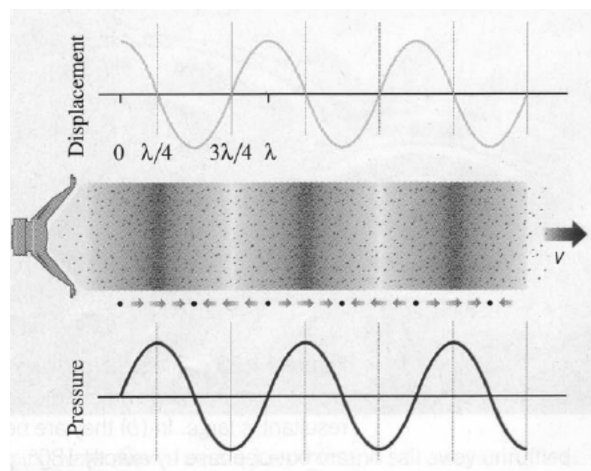
We've been using the form $D = A\sin(kx - \omega t)$ which describes the **displacement** of molecules in the medium as the wave passes by.

Sound is a longitudinal wave, meaning the displacement is in the same direction as the wave is travelling. The molecules in the medium (whether gas or liquid) get alternately compressed and rarified as the wave passes through a given location.

Suppose we have a continuous sine wave displacement function and see what effect it's having on the molecules. We'll arbitrarily pick some time and call it $t = 0$ so the longitudinal displacement of the molecules will be $D(x) = A\sin(kx)$ as shown here.

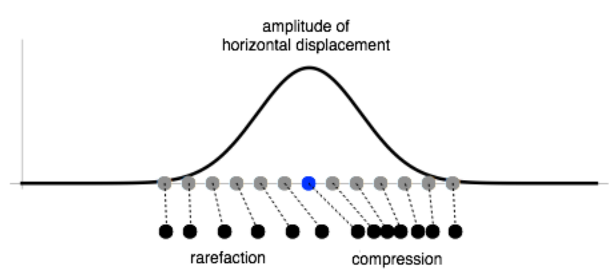
The top graph shows what the displacement of a molecule at that location would be at this moment in the wave's travels.

Look at the point labelled $\lambda/4$: molecules just to the left of that point have a positive displacement which means they've moved to the right (the $+X$ direction). Molecules just to the right of that point have a negative displacement which means they've moved to the left. The net result is that molecules are 'bunching up' at that point, resulting in a **higher pressure**.
Now, look at the point labelled $3\lambda/4$. Here, the molecules are doing the opposite. Those just to the left of that point have $D < 0$ so they've moved to the left. Molecules just to the right of that point have $D > 0$ so they've moved to the right. Here, they're 'thinning out', resulting in a **lower pressure**.



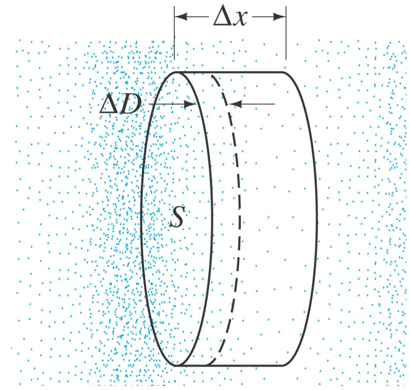
This is reflected in the lower part of the figure showing the pressure as a function of position at this particular snapshot in time.

Here we look at a Gaussian-shaped wave shape that's passing through the medium. The dots along the X axis are the original locations of a few molecules, and the dots below that show where they'll be at this instant due to the wave: again, the wave we've been writing as a **displacement** is resulting in a related **pressure** wave. As the displacement shape propagates at some velocity v through the medium, the corresponding pressure changes do the same.



Relating Displacement to Pressure Directly

Consider a little infinitesimal cylinder of air of some area S and length Δx. At a particular snapshot in time, the wave causes the left end of this cylinder to move with a displacement of $D(x)$, while the right end of this cylinder will move with a displacement of $D(x+\Delta x)$. The net change in the 'length' of this little cylinder will be $\Delta D = D(x+\Delta x) - D(x)$.



That means that the **volume** of the cylinder has changed, and we have a relationship between pressure changes and volume changes: $\Delta V = -\frac{1}{B} V \Delta P$ so this gives us a path to turn the displacement function (how individual molecules are being affected by the wave) to the pressure fluctuations that are also connected to the wave.

Now, $\frac{\partial D}{\partial x} = \lim_{\Delta x \to 0} \frac{D(x+\Delta x) - D(x)}{\Delta x}$ which means we can write $\Delta D = \frac{\partial D}{\partial x} \Delta x$.

Volume change will be the area times the change in D from one end to the other, so $\Delta V = S\Delta D$ and using what we just found, we can write this as: $\Delta V = (S)(\frac{\partial D}{\partial x})\Delta x$.

Now, $(S)(\Delta x)$ is just the original volume of this little cylinder, so making that substitution:

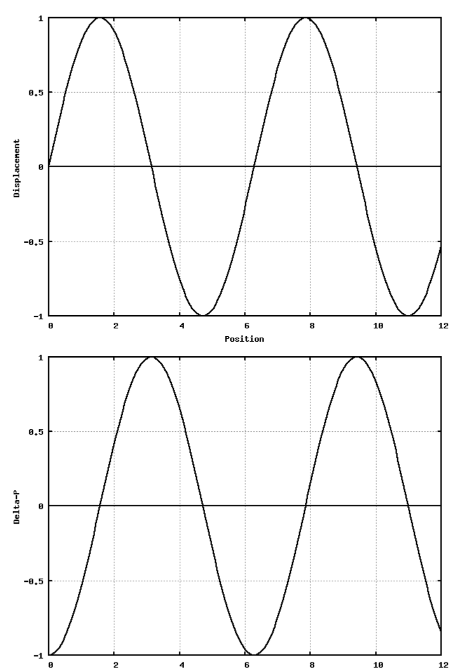
$\Delta V = V \frac{\partial D}{\partial x}$ or $\frac{\Delta V}{V} = \frac{\partial D}{\partial x}$

From the definition of the bulk modulus for a gas or liquid: $\frac{\Delta V}{V} = -\frac{1}{B} \Delta P$

Combining those and rearranging terms: $\boxed{\Delta P = -B \frac{\partial D}{\partial x}}$. That's the relationship we were looking for. Given some **displacement wave function** $D(x,t)$, we can immediately find the corresponding **pressure wave function** related to this wave.

Using $D(x,t) = A \sin(kx - \omega t)$ (our travelling sine wave) we find that the **pressure** wave (as a function of x and t) is: $\boxed{\Delta P = -BAk \cos(kx - \omega t)}$

These figures show the molecular displacement and corresponding pressure change at some snapshot in time.



The top figure shows the molecular **displacement**, with amplitude A.

The bottom figure shows the corresponding **pressure fluctuation** ΔP (high or low, relative to the ambient pressure present). The amplitude of these fluctuations will be $\Delta P_{max} = BAk$ where B is the bulk modulus of the medium, A is the molecular displacement amplitude, and k is the wave number $k = 2\pi/\lambda$.

Note that since the pressure wave is so closely connected to the displacement wave, they'll both have the same wavelength, frequency, and wave speed.

The pressure will fluctuate between $\pm BAk$ so the **amplitude** of the pressure fluctuations is usually written as $\boxed{\Delta P_{max} = BAk}$

Using $v = \sqrt{B/\rho}$ and $k = \omega/v = 2\pi f/v$ we can produce some other useful relationships:

$$\boxed{\Delta P_{max} = BAk = \rho v^2 Ak = 2\pi\rho v A f}$$

We can also morph the intensity equation we had earlier into a form relating intensity to the pressure fluctuations. Combining $\Delta P_{max} = 2\pi\rho v A f$ and the intensity equation: $I = 2\pi^2 \rho v A^2 f^2$ yields:

$\boxed{I = (\Delta P_{max})^2/(2v\rho)}$ (bypass worrying about f in this form)

Speaker Example Redux

Earlier we looked at a 40 W speaker putting out a pure tone of $f = 1000\ Hz$. In the ch15-lecture05 pdf example I had the speaker only putting out sound 'ahead' of the speaker, representing a quarter of a sphere, but in class I assumed an idealized omnidirectional speaker, so that's what we'll do here too.

At $r = 2\ m$ from the speaker, the 40 W of sound is being distributed over an area of $S = 4\pi r^2 = 50.265\ m^2$. The **intensity** then is $I = P/S = 40/50.265 = 0.79577\ W/m^2$. (We'll see later this is likely painfully loud.)

In the previous chapter we related the intensity to the displacement amplitude: $P = 2\pi^2 \rho S v f^2 A^2$ or $I = P/S = 2\pi^2 \rho v f^2 A^2$. At STP, the air density is about $\rho = 1.2\ kg/m^3$ and $v = 343\ m/s$ and here $f = 1000\ Hz$, so we find that $A = 9.897 \times 10^{-6}\ m$ or 0.009897 mm (about 0.01 mm).

What pressure fluctuation does this represent?

- $\Delta P_{max} = BAk$ where $B = 1.42 \times 10^5\ N/m^2$. We'll need the wavenumber. One path is $v = \omega/k$ so $k = \omega/v = 2\pi f/v = (2)(\pi)(1000)/(343) = 18.318\ m^{-1}$ so $\Delta P_{max} = (1.42 \times 10^5)(9.897 \times 10^{-6})(18.318) = 25.7\ N/m^2$.
- A simpler path is: $I = (\Delta P_{max})^2/(2v\rho)$ where $I = P/S = 0.79577\ W/m^2$ (found above). Rearranging: $\Delta P_{max} = \sqrt{2v\rho I} = \sqrt{(2)(343)(1.2)(0.79577)} = 25.6\ N/m^2$.

(The second one relied in fewer previous calculations, so it's probably 'better'.)

That sounds like a lot of pressure, but it really isn't.

Compare to atmospheric pressure : the weight of the atmosphere pushing down on us creates an ambient pressure called '1 atmosphere' or 1 ATM and that's about about 14.7 $pounds/in^2$ or $101,325\ N/m^2$. (Note: the units of N/m^2 is also called a **pascal**.)

So, our (roughly) 25.6 N/m^2 pressure represents a tiny deviation (about 1 part in 4000) on top of the normal atmospheric pressure we're already under.

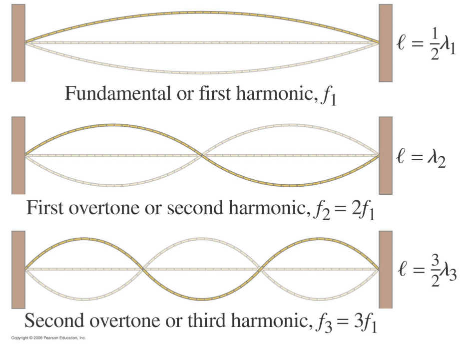
Force on the Eardrum : A typical adult eardrum is roughly a circular disk around 8 to 10 millimeters in diameter, so let's say a radius of 5 $mm = 5 \times 10^{-3}\ m$. That's a surface area of $S = \pi r^2 = 7.85 \times 10^{-5}\ m^2$. The force on the eardrum will be the pressure times the area, so this painfully loud sound is only exerting a force of $(force) = (force/area) \times (area) = (25.6\ N/m^2)(7.85 \times 10^{-5}\ m^2) = 2 \times 10^{-3}\ N$.

16.4 : Sources of Sound

Vibrating Wires/Strings : quick review

We saw last time that two identical waves travelling in opposite directions can create **standing waves** and one scenario where these appear is in vibrating strings or wires, which appear in stringed musical instruments. These standing wave patterns imply that the length of the string is some integer multiple of the wavelength of the waves: $L = N(\lambda/2)$ which means that only select wavelengths will 'fit' on the wire - wavelengths such that:

$\lambda_N = \frac{2L}{N}$ with $N = 1, 2, 3, ...$



Since $v = \lambda/T = \lambda f$ that implies that the wires vibrating in these modes represent frequencies of:

$f_N = N(\frac{v}{2L})$ for $n = 1, 2, 3, \cdots$.

where $v = \sqrt{F_T/\mu}$ (the tension in the wire divided by it's mass-per-length).

The frequencies are all integer multiples of the wire's lowest frequency (called the **fundamental**).

$\boxed{\textbf{An Old Test Problem}}$: The spokes in a bicycle wheel need to be adjusted to the proper tension. One way bike shops do this is to bang on each spoke and check what frequency of sound it emits. If the note is too low or high, they'll adjust the tension accordingly. Suppose we have a spoke that is 26.2 cm long and made of 15-gauge stainless steel wire (which means it has a diameter of 1.8 mm and a density of 8000 kg/m^3. What spoke tension is needed so that, when vibrating at its fundamental frequency, it produces sound with a frequency of 440 Hz?

1. (20) Tension = __________ N

This 'wire' will vibrate at frequencies of $f_n = n(\frac{v}{2L})$. The bike spoke is vibrating at it's fundamental ($n = 1$) so here $f_1 = \frac{v}{2L}$. Here then, $440 = \frac{v}{(2)(0.262)}$ so $v = 230.56$ m/s is the wave speed on this wire.

The wave speed $v = \sqrt{F_T/\mu}$ and we know v and can find μ so $F_T = v^2\mu$.

The mass of the wire will be its density times its volume. The wire is essentially a long thin cylinder with $L = 0.262$ m and $d = 1.8$ mm so $r = 0.9$ $mm = 0.9 \times 10^{-3}$ m. The cross-sectional area will be $S = \pi r^2 = 2.54469 \times 10^{-6}$ m^2 so we have a volume of $V = (S)(L) = 6.667 \times 10^{-7}$ m^3 and a mass of $M = \rho V = 5.3336 \times 10^{-3}$ kg and finally a mass per length of $\mu = M/L = 2.036 \times 10^{-2}$ kg/m.

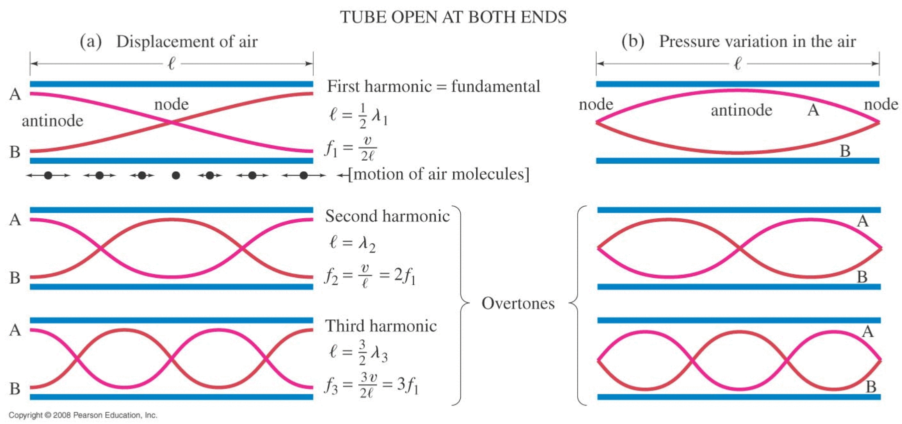
Finally then $F_T = v^2\mu = (230.56\ m/s)^2(0.02036\ kg/m^2) = 1082\ N$.

(That sounds high but apparently typical tensions range from 980 to 1200 Newtons. Apparently some spokes are actually hollow to reduce the total weight of the bike. That would make μ (the mass/length) value smaller which would reduce the tension $F_T = v^2\mu$.)

Vibrating Air Columns

The same standing-wave situation can occur with pressure waves, but figuring out what patterns of waves have to 'fit' in a given length is slightly more tricky.

Open Pipe : Consider a hollow tube, **open at both ends** as shown in the figure below. The air molecules at each end can move left or right in the figure - there's nothing stopping them from doing so. The important figure is on the right though where we look at what patterns of pressure fluctuations are 'allowed'.



At an open end, the air in the tube is basically in contact with an infinite reservoir of air where $\Delta P = 0$ so the standing wave patterns will be those wave shapes where a **pressure node** exists at each end of the tube.

Focusing on the right side, this works if the length of the tube is any integer multple of $\lambda/2$ so that each end of the tube can be a 'pressure node'.

That's the same 'pattern' we had with the transverse standing waves on a wire, so we end up with the same equations for the standing wave frequencies and wavelengths: $\boxed{\lambda_N = \frac{2L}{N}}$ and $\boxed{f_N = N\frac{v}{2L}}$ for $N = 1, 2, 3, ...$ and where v is the speed of sound in the medium.

Example

(a) How long would an organ pipe (open at both ends) need to be to resonate at middle C (a frequency of $f = 261.626\ Hz$)?

$f_n = n\frac{v}{2L}$ and if this is the fundamental frequency of the pipe then $f_1 = \frac{v}{2L}$.
The speed of sound at $20^o\ C$ is $v = 343\ m/s$ so rearranging:
$L = \frac{v}{2f_1} = \frac{343\ m/s}{(2)(261.626\ s^{-1})} = 0.6555\ m$.

(b) Suppose it's 38^oC (about 100^oF) in the room. What frequency will the pipe produce?
$v = 331 + 0.6T = 331 + (0.6)(38) = 353.8\ m/s$
so $f_1 = \frac{v}{2L} = \frac{353.8}{(2)(0.6555)} = 269.9\ Hz$ (about a half-note too high).

TABLE 16–3 Equally Tempered Chromatic Scale†

Note	Frequency (Hz)
C	262
C♯ or D♭	277
D	294
D♯ or E♭	311
E	330
F	349
F♯ or G♭	370
G	392
G♯ or A♭	415
A	440
A♯ or B♭	466
B	494
C′	524

†Only one octave is included.

This is an issue with all wind instruments. The frequencies they produce depend on the speed of sound, which in turn depends on the temperature:

$f_1 = \frac{v}{2L}$: the fundamental
$v = 331 + 0.6T$: speed of sound in m/s depends on temperature T (deg C here)

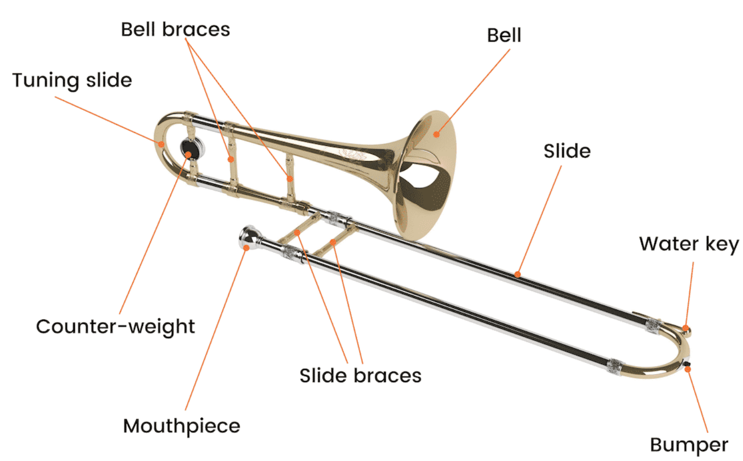
As the air inside the instrument changes temperature (either because the ambient temperature in the vicinity is changing, or as the instrument reacts to the hot air being blown into it), it's frequencies will change. How is this addressed in actual wind instruments then?

In the case above (going from normal room temperature to an unpleasantly hot room) the fundamental went from the required $261.626\ Hz$ to an actual $269.9\ Hz$, which means the frequency increased by a factor of 1.0316. If we increase the length by the same percentage, it will return to producing the correct frequency. (I don't know if pipe organs can be adjusted like this - maybe there's a part of the pipe at the bottom that can be adjusted in and out to change the length of the tube...)

A couple of students familiar with specific instruments provided the answer here. Since $f_1 = \frac{v}{2L}$, as v changes, we can 'fix' the problem by changing the length of the pipe.

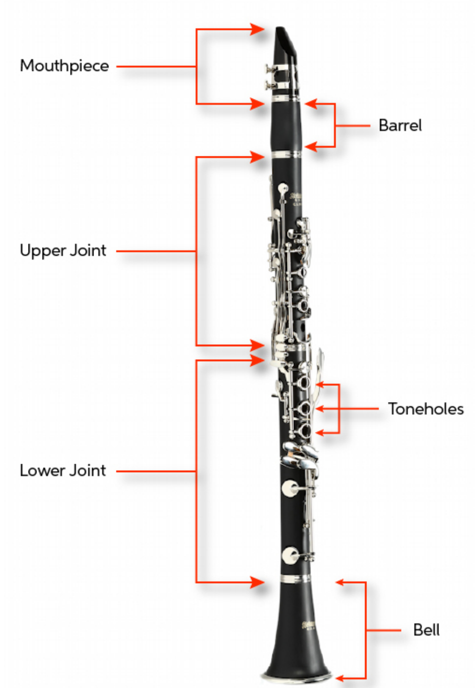
Length-adjustment components in wind instruments

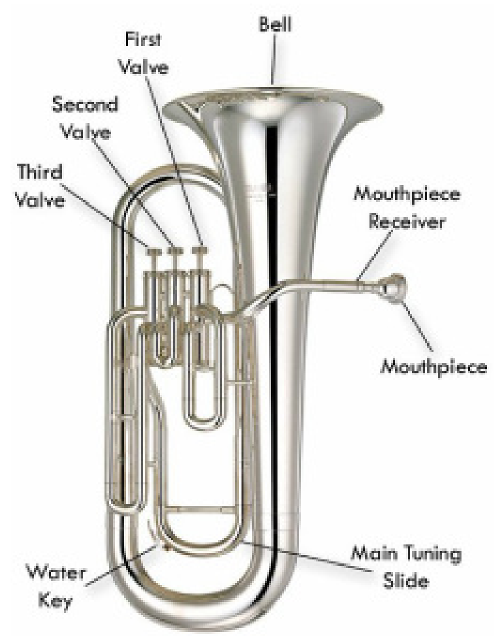
TROMBONE : This is somewhat automatic in the case of an instrument like a trombone where the length of the pipe is continuously variable by moving the slide in and out. In the diagram on the left, there's also a section called the 'tuning slide' which can be moved to alter the length of the pipe so that a particular position of the slide itself will always represent a given note.



CLARINET (left figure) : the part labelled the 'barrel' can be rotated to alter the distance between the two parts it's connected to, resulting in changing the overall length of the pipe.

TUBA (right figure) : note the part labelled the 'main tuning slide' which can be moved in and out to alter the overall length of the pipe.





PIPE ORGAN (below) : Since our example involved a pipe organ I tried to find how those might be adjusted but was not very successful. It *looks* like the top part of the pipes in this figure are different pieces and *maybe* they can slide up or down to alter the fundamental frequency of the pipe but I'm not sure that's how it's actually done.

It's also possible that pipe organ is taken as the reference and the other instruments are then tuned to match whatever frequency it's producing.