PH2233 Fox : Lecture 12
Chapter 32 : Light Reflection and Refraction

(32.4) : index of refraction

The speed of light in a vacuum is $c = 299,792,458\ m/s$. In fact the meter is now defined in such a way that this is taken to be an **exact** value. (For any work we do in this class, using $c = 3 \times 10^8\ m/s$ is close enough.)

When light travels through other media (air, water, glass, etc) it's speed is less. As a result, when light passes from one medium to another with different wave speeds, it 'refracts' at that interface, changing direction. The ratio $\boxed{c/v = n}$ is called the **index of refraction** for a material. Since $v <= c$ always, then $n >= 1$ always.

This table shows some common n's that we'll be using. For naturally occuring substances, diamond appears to have the largest index of refraction at visible frequencies but lab-created materials constructed down at the molecular level have reached indices as high as 38.6 now apparently.

TABLE 32–1 Indices of Refraction†

Material	$n = \frac{c}{v}$
Vacuum	1.0000
Air (at STP)	1.0003
Water	1.33
Ethyl alcohol	1.36
Glass	
Fused quartz	1.46
Crown glass	1.52
Light flin	.58
Lucite or Plexiglas	1.51
Sodium chloride	1.53
Diamond	2.42

† $\lambda = 589$ nm.

Common Shortcuts:

- Air has $n = 1.0003$, but it's common to just use $n = 1$ for air since the error introduced is so small.
- Water is about $n = 1.33$ but that varies a bit depending on impurities in the water, so it's usually fine to use $n = 4/3$ for water.
- Glass varies a bit, but if something is just called 'glass', it's probably ok to approximate it with $n = 1.5$.

(32.5) : snell's law

Light is an electromagnetic wave, with a ray or photon having a particular frequency associated with it. $v = \lambda/T = \lambda f$ so as light moves from one material into another, its frequency remains the same but that means its wavelength must change.

Suppose light of some frequency f is moving through medium 1 which has an index of refraction of n_1. That means it's moving at a speed of $v_1 = c/n_1$ and will have a wavelength of $\lambda_1 = v_1/f = \frac{c}{n_1}\frac{1}{f}$

If this light now moves into a different material with an index of refraction n_2, it's frequency stays the same but it's wavelength changes to $\lambda_2 = \frac{c}{n_2}\frac{1}{f}$.

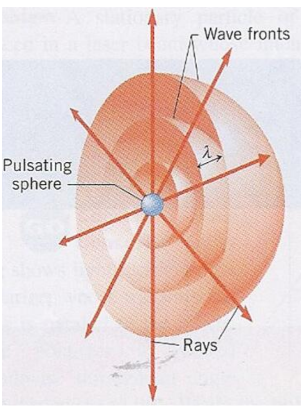
The higher the index of refraction, the smaller the wavelength becomes.

RAY vs WAVE models :

Back when we started chapter 32, I talked about two models we can use to analyze light, sound, and other wave-based phenomena.

Consider a point source of waves (sound, light, etc) putting out waves uniformly in all directions. The left figure below shows these waves spreading out from the source. We can take a point on each wave (where the pressure is equal to it's maximum value, for example, or in the case of light where the underlying $\vec{E}$ field has a peak) and call that a **wave front**. These waves are spreading spherically from the source so each sphere (each 'wave front') is perpendicular to the radius vector from the source, and we'll call that vector perpendicular to the wave front a **ray**.

The figure on the right is a cross section through the left figure more clearly showing the 'rays' and 'wave fronts' that describe how the sound is spreading out from the source.



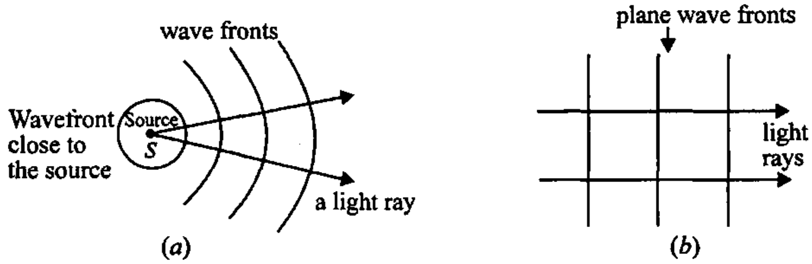


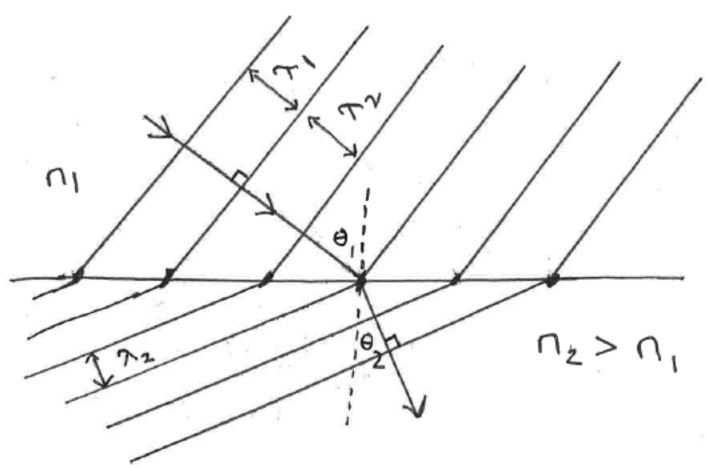
Fig. 20.1

We can analyze waves in two ways then: looking at the **waves** themselves (via the wave equation), or following the **rays** since we know the wave fronts will be perpendicular to those rays.

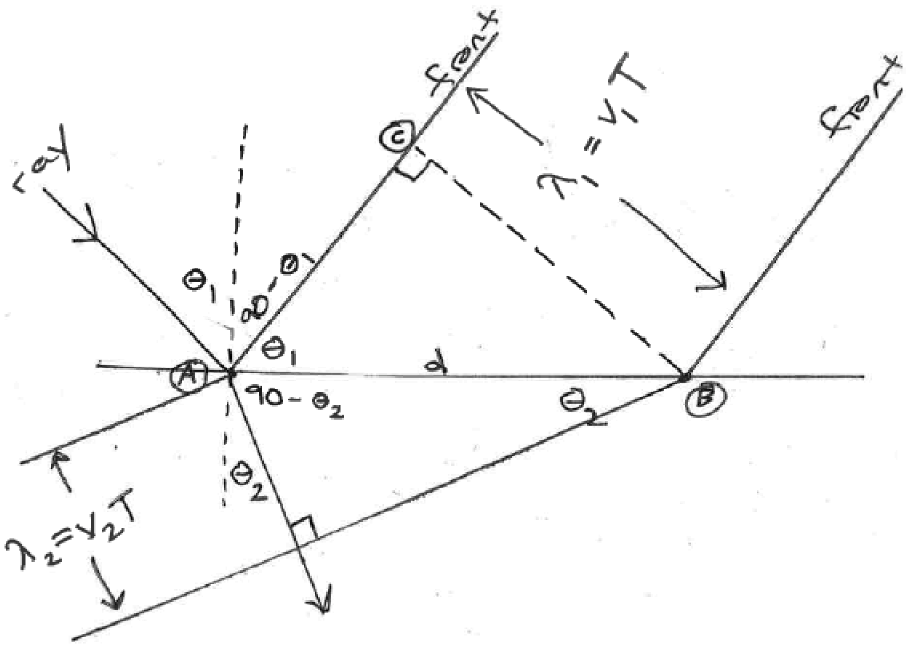
Suppose we're far from the source of light so the wave fronts are parallel, with the 'rays' perpendicular to those fronts.

This figure shows light with some wavelength λ_1 coming in from the top left, with the various parallel lines representing the wave fronts, each separated by $\lambda_1 = v_1 T$ where $T = 1/f$ is the period of the wave and $v_1 = c/n_1$.

These waves now enter a different medium with $v_2 = c/n_2$ and we'll let $n_2 > n_1$ here. At each point where a wave from medium 1 reaches medium 2, another wave in sync with the incoming wave is created in the new medium. That means their frequency (and period) will be the same. BUT since v_2 is smaller than v_1, it's wavelength $\lambda_2 = v_2 T$ will be shorter. The only way to make this work is for the waves to change direction as they enter the new medium.



Here, we zoom in on a pair of fronts as they move from medium 1 into medium 2. We'll define θ as the angle the **ray** makes with a normal to the interface, and propagate θ_1 and θ_2 around a bit. Looking at the two triangles in the figure, they each share side AB, so:



- $\sin\theta_1 = \frac{\lambda_1}{d} = \frac{v_1 T}{d}$
- $\sin\theta_2 = \frac{\lambda_2}{d} = \frac{v_2 T}{d}$

Taking a ratio: $\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2 T/d}{v_1 T/d} = \frac{v_2}{v_1}$.

Rearranging: $$\boxed{\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}}$$ **SNELL'S LAW (general)**

In the case of electromagnetic waves like light, we can write $v = c/n$ so taking this one step farther:

$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \frac{c/n_2}{c/n_1} = \frac{n_1}{n_2}$.

Rearranging: $$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$ **SNELL'S LAW (EM waves)**

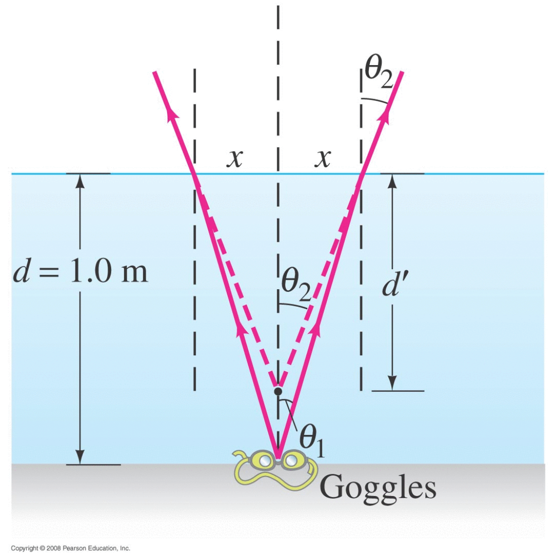
The two boxed equations are both versions of **Snell's Law**. The first is the more generic version that relates the angles to the actual wave speeds involved (useful for **sound** and other types of waves). The second one is specifically for EM waves like light since we usually find tables of n (the index of refraction) for materials directly instead of what the speed of light is in that material.

Example : Apparent Depth of a Pool

Suppose we're looking pretty much straight down into a pool of water where a pair of goggles is resting on the bottom of the pool. How far away does the bottom of the water appear to be?

When light hits the goggles, photons scatter off and head in all directions, including towards the surface so they can reach our eyes. Consider a ray from the goggles to the surface with the ray making an angle of θ_1 relative to the vertical. That ray will hit the surface making the same angle relative to the normal, but will exit the water at a different angle we can determine via Snell's Law:

$$n_{water} \sin\theta_1 = n_{air} \sin\theta_2$$



The ray now **appears** to be coming from a depth d' that's higher up than the actual depth d of the water.

n_{air} is essentially 1, and the angles are pretty small here so we can approximate $\sin\theta \approx \tan\theta$, so: $\sin\theta_1 \approx \tan\theta_1 = x/d$ and $\sin\theta_2 \approx \tan\theta_2 = x/d'$

Snell's law here becomes: $n_{water}\theta_1 \approx \theta_2$ or $n_{water}\frac{x}{d} \approx \frac{x}{d'}$ or $d' \approx d/n_{water}$ and with $n_{water} = 1.33..$ we have $d' \approx (\frac{3}{4})d$. (In this final form, the angle doesn't appear, so 'all' the rays appear to be coming from this point but remember we assumed the angles were all small, meaning we're looking vertically down through the water.)

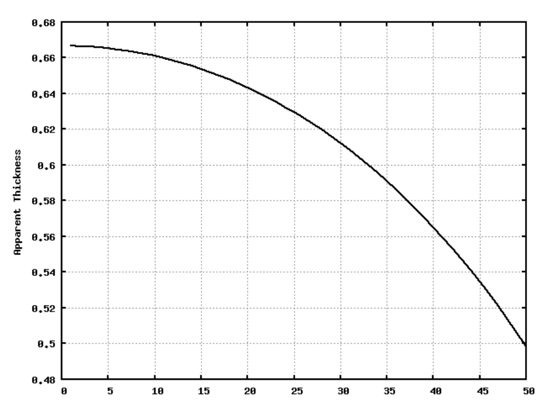
The water appears only about 3/4 as deep as it actually is.

Example : Document Under Glass

The same thing occurs if we place a document under a thick layer of glass. It's the same geometry as above, but $n_{glass} \approx 1.5$ so $d' \approx (\frac{2}{3})d$. The glass appears to be only about 2/3 as thick as it actually is.

We can see the actual thickness of a thick glass block by looking at its sides (those edges are highlighted with a thick black line), but notice the bottom back edge of the block appears much closer when we look at that edge through the glass itself.

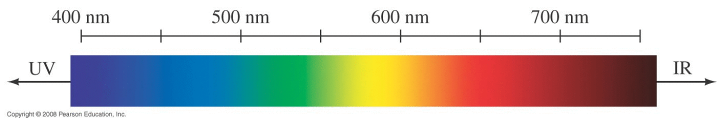
With $n = 1.5$, the block **should** look to be about 2/3 it's actual thickness but that's only if we're viewing it by looking straight down onto the face of the glass. Here we're not. The graph on the right shows the apparent thickness factor as we change our viewing angle.



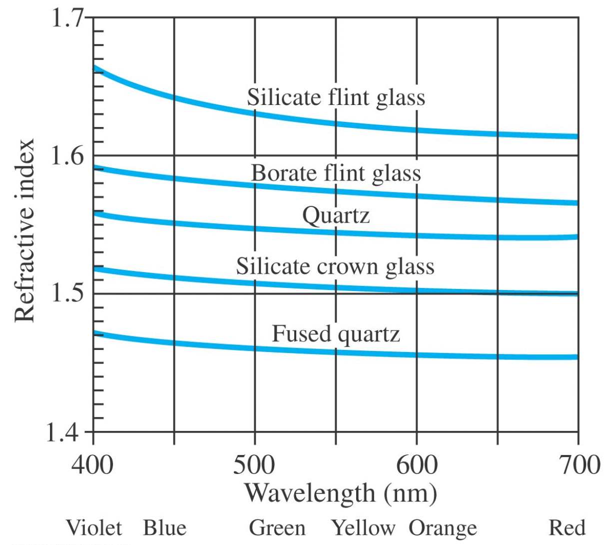
(32.6) : visible spectrum; dispersion

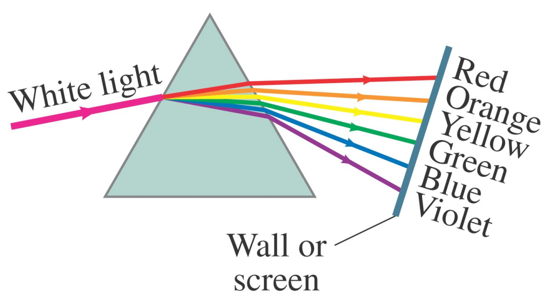
Electromagnetic waves can occur over a vast range of frequencies, from radio waves to X-rays and beyond. Our eyes respond to a fairly narrow range of frequencies covering wavelengths between about 400 nm and 750 nm. (You'll see slightly different ranges quoted in various sources.)

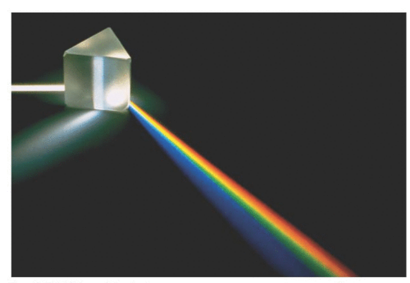
We 'see' these wavelengths as colors, with the shorter wavelengths at the blue end of the spectrum and the longer wavelengths at the red end of the spectrum.



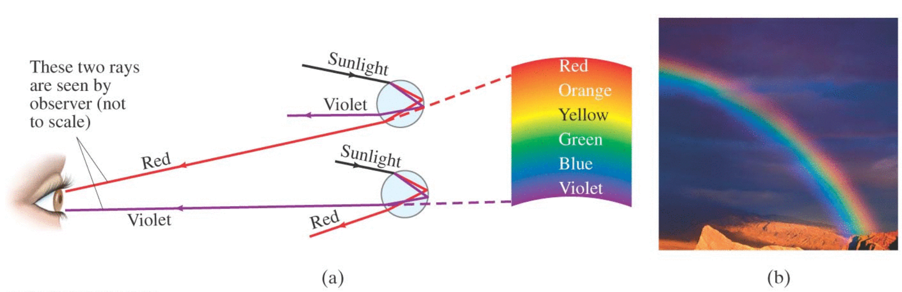
The index of refraction of real materials isn't constant but can vary slightly with wavelength, typically being slightly lower as the wavelength increases.
This figure shows how n varies with λ for a few materials.
The net result is that if a beam of light consisting of many colors enters a material, the different wavelength photons are refracted by different amounts. This is how a prism works - breaking a source of light into its constituent colors.





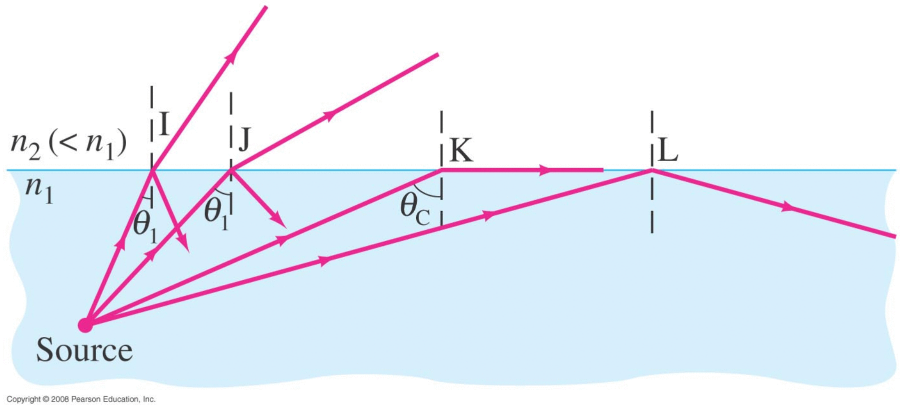


This dispersion has nice side effects, like rainbows and useful effects like breaking light into its constituent colors using a prism (allowing a spectral analysis of a source of light like a remote star), but we'll see in the next chapter it also has bad side effects when lenses are involved, sometimes creating halos of color around objects.



(32.7) : Total Internal Reflection; fiber optics

One of the useful features of refraction occurs when waves try to travel from a lower speed medium (i.e. higher n) into a higher speed medium (i.e. a lower n).



In this figure, we have a source of light (say) in a medium with index of refraction n_1. Let's follow some of the rays (photons) as they encounter the interface between this material and one that has a lower index of refraction n_2.

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ so the refracted ray trying to enter the upper material will have an angle of:

$$\sin\theta_2 = (\tfrac{n_1}{n_2})\sin\theta_1.$$

Now in this scenario, $n_1 > n_2$ which means that θ_2 will be larger than θ_1, and that's what we see happening with the first two rays. Eventually we get to a high enough θ_1 that the RHS of that equation becomes larger than 1 and we can no longer do the inverse sine to find θ_2. The last angle θ_1 where we can still find a solution occurs when $\theta_2 = 90^o$ or when $\sin\theta_1 = \frac{n_2}{n_1}$. This last angle for which the rays can escape is called the **critical angle**, so we can write this as $\boxed{\sin\theta_c = \frac{n_2}{n_1}}$. **CRITICAL ANGLE**

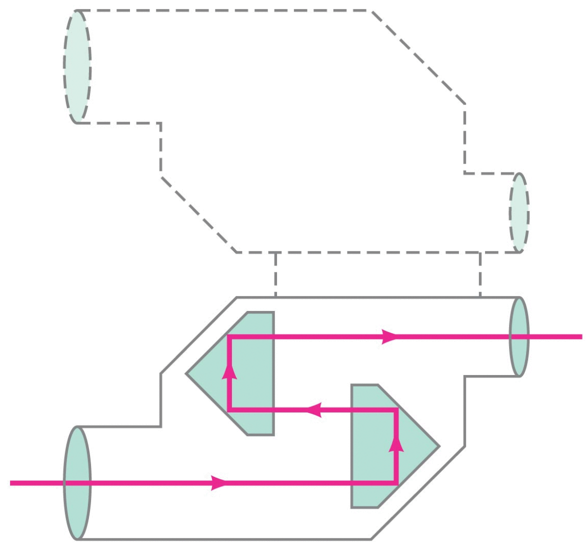
Some of the light leaving the source can make it out into the upper medium, but some is trapped. This scenario is referred to as **total internal reflection**.

Example: pool lights

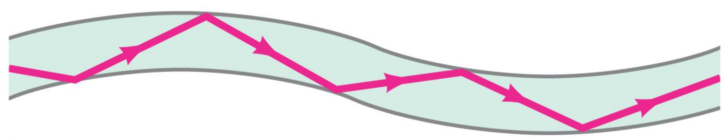
Suppose we have a pool that is 3 m deep, filled with water, and the item marked 'Source' in the above figure is a light on the bottom of the pool. Find the critical angle and determine how large the circle of light will be on the surface of the pool?

$n_1 = 1.33$ (water) and $n_2 = 1.00$ (air) so $sin\theta_c = \frac{n_2}{n_1} = 1/1.33 = 0.75$ yielding $\theta_c = 48.59^o$. Light heading vertically upward from the lamp makes it out, and measured from there light out to θ_c makes it out into the air, but no other light will. This creates a circle of radius: $\tan\theta_c = r/d$ where d is the depth of the pool, so $r = d\tan\theta_c = 1.134d$. If the pool is 3 m deep, there will be disks of light of radius 3.4 m above each light source. (Calculations like these can be used to determine how many lights need to be placed on the bottom of the pool to completely light it up.)

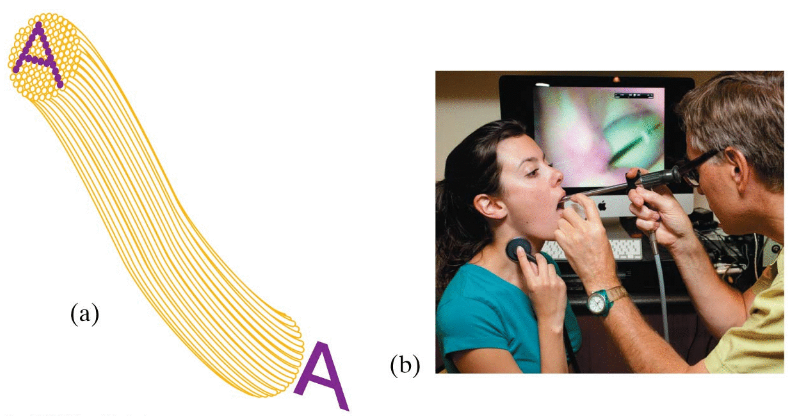
Example: Glass Prisms in Binoculars
Often the light-gathering parts of binoculars are farther apart than our eyes, so they need a way to move the 'rays' closer in. Cheap binoculars usually do this with mirrors that can degrade over time but the same effect can be created using a **clear glass prism**. In this figure, a ray entering the binoculars hits the back face of the prism at a 45^o angle. Why doesn't this ray just exit out into the air there? What is the critical angle at this point? The ray is in glass with $n = 1.5$ say so θ_c for this glass-air interface is $\sin\theta_c = 1.00/1.50 = 0.667$ from which $\theta_c = 42^o$. That 45^o ray exceeds this, so will be totally reflected at that surface, even though it's perfectly clear glass!



Example: Fiberoptics
Another place we find these is in thin fibers of glass or plastic (such as used in high speed fiberoptic internet). The light rays travelling through this pipe hit the side of pipe at an angle that's far larger than θ_c for the materials involved, meaning that none of the light can escape.



Example: Medical Fiberoptics
Finally, these can be used in medical scenarios. A large number of these fibers can be bundled as shown in the upper figure. Some are used to deliver light into the body and the others are used to 'see' inside us.



(a)
(b)