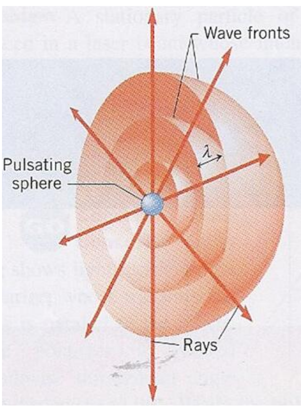
# PH2233 Fox : Lecture 10
# Chapter 32 : Light Reflection and Refraction

## 32.1 : the ray model of light

**RAY vs WAVE models** :

Consider our little spherical carbon nanotube speaker, putting out sound uniformly in all directions. The left figure below shows these sound waves spreading out from the speaker. We can take a point on each wave (where the pressure is equal to it's maximum value, for example) and call that a **wave front**. These waves are spreading spherically from the source so each sphere (each 'wave front') is perpendicular to the radius vector from the source, and this vector perpendicular to the wave front is called a **ray**.

The figure on the right is a cross section through the left figure more clearly showing the 'rays' and 'wave fronts' that describe how the sound is spreading out from the speaker.



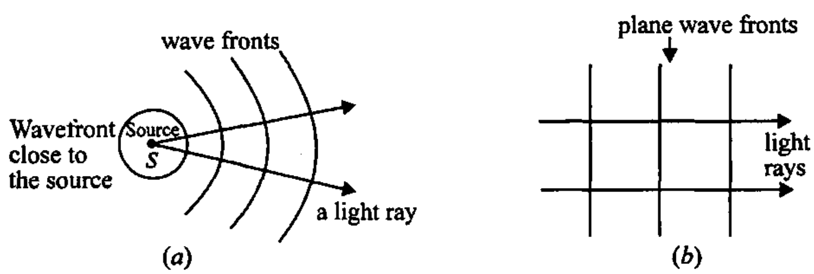




**Fig. 20.1**

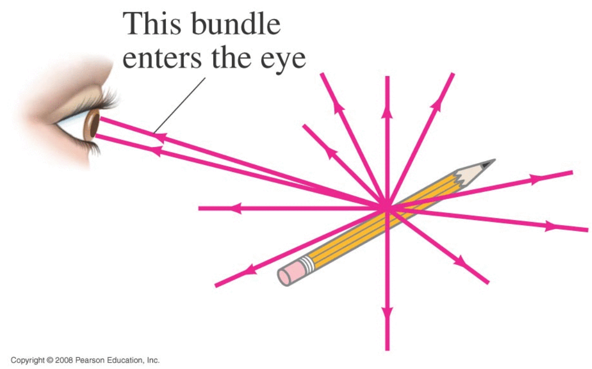
We can analyze waves in two ways then: looking at the **waves** themselves (via the wave equation), or following the **rays** since we know the wave fronts will be perpendicular to those rays.

Both approaches (models) have their uses.

**Light** can also be analyzed both ways. There is an underlying wave equation that connects electric and magnetic waves travelling at the speed of light, and chapters 34 and 35 will focus on the wave model for electomagnetic waves (light, x-rays, radio and TV waves, etc) but for the next two chapters we'll focus on the ray model, where we treat light as if it were made of tiny massless particles (**photons**) travelling along the rays at the speed of light. (Sound and other P and S wave disturbances travelling through a medium can also be treated as particles following raypaths, in which case these 'particles' are sometimes called phonons.)

The fact that we'll be focusing on these 'rays' (the nominally straight-line paths taken by a photon) leads to this type of analysis being called **geometric optics**.

When we look at an object, how do we 'see' it? Focusing on the 'ray' model, ambient light in the room hits the object and is scattered in 'all' directions, with some small fraction of those photons making it through our pupil to our retina.




Ray Model for Light (photons)

Where did the light illuminating the pencil come from? Some probably came directly from the lights overhead, some probably came from the Sun, passed through a window, bounced off a wall, then maybe bounced off a desk and someone's glasses and ultimately reached the pencil, before finally reaching my eye.

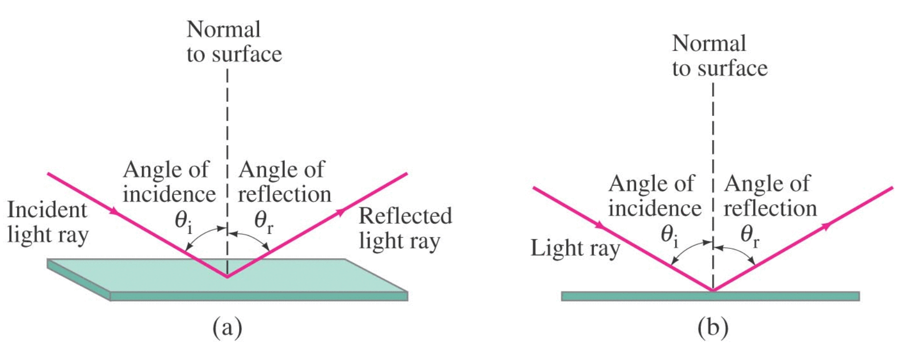
Let's focus on these interactions: the 'reflections' that the photon underwent on it's way to us.

## 32.2 : reflection : image formation by a plane mirror

The figure below shows a ray of light bouncing off a mirror. Consider a tiny particle bouncing off the perfectly flat surface of a vastly heavier object. If there's no friction present and we have a perfectly elastic collision, look at the (vector) conservation of momentum where the only force acting on the particle is perpendicular to the surface.

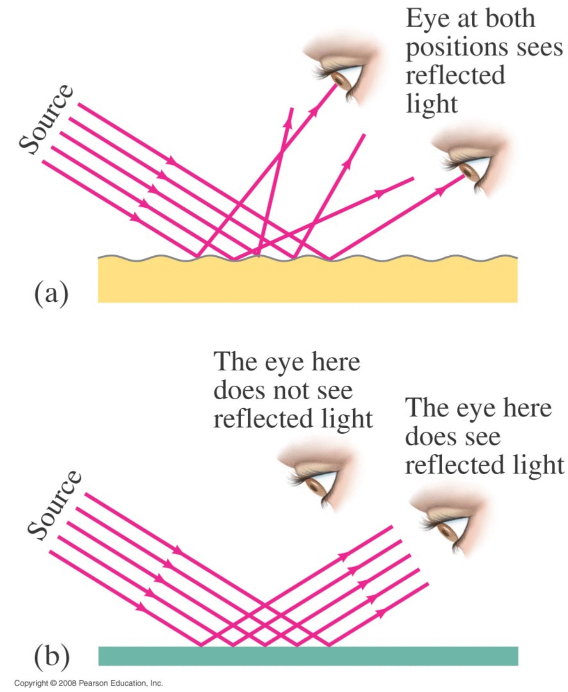
The only two solutions are that the particle either continues moving in the same direction, passing through the surface, or it's component of momentum normal to the surface gets reversed.

Photons are particles that carry momentum even though they're massless and they behave the same way. An X-ray photon (or one representing radio or TV frequencies) would pass through a sheet of paper as if it weren't even there. A photon of visible light encountering a smooth metal surface will 'bounce' as shown here though.




The figure is annotated with some terms we'll use to describe this effect. In particular note that the **angle of incidence** and **angle of reflection** are measured relative to a line that's **normal** (perpendicular) to the surface at the point where the photon strikes the surface.

Most materials aren't perfectly smooth or even perfectly reflective, and incoming rays get scattered in other directions. This is called **diffuse reflection**.
The smoother and more reflective a surface is though, the less the scattering is, ultimately yielding **specular reflection** as shown on the right below.
If we hit a sheet of paper with a laser, we can see the bright spot from any direction (diffuse reflection).
If we hit an ideal mirror with a laser, the reflection can only be seen from a 'single' angle.




DIFFUSE vs SPECULAR reflection

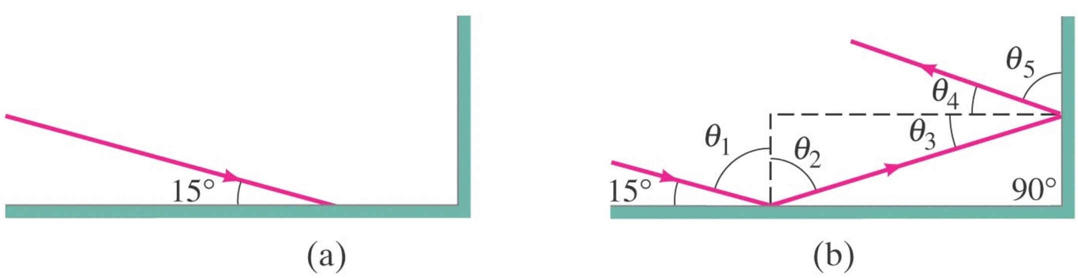
**Example 32-1 Corner Reflector**

A device that cleverly exploits specular reflection is a **corner reflector**. You've probably seen these at night: little disks with jagged-looking surfaces that reflect headlights and are used to warn of obstacles you might not be able to see directly (or denote driveway locations on dark roads).

Let's start with a simpler geometry, as shown in the figure below. A photon strikes the horizontal surface (let's call that the X axis) making an angle of $15^o$ relative to the surface, meaning an angle of incidence of $\theta_1 = 90 - 15 = 75^o$. It reflects at $\theta_2 = 75^o$.

The photon now hits the vertical (Y axis) part. Propagating angles, $\theta_3 = 90 - 75 = 15^o$ and will reflect at $\theta_4 = 15^o$.

Note that this direction is exactly the same angle above the horizontal that the incoming ray was travelling below the horizontal. The reflected ray is heading back parallel to the incoming ray, just in the exact opposite direction.

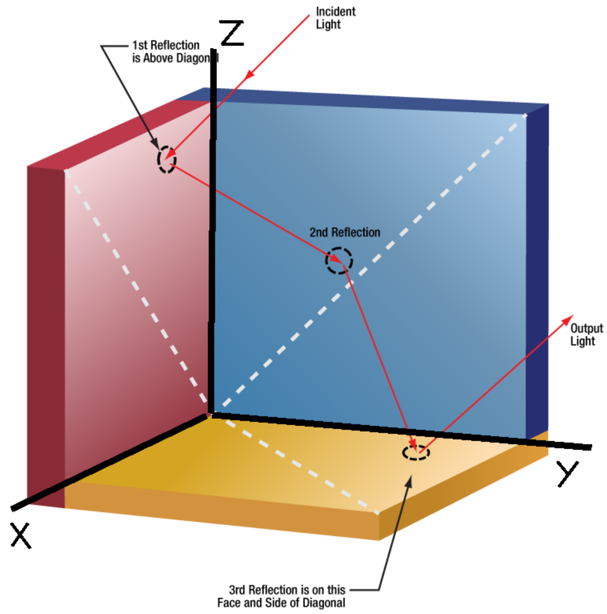


Let's extend this to a full 3-D corner reflector. Note from the above figure than when the ray (i.e. the photon) hit the mirror aligned with the X axis, it's $v_x$ didn't change by it's $v_y$ (the component of velocity perpendicular to the surface) flipped its sign.
Consider the vector $\vec{v}$ velocity of the photon.

1. Reflects off the XZ plane, resulting in:
   $\mathbf{v_y}$ being reversed.

2. Reflects off the YZ plane, resulting in:
   $\mathbf{v_x}$ being reversed.

3. Reflects off the XY plane, resulting in:
   $\mathbf{v_z}$ being reversed.

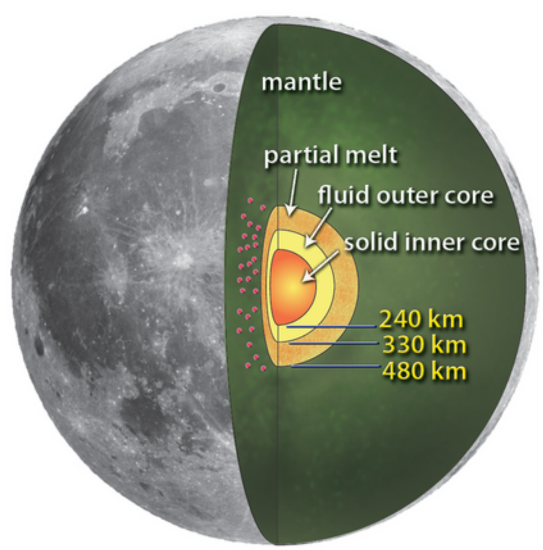



The net effect is that each component of the incoming $\vec{v}$ has now been reversed, resulting in the outgoing ray having a velocity of exactly $-\vec{v}$ : i.e. it's travelling in the exact opposite direction. (It was shifted over slightly, but at least it's heading back towards the source.) The outgoing beam is returned parallel to the incoming beam.

Besides driveway reflectors, these are also used in **surveying** to measure distances. A pulse of light is sent out, and reflects back to the device which measures the travel time. Light in air travels at very nearly the speed of light of a vacuum, so the travel time can be converted into the distance from the laser and the reflector.

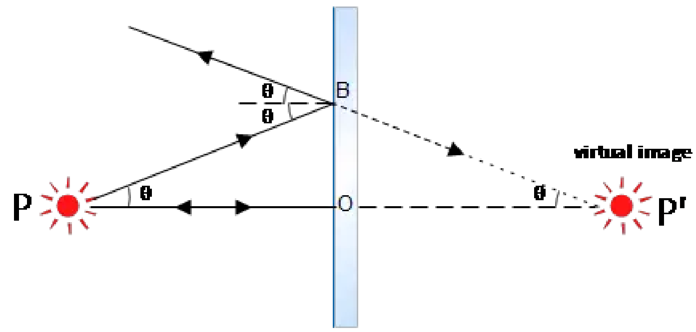
Several of these **retro-reflectors** have been placed on the moon, starting with the original Apollo 11 mission in July 1969. Apparently 6 are still operational, including that first one, and including one deployed recently (August 2023) by India's Chandrayaan-3 lunar lander. Here's a wikipedia article about them: `https://en.wikipedia.org/wiki/List_of_retroreflectors_on_the_Moon`

**Lunar Range Experiment** : by measuring the travel time from the Earth to the Moon and back, the distance to the Moon can be measured to within a few **millimeters** now. Among other things, these reflectors have shown that the moon is gradually moving away from the Earth at a rate of about 3.8 $cm/year$, and by measuring the slight wobbling of the Moon, it's been determined that the Moon may contain a small molten core like the Earth (even though it's not producing a magnetic field like we have on Earth).



## Image Formation with a Flat Mirror

Let's look at the light coming from a single point (labelled P) on an object that is located in front of a mirror.
Light from some (or multiple) source(s) hits the object. Photons are scattered from that point and (probably) fly out in 'all' directions each travelling in a straight line (a 'ray').



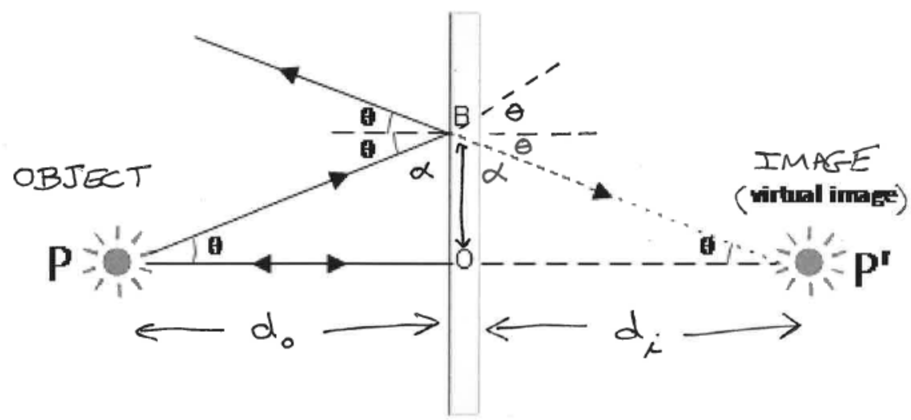

A ray (photon) that heads directly towards the mirror reflects right back along the same path (the ray from P to O and back).

A ray heading off at some angle $\theta$ above that direction (which is perpendicular to the mirror) hits the mirror further away. It's reflected such that $\theta_r = \theta_i$ but note that here all those angles are the same as $\theta$.

In the lower figure, I've extended the lines defining the actual photon path into dotted lines behind the mirror. Notice from geometry that we have the same $\theta$ angles on that side also. That means the angle I labelled $\alpha = 90 - \theta$ on the left is the same angle also labelled $\alpha$ on the right.

Ultimately, triangle BOP is **similar** to triangle $BOP'$.

More than that though, since those two triangles **share a side** (the line segment $BO$) they're not just similar, they're **identical** and the distance $d_o$ (how far the point $P$ is from the mirror) is the same as $d_i$ (how far the point $P'$ is behind the mirror).

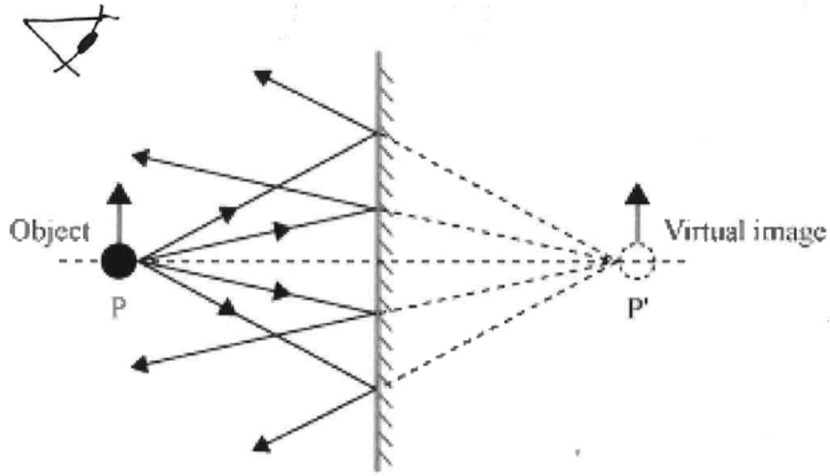


The location of point $P'$ didn't depend on $\theta$, so **every** ray (photon) that leaves P and hits the mirror appears to be coming from that same 'mirror image' point $P'$.

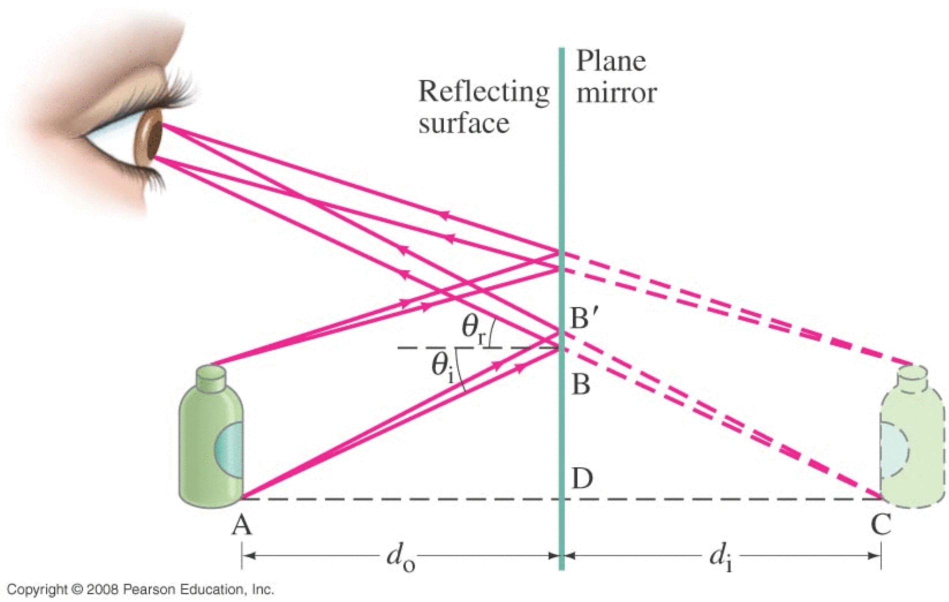
We see something because photons (rays of light) appear to be coming from it. Maybe they actually **are** coming from it (when we look directly at something) but here we also see the copy of the object that appears to be behind the mirror.

Our eye (or a camera placed there) 'sees' rays that seem to be coming from an object located behind the mirror.

Since no photons are really coming from that point, they just **appear** to be, the image is called a **virtual image**.

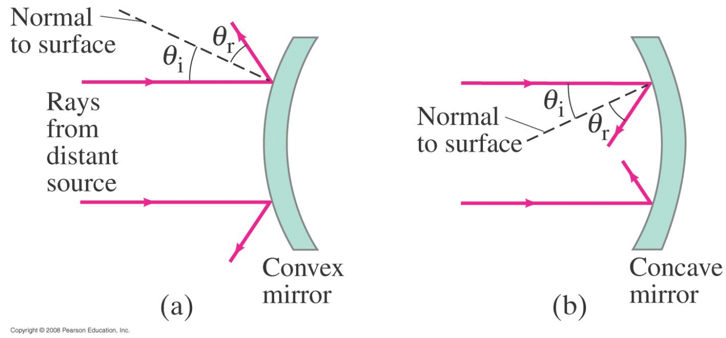



The same process occurs for every point on an object. Rays coming from a point on the top of the bottle below appear to be coming from the top of the mirror-image bottle behind the mirror. Ditto the rays coming from any other point that makes up the complete object.



## 32.3 : formation of images by spherical mirrors

Consider a mirror in the form of a section of a sphere of radius $r$. Rays coming in from the left will reflect off the mirrored surface. On the left is a **CONVEX** mirror, with rays being reflected outward. On the right, we see a **CONCAVE** mirror, with the rays being reflected inward.
The sign of the radius of curvature $r$ is taken to be positive for the concave mirror since the center of the sphere is on the same side of the mirror as the actual rays (photons). In the convex case, the center of curvature is 'behind' the mirror, where the rays (photons) don't physically visit and we'll call that a negative $r$.

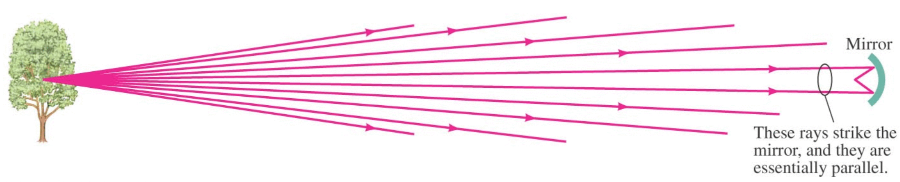



CONVEX vs CONCAVE mirrors. Note that in each case $\theta_i = \theta_r$

For most of the real-world cases we'll be considering, the mirror is only slightly curved, with objects fairly far away compared to the physical size of the mirror.

The effect is that we'll almost always be dealing with very small angles.

The Hubble telescope is (in effect - we'll see more on this later) a concave mirror with $r = 115.2\ m$ but the mirror itself is only 2.4 $m$ across from edge to edge. The rays of light (photons) coming in from a distance object differ in direction only very slightly. Given the magnitudes here, we can use $l = r\theta$ where $r$ in this case is how far away the object is that we're looking at. Mars at its closest approach to the Earth is about $5.45 \times 10^{11}\ m$ away, so the maximum difference between the rays coming from Mars into the telescope is about $\theta = (2.4\ m)/(5.45 \times 10^{11}\ m) = 4 \times 10^{-11}\ radians$ or about $2.5 \times 10^{-9}\ degrees$.

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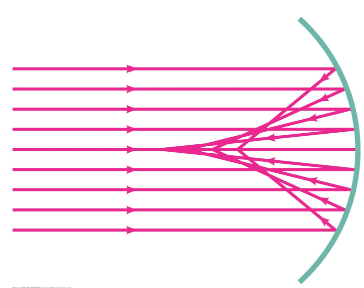
Consider a perfectly spherical concave mirror with light rays coming in from 'far away' (i.e. parallel incoming rays). What will happen to them?
The left figure shows several such rays. A ray travelling along a path that passes through the center of the sphere (this path is called the **principal axis** of the mirror) bounces straight back. As we move away from this axis, the rays intersect the principal axis at points there are moving closer to the mirror.



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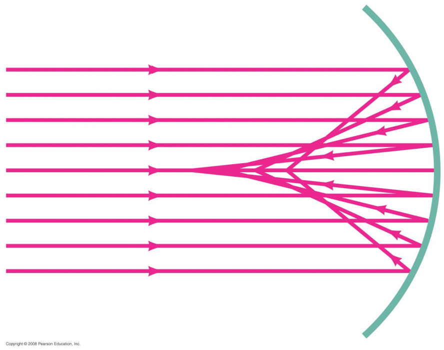
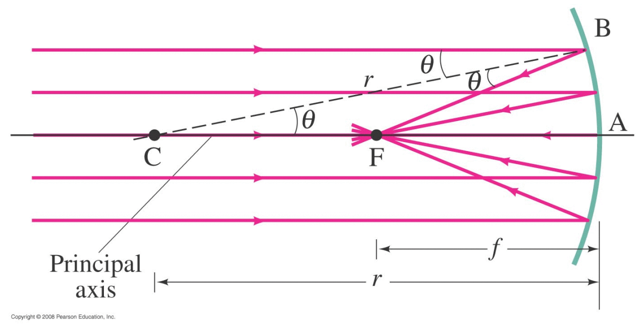
Let's 'do the math' here and see **exactly** where the rays go.



Actual Ray Paths




Used for the derivation we're about to do!

Consider a ray that is some distance $y$ above the principal axis. That hits the mirror at some point (B), reflects (with an angle of incidence $\theta$ equal to an angle of reflection, also $\theta$), and the ray then passes through the principal axis at some point (F). Let $x$ be the distance from C to F.

Note the equilateral triangle being formed here: CBFC. Let's cut that triangle in half by dropping a perpendicular from F to the line CB. Then $\cos\theta = (r/2)/(x)$ or $x = \frac{r}{2}\frac{1}{\cos\theta}$.

If we drop a perpendicular from point B to the principal axis, that segment will have a length of $y$, so we see that $\sin\theta = y/r$.

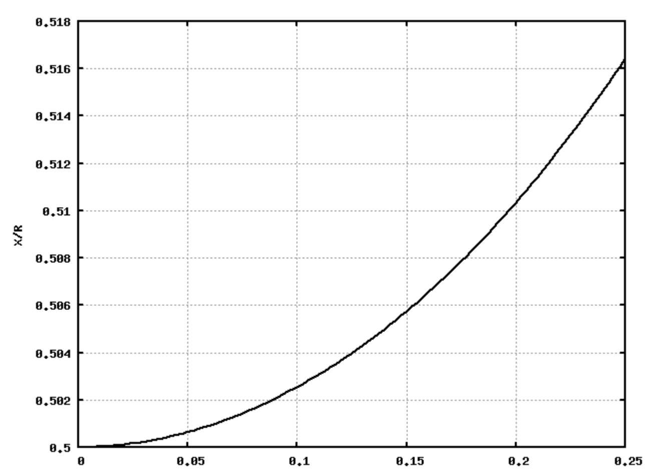
Now $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-(y/r)^2}$

So finally: $\boxed{x = \frac{r}{2}\frac{1}{\sqrt{1-(y/r)^2}}}$

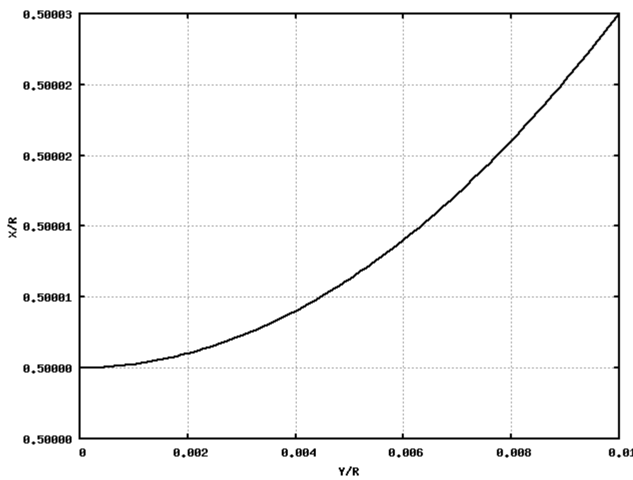
This tells us where each ray, parallel to the principal axis but $y$ away from it, will intersect the principal axis (and we made **no** approximations (yet), so this is an exact result).

Here are a couple of plots showing how that function behaves. Moving an $r$ from the right to the left side, we can write the equation as: $\frac{x}{r} = \frac{0.5}{\sqrt{1-(y/r)^2}}$ so let's plot $x/r$ as a function of $y/r$. Remember $y$ is how far the incoming ray is away from the principal axis, so $y/r$ can't be any larger than 1. It's likely much smaller than that, so here we plot $x/r$ out to $y/r = 0.25$.

With this geometry, rays near the pricipal axis certainly hit near $x = r/2$ but as we move away from there, the rays start hitting further away from that point, moving towards the vertex of the mirror. Even with this (unrealistic) geometry though, they're only a few percent off from the $x = r/2$ location.



Here's a similar diagram for the Hubble telescope which has a diameter of 2.4 $m$ and an effective radius of curvature of $r = 115.2\ m$. The largest that $y/r$ can be now is $1.2/115.2 \approx 0.01$ so let's make the same plot using that as the 'worst case' value for $y/r$.




We see that $x/r$ varies only very slightly from 0.5, reaching a worse-case of just 0.50003. Rays near the principal axis hit at $x/r = 0.5$ or $x = 57.600..\ m$ and the rays hitting the outer edge of the mirror end up at $x = (0.50003)(115.2\ m) = 57.603\ m$. That's only 0.003 $m$ or 3 $mm$ away from the ideal $x = r/2$ location.

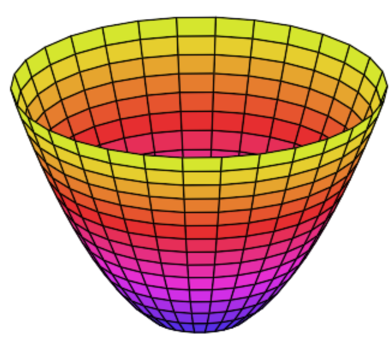
If we put a detector or piece of film at $x = r/2$, light rays coming from a point on some distant object (say Mars) still don't all focus at the same point exactly, resulting in a (minute) level of blur. We'll see how we can 'fix' this in a moment.

Since the rays very closely all pass through the same point $x = r/2$, that point is called the **FOCAL POINT** of the mirror (which is why it got the label F).

**If we're going to spend a lot of money on a space-based telescope, can we do better with the mirror and find a geometric shape for which all the parallel rays actually do exactly pass through a single point? If we built our mirror in that shape, we could eliminate this problem.**

(Careful here - in what follows I've switched what I'm calling X and Y from the previous discussion.)

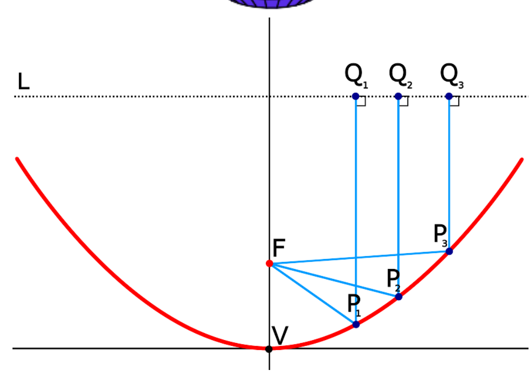
What if we make the mirror in the shape of a parabola (technically a **paraboloid**, where we rotate the parabola around it's axis):



$y = Ax^2$ or write as $y = \frac{1}{4f}x^2$ where $x$ will be the 'focal point' of this mirror.

Note that this means if $x = R$ (the radius of the mirror) and $D$ is its 'depth' (from the rim to the bottom of the parabola) then $4DF = R^2$ : convenient shortcut to determine how 'deep' a given parabolic reflector is given its focal length and diameter (radius).

For Hubble, $R = 1.2\ m$ and $F = 57.6\ m$ so $D = R^2/(4F) = (1.2)^2/(4 \times 57.6) = 0.0063\ m$ or 0.63 $cm$.




The outer rim of the mirror is less than a centimeter above the lowest point in the mirror. If you looked at the mirror from the side you'd just be able to make out this very slight curvature.

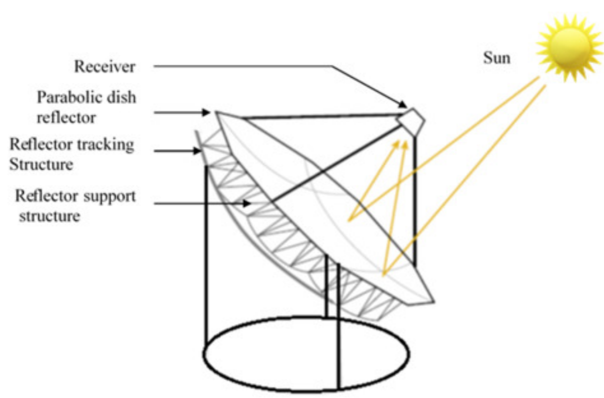
To first order, it turns out we can convert the equation for a sphere into one for a paraboloid. Let's do this with a circle in the XY plane, with the vertex at the origin and the center of the circle at $x = 0, y = r$. Then the equation for the circle would be $(y - r)^2 + (x)^2 = r^2$.

Expanding out the first term: $y^2 - 2ry + r^2 + x^2 = r^2$ or $y^2 - 2ry + x^2 = 0$ or: $2ry = x^2 + y^2$.

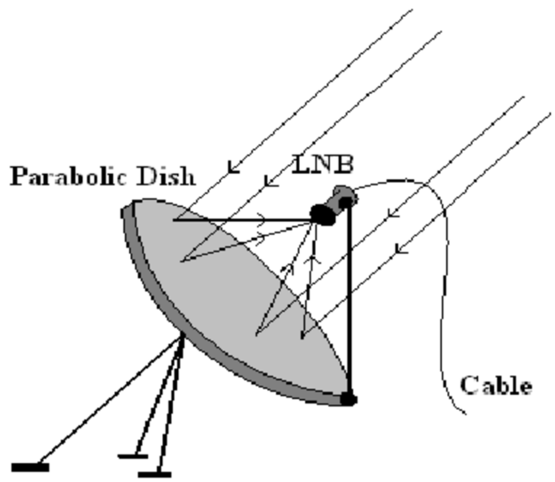
Now, with real telescope mirrors, we already know that $y$ is going to be tiny, even at the outer edge of the mirror, so to first order we can write this as $2ry \approx x^2$ or $y \approx \frac{x^2}{2r}$ and with $f = r/2$ finally: $y \approx \frac{x^2}{4f}$ which is exactly the form we had for the parabola.

With a parabola, the parallel incoming rays all **exactly** go through the focal point F. With a sphere, they do to first order but don't **really** if we include all the terms. The fact that a true spherical mirror doesn't exactly focus the rays to a single point is called **spherical abberation**.

**Non-mirror Examples of Parabolic Reflectors**




**Solar Collector**




**Satellite TV antenna (cartoon)**


**Satellite TV antenna (actual)**

**Sound: a way to clearly hear distance conversations**