Physics 2233 : Chapter 32 Examples : Light Reflection and Refraction

Note: some of these came from an earlier textbook that used different symbols for the object and image distances. I've tried to track them all down, but just in case:

Symbol Conventions		
Variable	Current book	Old Book
Object Distance	d_o	s
Image Distance	d_i	s'
Focal distance	f	f
Object size	h_o	y
Image size	h_i	y'
Corresponding Equations		
Mirror Equation	$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$	$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$
Magnification	$m = h_i/h_o = -d_i/d_o$	$m = y'/y = -s'/s$
inverted image	$m < 0 \Rightarrow h_i < 0$	$m < 0 \Rightarrow y' < 0$

Spherical Mirror

$f = \frac{1}{2}R$ where R is the radius of curvature of the mirror.

Concave mirror : $R > 0$ so $f > 0$ (the vertex of the mirror is farther away from the object)

Convex mirror : $R < 0$ so $f < 0$ (the vertex of the mirror is closer to the object)

Real vs Virtual

An image is said to be **real** if the light rays actually pass through the location of the image (and therefore a piece of paper or film placed at that point would display or record the image). An image is said to be **virtual** if the light rays only **appear** to be coming from that point.

Flat Mirror

In effect r is infinitely large, so f is also infinity and the mirror equation reduces to $d_i = -d_o$ and $m = +1$. (In words: an upright image the same size as the object, located as far behind the mirror as the object is in front of the mirror.)

Ray Diagrams (see book and attached examples for pictures)

Concave Mirror	Convex Mirror
1. ray parallel to axis reflects through F	1. ray parallel to axis reflects as if from F
2. ray to vertex reflects with same angle	2. ray hitting vertex reflects with same angle
3. ray through F reflects parallel to axis	3. a ray heading towards F reflects parallel to axis

Index of Refraction

The speed of light (or any EM waves) is slower moving through any medium than it is in a vacuum. The ratio c/v is called the index of refraction and is always 1 or larger: $\boxed{n = c/v}$. The index of refraction of various common materials is given in table 32-1 on page 939.

Indices of Refraction	
Material	$n = c/v$
Vacuum	1.0000...
Air (STP)	1.0003
Water	1.33
Ethyl alcohol	1.36
Glass (fused quartz)	1.46
Glass (crown glass)	1.52
Glass (light flint)	1.58
Lucite, plexiglas	1.51
Sodium chloride	1.52
Diamond	2.42

Snell's Law relates the angle of an incoming ray to the angle of the outgoing reflected and refracted rays. The angle here is defined to be the angle with respect to a line perpendicular to the surface where the ray hits.

Reflected ray: angle of incidence is equal to the angle of reflection

Refracted ray: let θ_1 be the angle of the incoming ray in a material with index of refraction n_1, and θ_2 be the angle of the refracted ray in a material with index of refraction n_2. Then:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Equivalently: $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$ where v_i is the speed of the waves in the two media. This version is more generic and applies to other waves as well, such as **sound**.

Dispersion

The index of refraction usually depends (slightly) on the wavelength of the waves, so light of different frequencies will be bent at slightly different angles. An incoming white light (being made up of many colors) will 'split' into a rainbow spectrum. Details of this spectrum (what colors are present or absent) can be used to determine the chemical makeup of the source of the light.

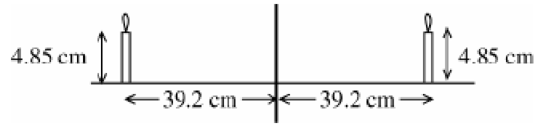
Total Internal Reflection

If the light ray is moving from a material with a higher index of refraction into one with a lower index of refraction (i.e. $n_1 > n_2$), there will be an angle of incidence where none of the light is refracted, meaning all of the light will be reflected back into the material. This is called the critical angle: $\sin\theta_c = \frac{n_2}{n_1}$.

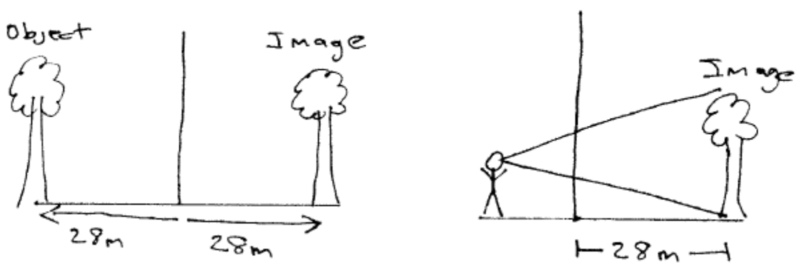
MIRROR EXAMPLES

Example 1 : A candle 4.85 cm tall is 39.2 cm to the left of a plane mirror. Where is the image formed by the mirror, and what is the height of this image?

This is a plane mirror, so $d_i = -d_o$ and $m = h_i/h_o = -d_i/d_o = +1$. Here we have $d_o = 39.2\ cm$ and $h_o = 4.85\ cm$ so the image will be located at $d_i = -39.2\ cm$ with a height of $h_i = mh_o = (+1)(4.85\ cm) = 4.85\ cm$. The image is located 39.2 cm behind the mirror, has the same height as the object, is erect (and although the problem didn't ask for it, will be virtual since the light rays don't actually pass through the image itself.)

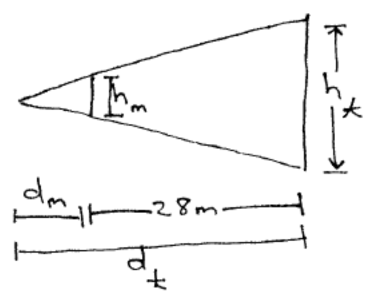


Example 2 : The image of a tree just covers the length of a plane mirror 4.00 cm tall when the mirror is held 35.0 cm from the eye. The tree is 28.0 m from the mirror. What is its height?



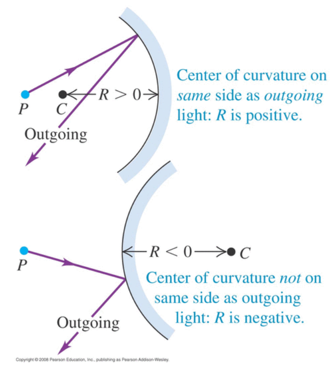
Imagine we had a huge mirror, not just the tiny mirror described here (above figure). Then the image of the tree would be located (per question 1 above) exactly 28.0 m behind the mirror and will have the same height as the actual tree. If we place our eye 35 cm on the left side of the mirror and draw rays from our eye to the top and bottom of the image, the image of the tree will only take up a small part of the mirror. In fact, we're told that we can cut the mirror down to be only 4 cm tall and image will completely fit.

We have a case of similar triangles here, then. The ratio of the height of the image h_t to the distance from our eye to the image d_t will be equal to the size of the mirror h_m divided by the distance of our eye to the mirror d_m. But d_t will be equal to the distance from our eye to the mirror plus the distance from the mirror to the image of the tree. Rearranging our ratio equation: $h_t/d_t = h_m/d_m$ becomes: $h_t = h_m d_t/d_m$ but $d_t = 28\ m\ +\ 0.35\ m$ so $h_t = (0.04\ m)(28.0 + 0.35)/(0.35) = 3.24\ m$, giving us the true height of the tree.



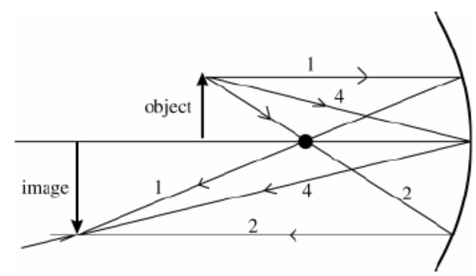
Example 4 : A concave mirror has a radius of curvature of 34.0 cm. (a) What is its focal length? (b) If the mirror is immersed in water (refractive index 1.333), what is its focal length?

(a) For a concave mirror, the center of curvature is on the same side as the outgoing rays, so $R > 0$ or here $R = +34.0\ cm$. The focal point for this shape is $f = R/2 = 17.0\ cm$.
(b) The rays never leave the water, so there is no opportunity for them to refract and they will just proceed in straight lines, reflecting off the mirror exactly as is the water were air. The ray diagrams are completely unchanged, so the focal point does not change either.



Example 5 : An object 0.600 cm tall is placed 16.5 cm to the left of the vertex of a concave spherical mirror having a radius of curvature of 22.0 cm. (a) Draw a principal-ray diagram showing the formation of the image. (b) Determine the position, size, orientation, and nature (real or virtual) of the image.

All of the rays that leave the arrowhead and reflect off the mirror will intersect at the spot where the **image** of the arrowhead will be. Most of the rays are difficult to track but there are some in particular (called the **principal rays**) that are easy to draw, so we can just use those to figure out where the image should be. In the figure, Ray 1 proceeds parallel to the axis, so when it reflects off the mirror it will pass through the focal point F. Ray 4 goes from the arrowhead to the vertex, where it will reflect with the same angle at which it was incident. Ray 2 passes through the focal point which means that when it hits the mirror, it will reflect off parallel to the axis.



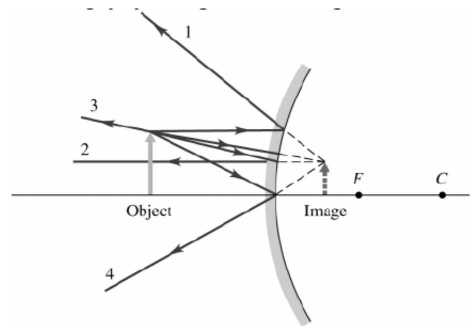
The center of curvature is on the same side as the outgoing rays, so $R > 0$ or $R = +22.0\ cm$. The focal distance for a spherical mirror is $f = R/2 = +11.0\ cm$. The object is on the same side as the incoming rays (incoming to the mirror), so $d_o = +16.5\ cm$. The image distance therefore will be: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or $\frac{1}{16.5} + \frac{1}{d_i} = \frac{1}{11}$ (all units in cm in those denominators) which yields $d_i = +33.0\ cm$. It is positive which means the image is on the same side as the outgoing rays, which here will be going to the left, so the image will be on the same side as the object. The magnification $m = -\frac{d_i}{d_o} = -\frac{33}{16.5} = -2.00$ so the image size will be $h_i = mh_o = (-2.00)(0.600\ cm) = -1.2\ cm$.

We would describe this image as being a 1.2 cm, inverted image. (And since the rays do pass through the image, it will be a **real** image: we could put a piece of paper there and the image will form on it.)

Example 6 : Repeat the previous problem for the case in which the mirror is convex.

Note that now the radius of curvature will be negative since the center of the sphere is on the opposite side from the outgoing rays, so $R = -22.0\ cm$ and $f = R/2 = -11.0\ cm$: the focal point will be on the right side of the image now, instead of being on the left as it was in the previous problem.

Here we draw three of the principal rays. Ray 1 proceeds parallel to the axis, so when it reflects off the mirror it will appear to be coming from the focal point F. Ray 3 heads towards the center point C of the mirror so that it hits it perpendicularly and bounces straight back off in the same direction it came in. Ray 2 heads towards the focal point F, which means that when it hits the mirror, it will reflect off parallel to the axis. Extending all these lines back as dotted lines in the figure, we see that image of the arrow head appears to be inside the mirror at the location shown.



We can find the image distance from: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or $\frac{1}{16.5} + \frac{1}{d_i} = \frac{1}{-11}$ (all units in cm in those denominators) which yields $d_i = -6.6\ cm$. It is negative which means the image is on the opposite side as the outgoing rays, which are going to the left, so the image will be on the opposite side as the object (which we see in our principal ray diagram already).

The magnification $m = -\frac{d_i}{d_o} = -\frac{-6.6}{16.5} = +0.40$ so the image size will be $h_i = mh_o = (+0.40)(0.600\ cm) = +0.24\ cm$.

We would describe this image as being a 0.24 cm, erect image. (And since the rays do **not** pass through the image, it will be a **virtual** image.)

Example 7 : The diameter of Mars is 6784 km, and its minimum distance from the earth is 5.58×10^7 km. When Mars is at this distance, find the diameter of the image of Mars formed by a spherical, concave telescope mirror with a focal length of 2.80 m.

See the figure from example 5 but now imagine that the object is located extremely far away. A mirror like this has a positive radius of curvature, hence a positive focal distance, so $f = +2.80$ m.

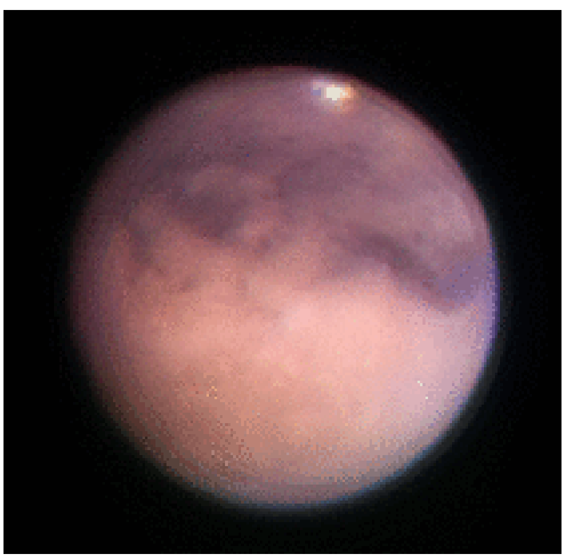
$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ so $\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$. Here, d_o is a billion times larger than f, so $\frac{1}{d_o}$ is a billion times smaller than $\frac{1}{f}$. Essentially, then, $\frac{1}{d_i} = \frac{1}{f}$ or $d_i = f = 2.80$ m. (The image is essentially being formed right at the focus of the mirror.)

The magnification factor $m = -\frac{d_i}{d_o} = -\frac{2.80\ m}{5.58 \times 10^{10}\ m} = -5.02 \times 10^{-11}$.

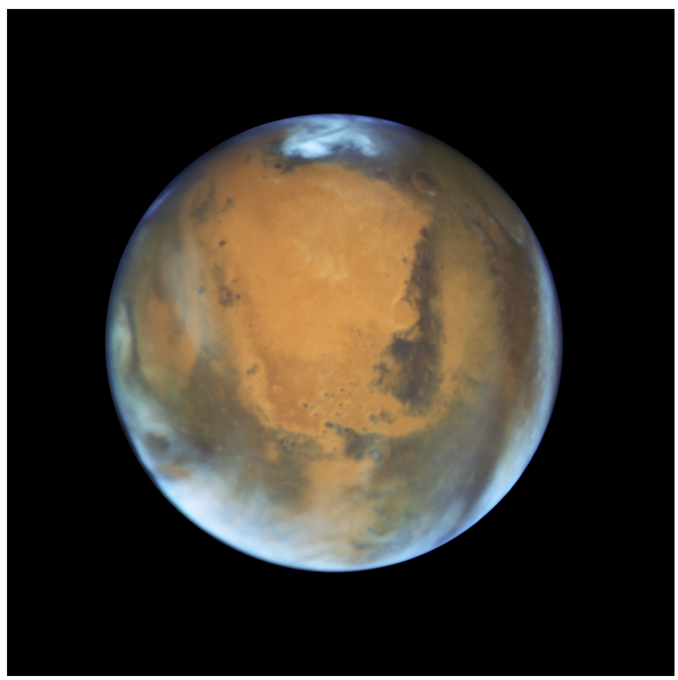
The image of Mars then will have a height of $h_i = mh_o = (-5.02 \times 10^{-11})(6.784 \times 10^6\ m) = -3.40 \times 10^{-4}$ m or about -0.340 mm. The image then will be 0.340 mm tall and inverted, but at least it's a real image since the rays do pass through the image, so we can put a camera there and take a picture that we could blow up further, as seen in the left figure below.

On the right is another picture of Mars taken from the Hubble telescope that has a focal length of about $f = 57$ m. That's about 20 times the focal length of the backyard telescope and ultimately creates an image about 20 times larger as well, or about 6.8 mm or 6.8 cm across.

These images weren't taken at the same time, so show different views of Mars. The point here is to note how much more details are visible using the telescope with the larger focal length (and much larger mirror as well - we'll see how that effects the resolution in a later chapter).



Backyard telescope with $f = 2.8$ m



Hubble space telescope with $f = 57$ m

Example 10 : You hold a spherical salad bowl 90 cm in front of your face with the bottom of the bowl facing you. The salad bowl is made of polished metal with a 35 cm radius of curvature. (a) Where is the image of your 2.0 cm tall nose located? (b) What are the image's size, orientation, and nature (real or virtual)?

(Go back a couple of problems to see a ray diagram for a convex mirror like this.)

The center of curvature is on the opposite side from the outgoing rays, so the radius of curvature will be negative: $R = -35.0\ cm$ and $f = R/2 = -17.5\ cm$. The nose is on the same side as the incoming rays, so $d_o = +90\ cm$. Then the image distance can be calculated from $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or $\frac{1}{90} + \frac{1}{d_i} = \frac{1}{-17.5}$ (all the denominators being in units of centimeters).

This produces $d_i = -15\ cm$. Negative means it will **not** be on the same side as the outgoing rays (which are on left, so the image is on the right, behind the mirror). (And since the light rays aren't passing through the image, it must be **virtual**.)

The magnification $m = -d_i/d_o = -(-15\ cm)/(90\ cm) = +0.167$ (positive telling us that the image will be erect and not inverted). The size of the nose in this image then will be $h_i = mh_o = (0.167)(2\ cm) = 0.33\ cm$.

So finally: the image will be 0.33 cm tall (smaller than the actual object), erect, and virtual.

Example 13 : A dentist uses a curved mirror to view teeth on the upper side of the mouth. Suppose she wants an erect image with a magnification of 2.00 when the mirror is 1.25 cm from a tooth. (Treat this problem as though the object and image lie along a straight line.) (a) What kind of mirror (concave or convex) is needed? (b) What must be the focal length and radius of curvature of this mirror? (c) Draw a principal-ray diagram to check your answer in part (b).

$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or $\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}$. Putting the right side over a common denominator: $\frac{1}{d_i} = \frac{d_o - f}{d_o f}$ or rearranging: $d_i = \frac{d_o f}{d_o - f}$. The magnification factor $m = -d_i/d_o$ so we see that we can write this as $m = -\frac{f}{d_o - f}$.

For a **concave mirror**, $f = R/2$ is positive, so that denominator can be rigged to be less than 1, meaning that m overall can be larger than 1.

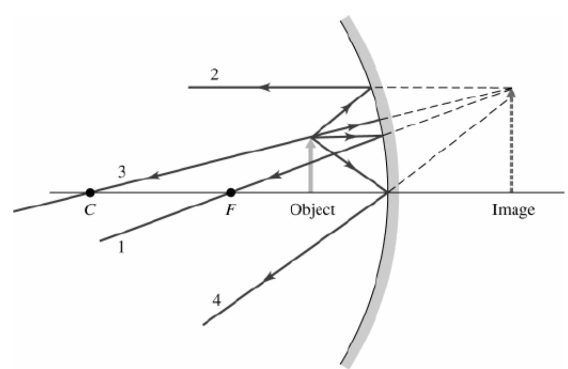
For a **convex mirror**, the radius of curvature of the mirror is negative, so $f < 0$ also. Let's write $f = -|f|$ to emphasize it being negative. Then $m = -\frac{f}{d_o - f}$ becomes: $m = -\frac{-|f|}{d_o - (-|f|)} = +\frac{|f|}{d_o + |f|}$.

The object is always on the same side as the incoming light though, so d_o is always positive. That means the denominator will always be larger than the numerator and m will always be less than unity. This gives us a general result for convex mirrors:

The **image** in a convex mirror will always appear to be **smaller** than the true object.

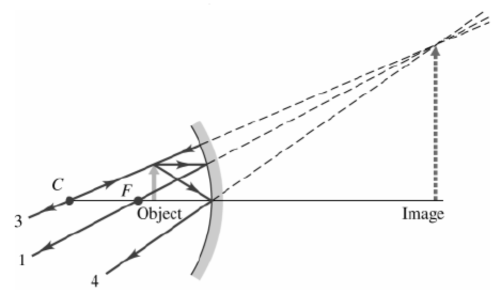
We thus can't make this dental mirror from a convex mirror, since we desire the image to be twice the size of the object.

We can also determine the type of mirror needed from just applying the equations we have for image and object distances and magnification. We have $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ and $m = -d_i/d_o$. But we want m to be +2.0 (the image should be twice the size of the object, and it should be erect, which makes m positive.) We already know the distance from the tooth to the mirror (the object distance) to be $d_o = 1.25\ cm$ $m = -d_i/d_o$ or $(+2.00) = -d_i/(1.25\ cm)$ from which $d_i = -2.50\ cm$. (Negative, so the image is on the other side from the outgoing rays. Those rays are on the left, so the image will be on the right - i.e. behind the mirror.) Now that we have both d_o and d_i we can calculate the focal length: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or $\frac{1}{1.25} + \frac{1}{-2.50} = \frac{1}{f}$ from which $f = +2.50\ cm$. $f = R/2$ so $R = 2f = +5.00\ cm$. The radius of curvature being positive means the center C is on the same side as the outgoing rays, which means C is on the left of the mirror, which makes the mirror concave. Whew.



Example 14 : A spherical, concave shaving mirror has a radius of curvature of 32.0 cm. (a) What is the magnification of a person's face when it is 12.0 cm to the left of the vertex of the mirror? (b) Where is the image? Is the image real or virtual? (c) Draw a principal ray diagram showing the formation of the image.

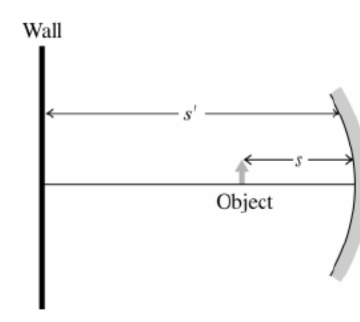
This mirror is concave, so C is on the same side as the outgoing rays, making R positive: $R = +32.0\ cm$ from which $f = R/2 = +16.0\ cm$. The object (the face) is 12.0 cm to the left of the vertex of the mirror, which puts in on the same side as the incoming rays so $d_o = +12.0\ cm$. We can now calculate d_i from $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ or $\frac{1}{12} + \frac{1}{d_i} = \frac{1}{16}$ or finally $d_i = -48.0\ cm$. Negative means it will be on the opposite side from the outgoing rays: that is, on the 'other' side (behind) the mirror. The light rays don't get behind the mirror, so it will be virtual. The magnification $m = -d_i/d_o = -\frac{-48}{12} = +4.00$. That's positive, so the image is erect. So finally: we see an erect, virtual image that is magnified by a factor of 4.



Ray 1 goes out parallel to the axis, hits the mirror and reflects back through the focal point F. Ray 3 hits the mirror perpendicularly and passes through the center of curvature C. Ray 4 hits the vertex and reflects with the same angle as the angle of incidence. Ray 2 (the one that we would draw from the arrow head through the focal point F) we can't draw since that line doesn't intersect the mirror anywhere.

Example 64 : A light bulb is 4.00 m from a wall. You are to use a concave mirror to project an image of the bulb on the wall, with the image 2.25 times the size of the object. How far should the mirror be from the wall? What should its radius of curvature be?

So here we have a light source that is not moving, and we're going to be adjusting the position of the concave mirror back and forth to achieve the desired effect. The figure sketches out where things are and what they should be labeled. (**NOTE** : this figure is from our old book which used s to represent the object distance d_o and s' for the image distance d_i.)
The object is on the same side as the incoming rays and the image is on the same side as the outgoing rays, so we have d_o and d_i both being positive. But since $m = -d_i/d_o$, that implies that the magnification must be negative, so $m = -2.25$. The distance from the object to the wall is 4.00 m so $(4\ m) + d_o = d_i$.



We have two equations now involving d_o and d_i. First: $m = -d_i/d_o = -2.25$ or $d_i = 2.25 d_o$. Second: $d_i = d_o + (4\ m)$. This implies that $2.25 d_o = d_o + (4\ m)$ or $d_o = 3.2\ m$. Now we know how far the mirror is behind the bulb. The distance from the mirror to the wall then is $d_i = 2.25 d_o = 7.2\ m$.

For a spherical mirror, $f = R/2$ and $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ so $\frac{1}{f} = \frac{1}{3.2\ m} + \frac{1}{7.2\ m} = 0.4514\ m^{-1}$ and $f = 2.215\ m$, from which $R = 2f = 4.43\ m$.

Example 66 : A mirror on the passenger side of your car is convex and has a radius of curvature with magnitude 18.0 cm. (a) Another car is seen in this side mirror. The car physically is behind you, 13.0 m from the mirror and is 1.5 m tall. What is the height of the image formed by the mirror? (b) These mirrors usually have a warning attached that objects viewed in it are closer than they appear. Why is this so?

What sign should f have? A convex mirror means when you look at it, it is bulging out a bit in the center. The center of curvature for this mirror then is somewhere beyond the mirror, making it NOT on the side of the outgoing rays (this is a mirror, not a lens), so $R = -18\ cm$. $f = R/2$ so $f = -9\ cm$.

So we have an object now (the car behind us) located at $s = 13\ m$ (positive since it's on the same side of the mirror as the light incoming from the object to the mirror). Where will be image be? $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ but we'll use this in the rearranged form: $d_i = \frac{d_o f}{d_o - f}$. Putting everything in terms of centimeters: $d_i = \frac{(1300)(-9)}{1300-(-9)} = -8.94\ cm$. (So the image of the car appears to be about 9 cm behind the mirror as I look towards it.)

The magnification of this image will be $m = -d_i/d_o = -(-8.94\ cm)/(1300\ cm) = 0.00688$. (Positive, so the image of the car is NOT inverted: that would be pretty disorienting.)

The image of the car then will have a height of $h_i = mh_o = (0.00688)(1.5\ m) = 0.010\ m$ or just 1 cm high.

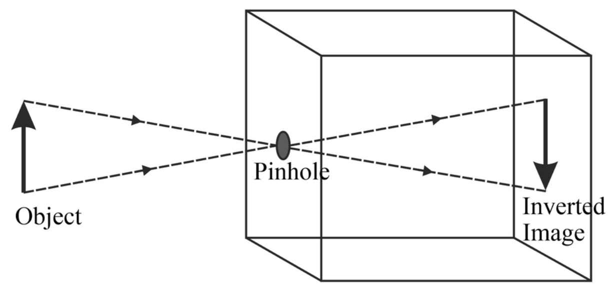
So why does the mirror carry a label about 'objects in mirror are closer than they appear?'

If the rear-view mirror is, say, 50 m away from my eyes, we found that the image of the car would be 9 cm behind the mirror and about 1 cm high. This represents an angle in my field of view of about: $\tan\theta = \frac{s}{r} = \frac{1\ cm}{50+9\ cm}$ or $\theta = 0.97^o$. If I were to just turn around and look at the car directly, it has a height of 1.5 m and is 13 m away, so the angle it subtends would be $\tan\theta = \frac{1.5\ m}{13\ m}$ or $\theta = 6.6^o$, which is nearly 6 times larger.

Thus the image looks like it's much farther away than the car actually is.

Example 77 : A pinhole camera is just a rectangular box with a tiny hole in one face. The film is on the face opposite this hole, and that is where the image is formed. The camera forms an image without a lens.

(a) Make a clear ray diagram to show how a pinhole camera can form an image on the film without using a lens. (Hint: put an object outside the hole and then draw rays passing through the hold to the opposite side of the box.) (b) A certain pinhole camera is a box that is 25 cm square and 20.0 cm deep, with the hole in the middle of one of the 25 cm $\times 25$ cm faces. If this camera is used to photograph a fierce chicken that is 18.0 cm high and 1.50 m in front of the camera, how large is the image of this bird on the film? What is the magnification of this camera?



Rays that pass through the hole are undeflected; all other rays are blocked. If we draw a ray from the top of the object, there's just one place it can go - through the hole in the box and then hitting the opposite side of the box, where we've placed the film. Clearly an inverted image is being formed, no matter how far the side is from the pinhole, and no matter how far the object is from the pinhole. So here the image distance will be $d_i = +20$ cm. (That's the distance between the pinhole and the film, and the sign is positive since the image is clearly on the same side as the rays 'outgoing' from our pinhole 'lens.') The source is located 1.5 m away from the lens, so the magnification of the image will be $m = -d_i/d_o = -(20)/(150) = -0.133$. The size of the chicken in the image will be $h_i = mh_o = (-0.133)(18\ cm) = -2.4\ cm$. (So the image is 2.4 cm tall and inverted.)

(Note: there is no opportunity for rays to focus here, so there really isn't a focal length for this type of 'lens.' (In fact based on the diagram, any object at any distance will form a clear, in-focus image on the film, so this type of 'camera' is considered to have an infinite depth of focus.) Also, since the pinhole needs to be very small for this to work, very little light gets inside the box and a long exposure time is required to form a picture (during which time objects probably moved, blurring the picture).)

Example 3 : A beam of light has a wavelength of 650 nm in vacuum. (a) What is the speed of this light in a liquid whose index of refraction at this wavelength is 1.47? (b) What is the wavelength of these waves in the liquid?

(a) The velocity of light in a medium of refractive index n is $v = c/n$, so for this material, $v = (3.00 \times 10^8\ m/s)/(1.47) = 2.04 \times 10^8\ m/s$.

(b) The frequency is the one thing that remains the same across the interface. If we stand on the air side of the interface, we count some number of waves passing by each second (i.e. the frequency). On the liquid side of the interface, the same number of waves must be passing by each second. Otherwise waves would be piling up or being destroyed, and neither occurs. $v = \lambda f$ so $f = v/\lambda$. If we have two materials, a and b we could write this as: $v_a/\lambda_a = f$ and $v_b/\lambda_b = f$ so $v_a/\lambda_a = v_b/\lambda_b$. But $v = c/n$ so we can also write this as: $\frac{c}{n_a}/\lambda_a = \frac{c}{n_b}/\lambda_b$. Rearranging terms, we finally find that: $n_a\lambda_a = n_b\lambda_b$.

For our situation, call a the air side where $n_a = 1$ (essentially) then $(1)(650\ nm) = (1.47)(\lambda_b)$ or $\lambda_b = 442\ nm$.

Example 5 : A light beam travels $1.94 \times 10^8\ m/s$ in quartz. The wavelength of the light in quartz is 355 nm. (a) What is the index of refraction of quartz at this wavelength? (b) If this same light travels through air, what is its wavelength there?

(a) $v = c/n$ and here we have the speed of light in quartz, so $n = c/v = (3 \times 10^8\ m/s)/(1.94 \times 10^8\ m/s) = 1.546$.

(b) In the previous problem, we found that $n_a\lambda_a = n_b\lambda_b$. If here we call a the quartz and b the air, then: $(1.546)(355\ nm) = (1.00)(\lambda_b)$ or $\lambda_b = 549\ nm$ in the air.

Example 11 : Light passes through three PARALLEL slabs of different thicknesses and refractive indices. The light is incident in the first slab and finally refracts into the third slab. Show that the middle slab has no effect on the final DIRECTION of the light. That is, show that the direction of the light in the third slab is the same as if the light had passed directly from the first slab into the third slab. Generalize this result to a stack of N slabs. What determines the final direction of the light in the last slab?

In the first slab, we have a material with index of refraction n_1, and the light ray is at an angle of, say, θ_1 (measured with respect to a line perpendicular to the interface between 1 and 2 (i.e. the normal)).

It now enters material 2 with index of refraction n_2. The angles will be related by Snell's Law: $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

This ray now continues through material 2 and enters material 3 with index of refraction n_3. The interfaces are parallel, so the normals are also parallel. That means that the angle that the ray makes when it hits material 3 is also θ_2. In material 3, the angle will be found from Snell's Law again: $n_2 \sin\theta_2 = n_3 \sin\theta_3$.

But comparing these two Snell's equations, we see that $n_1 \sin\theta_1 = n_3 \sin\theta_3$ directly, which is exactly the equation we would have written if we removed the middle material and just had the first and third slabs of material in direct contact.

If we have n such PARALLEL slabs stacked on top of one another, the same situation occurs:
$n_1 \sin\theta_1 = n_2 \sin\theta_2$ and
$n_2 \sin\theta_2 = n_3 \sin\theta_3$ so $n_1 \sin\theta_1 = n_3 \sin\theta_3$. Then $n_3 \sin\theta_3 = n_4 \sin\theta_4$ but $n_1 \sin\theta_1 = n_3 \sin\theta_3$ so $n_1 \sin\theta_1 = n_4 \sin\theta_4$ and so on all the way down the stack.

This means that if we stack a series of slabs on top of one another, the angle the light makes in the last slab can be calculated directly using only the indices of refraction in the top and bottom slabs.

What IS different is WHERE the beam finally comes out. At each interface, the ray is changing direction, so the exact POINT where it intersects the next interface will move around. The direction of the final beam may be unaffected, but the offset from the original beam can be much different depending on if we have all n slabs, or just the top and bottom slabs in direct contact.

Example 13 : In a material having an index of refraction n, a light ray has frequency f, wavelength λ and speed v. What are the frequency, wavelength, and speed of this light (a) in vacuum and (b) in a material of index of refraction n'?. In each case, express your answers in terms of only f, λ, v, n and n'.

We've already argued in a previous problem that the frequency does not change. We also have the general relations: $v = c/n$ and $n_a\lambda_a = n_b\lambda_b$.

In the following, remember that what the problem is giving us is the values in the material with index of refraction n.

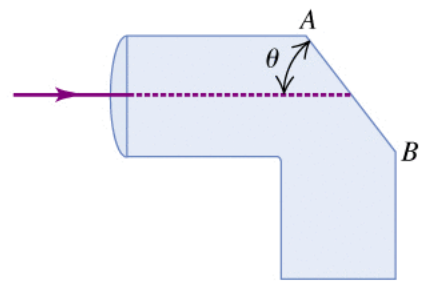
(a) In vacuum, then we can rearrange $v = c/n$ to say that $c = vn$, c being the velocity in vacuum.

From $n_a\lambda_a = n_b\lambda_b$, let's say that b represents the material and a the case we are interested in (i.e. in a vacuum). Then $(1)\lambda_a = n\lambda$ or the wavelength in a vacuum will be n times the wavelength in the material.

(b) Now we are in some other material with index of refraction n'. So now $v' = c/n'$ but from part (a), we were able to rewrite c as $c = vn$ so $v' = (vn)/n'$ or $v' = \frac{n}{n'}v$

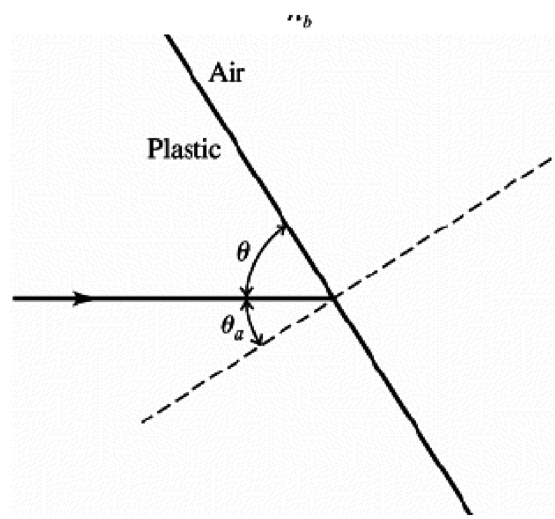
For the wavelength in this new material, we have generically $n_a\lambda_a = n_b\lambda_b$. Let a represent the original material and b be the new material. Then in terms of the variables they gave us for the original material: $n\lambda = n'\lambda'$ or $\lambda' = \frac{n}{n'}\lambda$.

Example 17 : Light enters a solid pipe made of plastic having an index of refraction of 1.60. The light travels parallel to the upper part of the pipe. You want to cut the face AB so that all the light will reflect back into the pipe after it first strikes that face. (a) What is the largest that θ can be if the pipe is in air? (b) If the pipe is to be immersed in water of index of refraction $n = 1.33$, what is the largest that θ can be?



Using the conventions of the book, we'll call a the material the light is traveling through and b the other material (which here will be either air or water). In material a, the light hits the interface with an angle of θ_a and will refract into material b with an angle we can find from Snell's Law. For **total internal reflection**, the angle the light ray makes WITH THE NORMAL (θ_a) should be the critical angle found from: $\sin\theta_{crit} = \frac{n_b}{n_a}$. At that angle, θ_b becomes 90^o and none of the light makes it out of the original material.

We are trying to find the angle in the original figure though, which is NOT measured with respect to the normal. Fortunately as the figure at right shows, they are simply related since $\theta + \theta_a = 90^o$. So if we can find the critical angle, we can relate that to the angle we need to cut the material.



(a) If the outside (b) material is air, then $n_b = 1.00$ so the critical angle becomes: $\sin\theta_{crit} = \frac{n_b}{n_a} = \frac{1}{1.60}$ from which $\theta_{crit} = 38.7^o$. The angle of the face then is $\theta = 90 - \theta_{crit} = 51.3^o$.

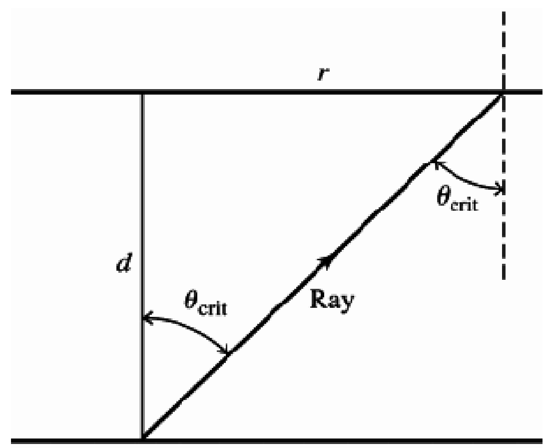
(b) If the outside (b) material is water, then $n_b = 1.333$ so the critical angle becomes: $\sin\theta_{crit} = \frac{n_b}{n_a} = \frac{1.333}{1.60}$ from which $\theta_{crit} = 56.4^o$. The angle of the face then is $\theta = 90 - \theta_{crit} = 33.6^o$.

Note: if we design this thing to operate in the air, will it work under water as well? If we use the angles from part (a), then under water light will be hitting the sloping surface at an angle of 38.7^o with respect to the normal. But this angle is less than the critical angle for the case of the material immersed in water. Light will happily refract out through the surface into the water.

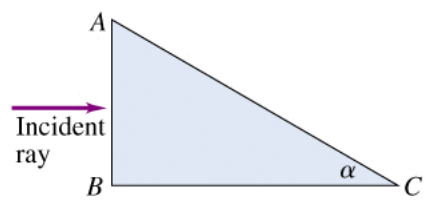
On the other hand, if we design it to operate under water, then in air light will still be hitting the sloping surface with an angle of 56.4^o which is well above the critical angle for material-vs-air, so the light will still be totally reflected even when this object is in the air.

Example 20 : A glowing ring is dropped into a river that is 10 m deep. Assuming the ring is small enough to be considered a point source of light, what is the AREA of the circle of light that is formed on the surface of the water?

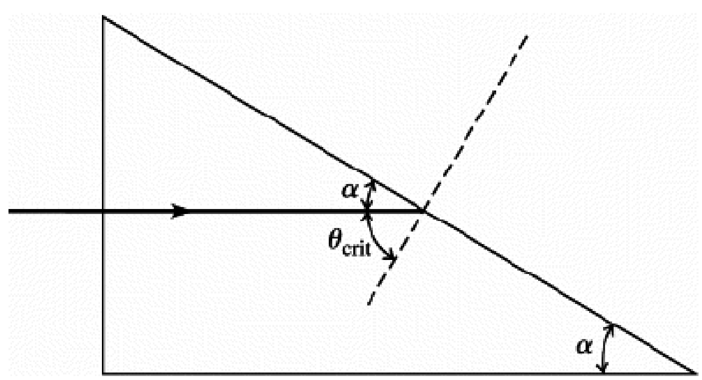
The largest angle of incidence for which any light can refract into the air (and therefore escape from the water and be seen from above the river's surface) is the critical angle (for the case of material a being water and material b being air). The figure shows a ray at the critical angle. The distance from the light source to the surface is 10 m. The radius of the lit-up circle formed on the surface is r (from which we can compute the area). From the figure, $r = d \tan\theta_{crit}$ but we can find θ_{crit} from $\sin\theta_{crit} = n_b/n_a = 1.00/1.333 = 0.75$ or $\theta_{crit} = 48.6^o$. So $r = d\tan\theta_{crit} = (10\ m)(1.134) = 11.34\ m$ The area of the lighted circle then is $A = \pi r^2 = 404\ m^2$.



Example 22 : Light is incident along the normal on face AB of a glass prism of refractive index 1.52 as shown in the figure. Find the largest value the angle α can have without any light refracted out of the prism at face AC if (a) the prism is immersed in air and (b) the prism is immersed in water.



If no light is to refract out of the glass at the glass-to-air interface, then the incident angle at that interface is θ_{crit}. The ray has an angle of incidence of 0^o at the first surface of the glass, so the light enters the glass without being bent. Using this figure, we can relate the critical angle to the desired angle of the edge of the glass α so find that $\alpha + \theta_{crit} = 90^o$. So if we can find the critical angle, we can then determine the desired angle α of this prism.

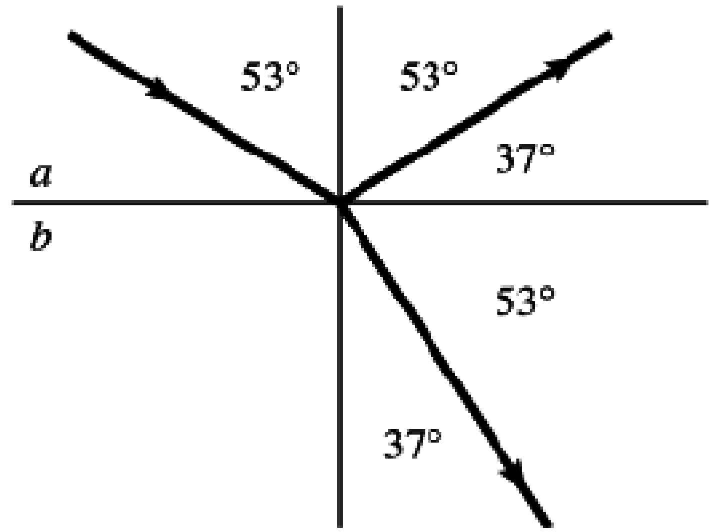


(a) For the glass-air interface, we have $\sin\theta_{crit} = n_b/n_a = (1.00)/(1.52)$ so $\theta_{crit} = 41.1^o$. The prism angle α then is $\alpha - 90^o - \theta_{crit} = 48.9^o$.

(b) For the glass-water interface, we have $\sin\theta_{crit} = n_b/n_a = (1.333)/(1.52)$ so $\theta_{crit} = 61.3^o$. The prism angle α then is $\alpha - 90^o - \theta_{crit} = 28.7^o$.

Example 26 : Light traveling in water strikes a glass plate at an angle of incidence of 53.0^o. Part of the beam is reflected and part is refracted. If the reflected and refracted portions make an angle of 90^o with each other, what is the index of refraction of the glass?

First, we draw a figure to see what's going on here. The angles of incidence and reflection will be 53^o. That angle is always measured with respect to the normal drawn at the interface. The light is then refracted into the second material at some angle. Moving clockwise around the figure, we can propagate angles around. The angle between the reflected light beam and the interface will be $90 - 53 = 37^o$. We were told that the refracted and reflected rays make an angle of 90^o so that tells us that the angle between the refracted ray and the interface must be $90 - 37 = 53^o$. Knowing that angle, we can determine the angle of refraction (the angle between the ray and the normal) to be 37^o.



$n_a \sin\theta_a = n_b \sin\theta_b$ where a is the water side (where $n_a = 1.333$) and b is the glass side with the unknown index of refraction. Using the angles we determined above, $1.333 \sin 53^o = n_b \sin 37^o$ from which we find that $n_b = 1.77$.

Example 38 : A light beam is directed parallel to the axis of a hollow cylindrical tube. When the tube contains only air, it takes the light 8.72 ns to travel the length of the tube, but when the tube is filled with a transparent jelly, it takes the light 2.04 ns longer to travel its length. What is the refractive index of this jelly?

Note the phrasing here. When we fill the tube with jelly, it takes 2.04 ns LONGER than when the tube was just filled with air, so the time it takes for light to pass through the jelly-filled tube is $(8.72\ ns) + (2.04\ ns) = 10.76\ ns$.

The speed of light in a medium with index of refraction n is $v = c/n$. In the 'air' case, $n = 1$ so $v = c$ and we can use the information provided to figure out the actual length of the tube: $d = vt$ so $d = (3.00 \times 10^8\ m/s)(8.72 \times 10^{-9}\ s) = 2.616\ m$.

The length of the tube hasn't changed, so when the light is traveling through the jelly, $d = vt$ so $(2.616\ m) = (v)(10.76 \times 10^{-9}\ s)$ from which $v = 2.431 \times 10^8\ m/s$. Finally, $n = c/v = 1.23$.

We can short cut this process though. Let's use a subscript of a to represent the air case, and j the jelly-filled case. Then $d = vt = v_a t_a = v_j t_j$. But $v_a = c$ so $c t_a = v_j t_j$. Rearranging this: $c/v_j = t_j/t_a$.

But c/v_j is just the index of refraction of the jelly so $n_j = t_j/t_a = (10.76\ ns)/(8.72\ ns) = 1.23$. (Doing it this way avoided the intermediate calculations and the prospect of introducing round off errors.)

Example 40 : In a physics lab, light with wavelength 490 nm travels in air from a laser to a photo-cell in 17 ns. When a slab of glass 0.840 m thick is placed in the light beam, with the beam incident along the normal to the parallel faces of the slab, it takes the light 21.2 ns to travel from the laser to the photo-cell. What is the wavelength of the light in the glass?

We can find the wavelength of the light in the glass if we can determine its index of refraction. See example 4 above, where we derived that: $n_a\lambda_a = n_b\lambda_b$ (where here a labels air ($n_a = 1.0$ and $\lambda_a = 490\ nm$).

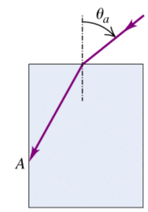
We could use the information provided in the air case to determine the total distance d between the laser and the photo-cell. Then when we add the slab of glass, we have the light traveling 0.840 m through glass and $d - .84$ through air. Each of those represents a time interval, the sum of which needs to be 21.2 ns.

A semi-shortcut here is to note that when we put the glass in place, it takes 4.2 ns LONGER for the light to travel that 0.84 m of distance. So if we just look at that section of the path, the time is takes light to travel in air that distance is $(0.84\ m)/c$ and the time it takes light to travel in the glass is $(0.84\ m)/v$ but $v = c/n$ so we can write this as $(0.84\ m)\frac{n}{c}$. The DIFFERENCE of these times is 4.2 ns so: $(0.84\ m)\frac{n}{c} - (0.84\ m)\frac{1}{c} = 4.2\ ns$. (Note here we put the time through the glass first, since we know the speed of light will be less there, meaning that the travel time will be larger.)

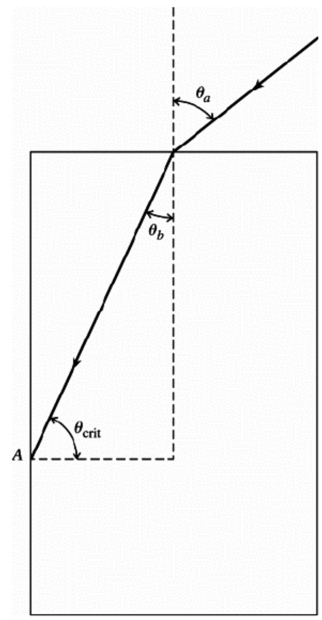
Rearranging the left side: $(0.84\ m)\frac{n-1}{c} = 4.2\ ns$ or $(n-1) = (4.2\ ns)(c)/(0.84\ m)$ or finally $n = 1 + (4.2\ ns)(c)/(0.84\ m)$. Inserting the value of c and converting the time from nanoseconds to seconds: $n = 1 + \frac{(4.2\times10^{-9}\ s)(3.00\times10^{8}\ m/s)}{0.84\ m} = 1 + 1.5 = 2.5$.

And now finally $n_a\lambda_a = n_b\lambda_b$ gives us $(1.000)(490\ nm) = (2.50)(\lambda_b)$ or $\lambda_b = 196\ nm$.

Example 41 : A ray of light is incident in air on a block of a transparent solid whose index of refraction is n. If $n = 1.38$, what is the **largest** angle of incidence θ_a for which total internal reflection will occur at the vertical face (point A in the figure)?

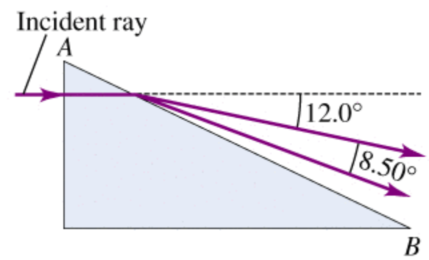


Sketching the figure at right, the situation described in the problem implies that the angle the light hits the vertical face (at point A) should be the critical angle. But that means that θ_b will be $90 - \theta_{crit}$ due to that triangle formed from the dotted lines (which are the normals to the two surfaces).



So, looking at point A, the glass-to-air interface, and applying Snell's law: $(1.38)\sin\theta_{crit} = (1.00)\sin 90$ or $\theta_{crit} = 46.4^o$.
But this implies that: $\theta_b = 90 - \theta_{crit} = 43.6^o$
Now we can apply Snell's law at the top surface to find the angle of incidence: $n_a \sin\theta_a = n_b \sin\theta_b$ or $(1.000)\sin\theta_a = (1.38)\sin 43.6 = 0.952$ which which $\theta_a = 72.1^o$.

Example 42 : A light ray in air strikes the right-angle prism shown in the figure. At point A, the angle of the prism is 60^o and at point B, the angle is 30^o. (Note: the angles were left out of the figure in the book but you can't solve the problem without them.) This ray consists of two different wavelengths. When it emerges at face AB, it has been split into two different rays that diverge from one another by 8.5^o. Find the index of refraction of the prism for each of the two wavelengths.



The trick here is to convert the angles provided into the angles we actually need to apply Snell's law, which are angles with respect to the normal.

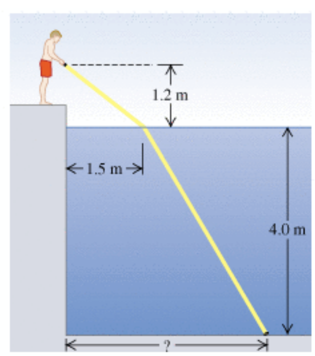
So first we draw a line normal to the AB line and intersecting the line where the incident rays are hitting that surface. The dotted line they drew in the figure is parallel to the base of the prism, so the angle between the dotted line and the line AB (in the clockwise direction) is also 30^o. The angle from the dotted line UP to the normal we just drew is thus 60^o. But looking on the inside of the prism now, that's exactly the angle the incident ray is making with the normal. (We will surely do this example in class to make this 'propagation of angles' more clear.) So the angle of incidence (for the light in the glass as it hits the glass-air interface along the line AB is $\theta_a = 60^o$.

Similar hand-waving converts the given angles into the angles of refraction for the two rays. The upper ray has $\theta_b = 60 + 12 = 72^o$ and the lower ray has $\theta_b = 60 + 12 + 8.5 = 80.5^o$.

Snell's law for the refracted rays: $n_a \sin\theta_a = n_b \sin\theta_b$. Here, n_a is the unknown index of refraction of the prism (n) and n_b is the index of refraction of air (1.00) so: $(n)(\sin 60) = (1.00)(\sin\theta_b)$ or rearranging: $n = \frac{\sin\theta_b}{\sin 60} = 1.155 \sin\theta_b$

For the upper ray: $n = (1.155)(\sin 72) = 1.098$ and for the lower ray: $n = (1.155)(\sin 80.5) = 1.139$.

Example 46 : After a long day of driving, you take a late-night swim in a motel swimming pool. When you go to your room, you realize that you have lost your room key in the pool. You borrow a powerful flashlight and walk around the pool, shining the light into it. The light shines on the key (which is lying somewhere on the bottom of the pool) when the flashlight is held 1.2 m above the water surface and is directed at a point on the surface of the water a horizontal distance of 1.50 m from the edge (see figure). If the water here is 4.0 m deep, how far is the key really from the edge of the pool?



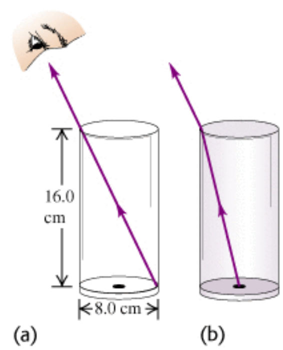
The light from the flashlight will be refracted when it enters the water. We can use the information provided to determine the angle of incidence. If we draw a rectangle around the air-part of the light ray with the top of the rectangle aligned with the dotted line in the figure, we can see that $\tan\theta_a = 1.5/1.2$ so $\theta_a = 51.34^o$.

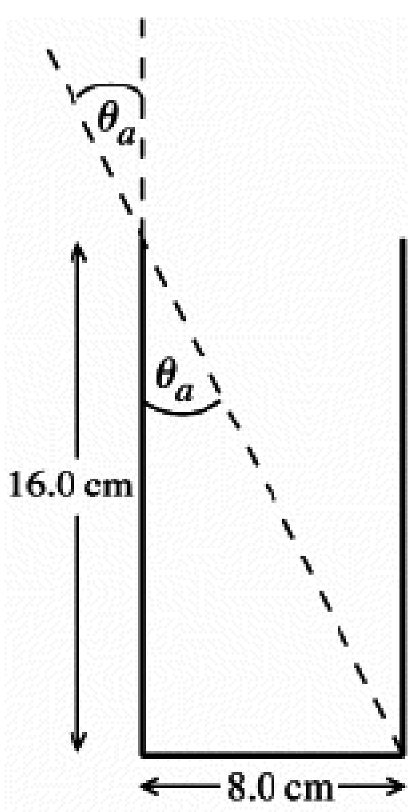
The angle of refraction, then, can be found from Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$ where a is the air side and b is the water side: $(1.00)\sin 51.34 = (1.333)\sin\theta_b$ from which $\theta_b = 35.86^o$.

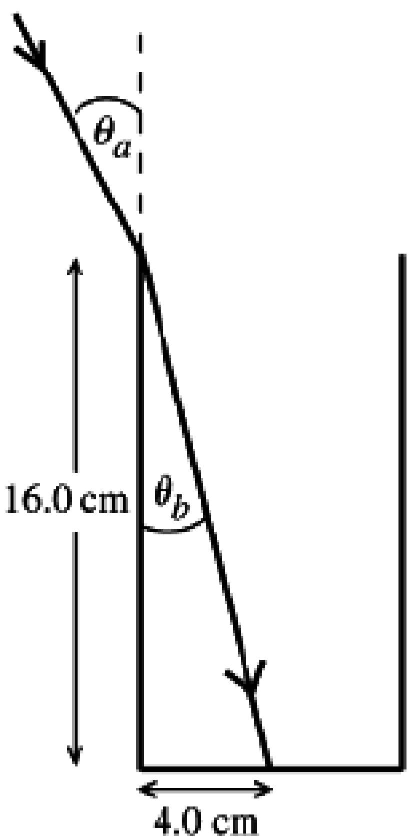
Forming a triangle with a hypotenuse representing the light ray in the water, we see that the bottom of that triangle and the side are related by $\tan\theta_b = x/4$ or $x = (4.00\ m)\tan 35.86 = (4.00\ m)(0.7228) = 2.89\ m$.

The full distance from the edge of the pool to the spot on the bottom of the pool then is $(1.5\ m) + (2.89\ m) = 4.39\ m$.

Example 47 : You sight along the rim of a glass with vertical sides so that the top rim is lined up with the opposite edge of the bottom. The glass is a thin-walled hollow cylinder 16.0 cm high with a top and bottom diameter of 8.0 cm. A dime is placed at the center of the bottom of the glass. Now, while you keep your eye pointing in the same direction, a friend fills the glass with a transparent liquid and you see the dime 'move' so that it is now directly in your line of vision. What was the index of refraction of the liquid?







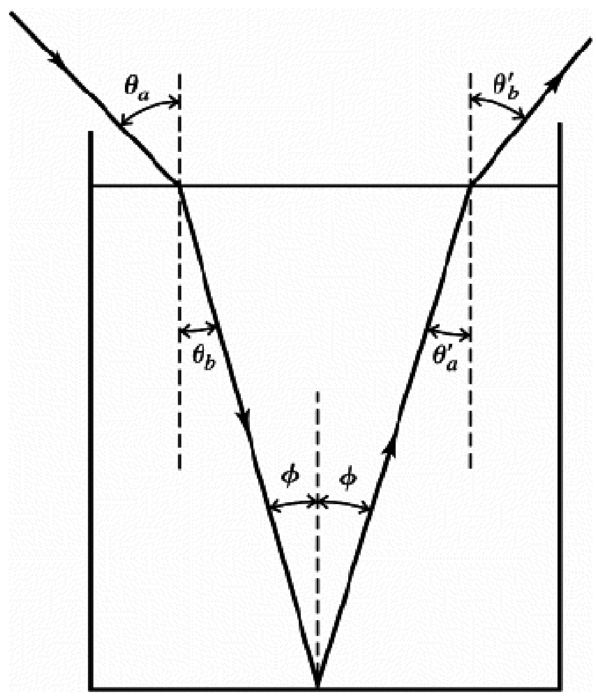
The left figure shows the situation when the glass is empty. The person does not move where they're pointing their eye so when the glass is later filled (right figure) we see that we can determine the angle of incidence from the information provided in the left figure. From that figure, $\tan\theta_a = \frac{8.0\ cm}{16.0\ cm} = 0.500$ or $\theta_a = 26.57^o$.

The right figure shows the path of the light ray when the glass is filled. We're told that we see the dime now, but the dime was really located at the middle of the glass, so the ray must be hitting the middle of the glass now. From this diagram, $\tan\theta_b = \frac{4.0\ cm}{16.0\ cm} = 0.250$ or $\theta_b = 14.04^o$.

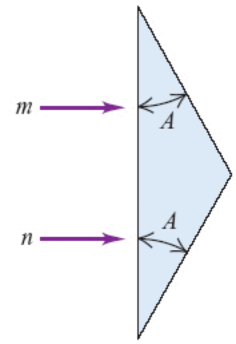
Now that we know the two angles, we can apply Snell's law to find the index of refraction the liquid must have had: $n_a \sin\theta_a = n_b \sin\theta_b$ where a is the air and b is the liquid, so: $(1.00)\sin 26.57 = (n)\sin 14.04$ from which $n = 1.84$.

Example 48 : A beaker with a mirrored bottom is filled with a liquid whose index of refraction is 1.63. A light beam strikes the top surface of the liquid at an angle of 42.5^o from the normal. At what angle from the normal will the beam exit from the liquid after traveling down through the liquid, reflecting from the mirrored bottom, and returning to the surface?

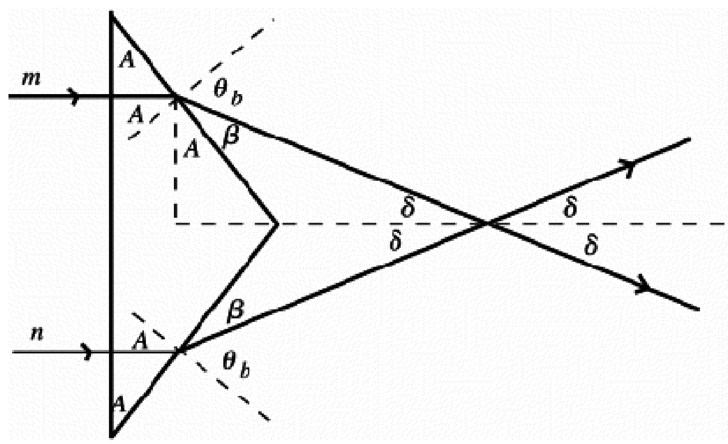
This can be done without doing any calculations at all. The ray has some angle of incidence θ_a. It will refract into the liquid at some other angle θ_b. That ray reflects off the bottom but we see from the diagram that that angle ϕ will be the same as the angle θ_b. On the mirror, the angle of reflection will be the same as the angle of incidence. The ray now proceeds with this angle to the surface but we see that the angle of incidence θ'_a is the same as ϕ, which in turn was the same as θ_b, so when we do Snell's law for the ray leaving the liquid, we end up with exactly the same equation we had when it originally entered the liquid and the angle it leaves with (θ'_b) will be the same as the original incidence angle.



Example 51 : The prism shown has a refractive index of 1.66 and the angles A are each 25^o. Two light rays m and n are parallel as they enter the prism (perpendicular to that surface). What is the angle between them after they emerge?



The problem does not specify the material the light rays are moving in when they are outside of the prism. We'll assume that it is air. Looking at the upper ray first, it is entering the prism on the back perpendicular to that surface, so the angle of incidence there is zero meaning that the angle of refraction will be zero as well. So the light continues in the original direction until it hits the sloping surface. It strikes this surface at an angle. Since the index of refraction of the prism is higher than air, the beam will be refracted in a way that the angle of refraction is larger than the angle of incidence (on that surface). That means that the upper ray will be bent downward. Similarly, looking at the lower ray, we can argue that it will end up being bent upwards after passing through the prism. This lets us draw the figure at right, which we can use to propagate angles around.



Looking at the angles in the triangle at the top, we can infer that the angle of incidence will be the same as the angle A at the top of the prism. We know the index of refraction of the material, so we can compute the angle of refraction θ_b. That angle plus β equals 90 degrees. β plus A plus 90 plus δ equals 180, so finally we can find that angle δ in the figure, and the angle between the two outgoing rays will be $2 \times \delta$. (Seems like a lot of steps but I couldn't find a shortcuts.)

In this particular prism, we were told that the angles at the top and bottom were both equal to 25^o so at least we only need to do all these calculations once.

Snell's law applied to the upper face where the ray is moving from the prism into air: $n_a \sin\theta_a = n_b \sin\theta_b$ where a is the glass and b is the air, so: $(1.66)\sin 25 = (1.00)\sin\theta_b$ or $\theta_b = 44.55^o$.

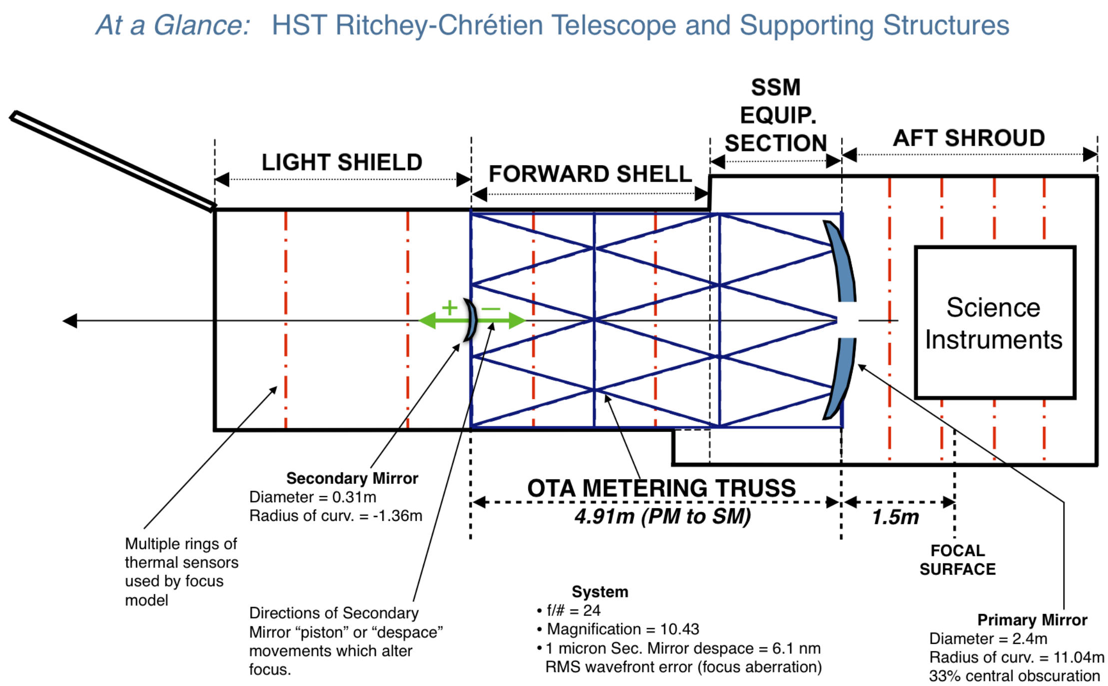
$\theta_b + \beta = 90$ so $\beta = 45.45^o$.

Finally $A + \beta + \delta = 90$ so $\delta = 19.55^o$ so the angle between the two rays is twice that or 39.1^o

Multiple Mirrors (Hubble Telescope)

Refer to the section on convex and concave mirrors.

The focal length of the Hubble space telescope is quoted as being just under 60 m, but that is far larger than the physical size of the telescope, so how is this accomplished? The trick is to use more than one mirror:



Light from remote objects enters the open end of the telescope tube (from the left in the figure) and falls on a large **concave** mirror (called the **primary**) at the back end. The radius of curvature of this mirror is $R = +11040\ mm$, creating a focal length of $f = R/2 = 5520\ mm = 5.520\ m$. (Which is nowhere close to the 60 meters it's supposed to have...)

Before the rays have a chance to converge on the focal point though, they encounter a second mirror (called the **secondary**). This **convex** mirror has a radius of curvature of $R = -1358\ mm$, creating a focal length of $f = R/2 = -679\ mm = -0.679\ m$.

Let's see what happens to light rays coming in from far away (and actually given the dimensions, radii of curvature, etc here, even pointing the telescope towards the earth would be considered 'far away).

The first mirror will (try to) form an image at: $d_i = \frac{d_o f}{d_o - f}$ and when the object distance d_o is vastly larger than the focal length, the image essentially forms right at the focal length: $d_i = f = 5.520\ m$.

Now, what happens if we stick that second mirror in the way. The image from the first mirror becomes the object for the second mirror. This 'object' is located **behind** the second mirror though (in the figure, that would be slightly to the left of the secondary mirror). The two mirrors are separated by $4.90597\ m$ so this 'object' is located $5.520 - 4.90597 = 0.61403\ m$ **behind** the secondary mirror. That is, when we apply the mirror equation to this 'object', we'll need to use an object distance of $d_o = -0.61403\ m$.

Where does the final image form then? For the secondary mirror, we have a focal length of $f = -0.679\ m$ and an object distance of $d_o = -0.61403\ m$ (relative to the mirror location), creating an image distance of: $d_i = \frac{d_o f}{d_o - f} = \frac{(-0.61403)(-0.679)}{(-0.61403)-(-0.679)} = +6.4172\ m$.

This will be $6.4172\ m$ to the right of the secondary mirror, which puts it $1.51\ m$ **behind** the primary mirror (in the figure, that would be slightly to the right of the primary mirror). To deal with this, the primary mirror actually has a **hole** cut out of its middle so that the light from the secondary mirror can pass through, where it can then be captured by the cameras and other instruments placed behind the primary mirror.

Effective Focal Length

So how does this turn into a focal length that's so much longer than the telescope itself?

The magnification factor for a mirror is $m = h_i/h_o = -d_i/d_o$, but for far objects, d_i is essentually just f, so $m \approx -f/d_o$.

Let's look at the two-mirror situation now.

For the first mirror, $m_1 \approx -f_1/d_o = -5.52/d_o$.

The secondary mirror takes that 'object' and further magnifies it by a factor of $m_2 = -d_i/d_o = -(6.4172\ m)/(-0.61403\ m) = +10.45$.

The overall magnification then is $m = m_1 m_2 = \frac{-5.52}{d_o} \times 10.45 = -57.7/d_o$.

If we look back a step though, the magnification for a single concave mirror looking at something far away is $m = -f/d_o$ so apparently this combination of two mirrors has an effective focal length of $f = +57.7\ m$ (over 4 times as long as the telescope itself).

Note 1 : The actual focal length for Hubble's two-mirror system is quoted as being $f = 57.6\ m$ so we're slightly off, which is almost certainly due to the fact that some of the measurements used in this calculation were rounded to the nearest millimeter, but those are the most accurate values I could find.

Note 2 : The next major space telescope is the James Webb Space Telescope (hopefully to be launched in the next year or two), which expands on this idea by using **three** curved mirrors to achieve an even longer effective focal length of $131\ m$.

Example: Sound Refraction

For light (and other electromagnetic waves), the incoming and refracted ray angles (relative to the normal) are related by: $n_1 \sin\theta_1 = n_2 \sin\theta_2$, where $n = c/v$ is the index of refraction of the materials.

Snell's Law applies to any propagating waves, though, not just light. The more generic version is: $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$ where v_1 and v_2 are the wave propagation speeds in the two media.

Consider a (point) source of sound waves located some distance above an air-water interface. How will the 'rays' of sound from this source refract when they enter the water?

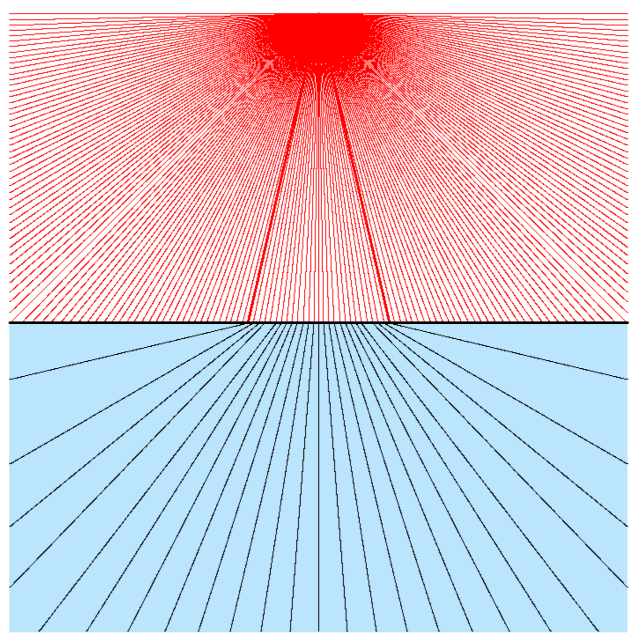
If we rearrange the more generic version of Snell's law, we have: $\sin\theta_2 = \frac{v_2}{v_1}\sin\theta_1$. In our scenario, the source is located in air, so $v_1 = 343\ m/s$ (speed of sound in air) but below the water, the speed of sound is closer to $v_2 = 1500\ m/s$, so: $\sin\theta_2 = \frac{1500}{343}\sin\theta_1 = 4.27\sin\theta_1$.

For any given incoming angle, we take the sine of that angle, multiple the result by 4.27 and then take the inverse sine to find the refracted ray angle, which will clearly be (much) larger now.

In the figure below, a point sound source is located at the center of the red blob at the top of the picture and the (red) lines represent rays travelling outward at 1 degree increments and the black lines represent the refracted rays these incoming rays turn into.

Since $v_2 > v_1$, where will be a maximum incoming angle θ_1 for which refraction can occur. Right at that critical point, the refracted ray reaches $\theta_2 = 90^o$ and any larger incoming angles can't refract into the water at all (which means any energy coming in at these higher angles is completely **reflected** back out into the air). Here, this critical angle would be $\sin\theta_c = 343/1500 = 0.2287$ from which $\theta_c = 13.2^o$. (This angle is shown with a thicker red line in the figure.)

The cone of sound that makes it into the water would be $2 \times 13.2 = 26.4^o$ across. Let's compare that to the full circle (360 degrees) and we see that only 0.073 of the energy makes it into the water. But actually it's much worse than that - the sound is actually spreading out **spherically**, so it's the **area** on the end of this cone that matters. This $\theta = 13.2^o$ 'cone' represents a fraction of the total spherical area of $\frac{1-\cos\theta}{2} = 0.0132$ so only a little over 1% of the intensity produced by the above-water sound makes it into the water.



Once it does so, it spreads out over an entire hemisphere, making the intensity at any point under the water even less.