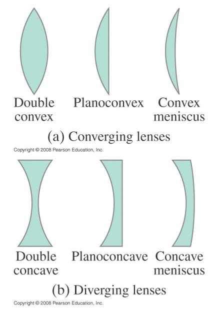
PH2233 Fox : Lecture 13
Chapter 33 : Lenses and Optical Instruments

(33.1) : Thin Lenses; Ray tracing

Lenses are typically relatively thin pieces of clear material (glass or plastic) with one or two curved surfaces.
A **converging** lens will take light coming from far away (parallel rays, say) and **focus** it at a point on the other side of the lens.
A **diverging** lens does the opposite: the incoming rays get spread out when they pass through the lens.

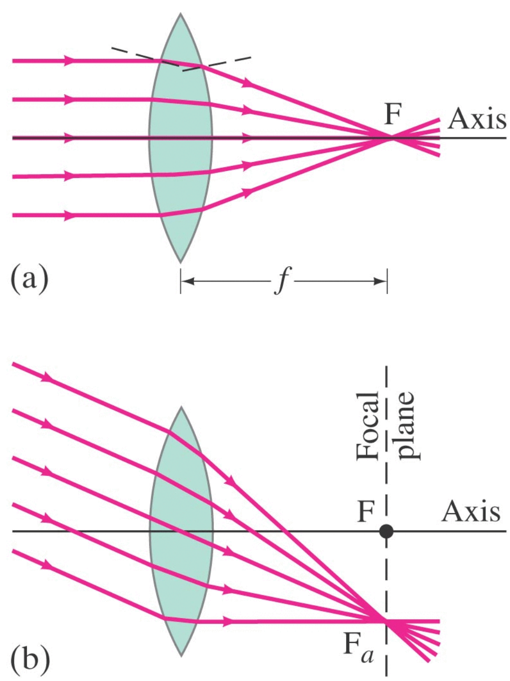


Let's use what we know about refraction to understand how this happens. The claim in the figures above is that the 'double convex' lens will be a converging lens, so let's zoom in on the upper part of the lens and send a ray parallel to the axis into it. Remember with Snell's law all angles are measured relative to the normal to the interface. The ray hits the left side of this lens at an angle θ_1. The lens is glass, so has a higher index of refraction than the air, so θ_2 will be larger than θ_1. The ray now hits the right side of the lens at some angle θ_3. The ray is now moving from a higher to a lower n, so will exit at an angle θ_4 which is less than θ_3. The net result though is that overall the ray has still been 'bent downward' by the lens.

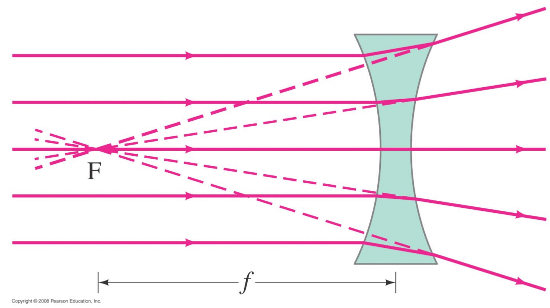
The claim is that a converging lens (upper figure) will result in incoming parallel rays being focused at a point F (called the focal point).

We haven't (and won't) show that the same happens to parallel rays coming in 'off axis'. Those rays will also focus and the nexus of these focal points is called the **focal plane**.

(Convenient since we can put a piece of film or an image sensor at that location and record an image.)



A diverging lens (lower figure) will spread out the incoming rays, but in such a way that they **appear** to be coming from a point on the 'incoming ray' side of the lens. That point is still called the focal point, although in this case the light isn't focused there but rather 'appears to be coming' from there.
We'll be able to show later that any lens will behave the same way if we just flip the lens around.



Terminology/Convention

Optometrists/ophthalmologists (eye doctors) don't typically use f to describe lenses, but instead use a term called the **POWER** of a lens, which is defined as $P = 1/f$. The standard in that field is to use a focal length in meters, resulting in a **lens power** in units of inverse-meters which are then called a **diopter** (D). If you have a prescription for eyeglasses or contact lenses, you'll see lens powers listed there, and most pharmacy's sell 'reading glasses' that are labelled like $+1\ D$, $+2.5\ D$ and so on, giving the diopters for the lenses (which you could convert into a focal length (in meters) by just inverting the number on the label).

Lens Power : $P = 1/f$ (with f in meters, giving P in **diopters**)

Example: A lens with a focal length of $f = 20\ cm$ would have a lens power of $P = 1/f$ but we need f in meters first, so $f = 0.2\ m$ and then $P = 1/0.2 = 5\ m^{-1} = 5.0\ D$.

NOTE: we'll almost exclusively use the focal length f (instead of the 'lens power') in our examples until we get to the section on corrective eyeglasses.

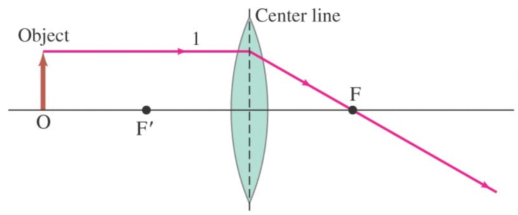
RAY DIAGRAM for converging lens

Since we know that a focal point exists, we **already have the tools** we need to see how a given lens creates an image from an object.

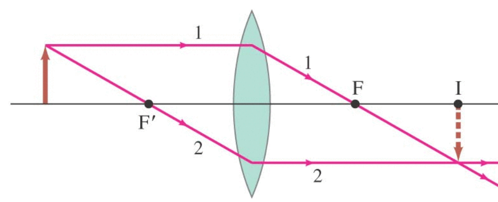
Let's start with a converging lens and an object farther away than F from the lens.

First, note we've marked two focal point here: the one labelled F is where parallel rays coming in from the left would focus. We 'know' the lens works both ways though, so rays coming in from the right would focus at F'.

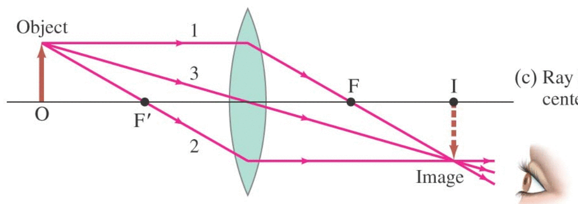
- **Ray 1** : rays (photons) from a point on the object that travel parallel to the axis will hit the lens and head towards F.
- **Ray 2** : A ray that passes through that 'other' focal point F' will refract into a ray parallel to the axis (because a photon coming in from the right along that exact path would refract that way).
- **Ray 3** : A ray (photon) that passes through the center (vertex) of the lens will ultimately exit at the same angle it came in with, appearing to just keep going in a straight line (we'll show why in class).



(a) Ray 1 leaves one point on object going parallel to the axis, then refracts through focal point behind the lens.



(b) Ray 2 passes through F′ in front of the lens; therefore it is parallel to the axis behind the lens.



(c) Ray 3 passes straight through the center of the lens (assumed very thin).

In this case, the image forms some distance over to the right of the lens and the photons are actually passing through that point so this is a **real** image. (A piece of film or an image sensor can be placed right there and record the image.)

Note that in order to 'see' this image, our eye would need to be located somewhere further to the right. If we move in too close, the rays haven't had time to focus and we'd just see a blur.

EXAMPLE : Suppose we have a converging lens with $f = 20$ and place an object 30 cm to the left of this lens. Use a ray diagram to determine where the image will form. Is it real or virtual? Upright or inverted? Larger or smaller than the object?

EXAMPLE : Suppose we use the same lens but place an object just 10 cm from the lens.

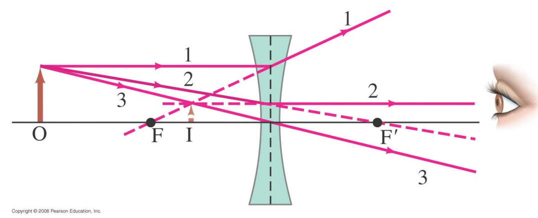
(See the last page of this pdf for the figure, along with the numerical solution via the 'lens equation'.)

RAY DIAGRAM for a diverging lens

What if we have a diverging lens? We 'know' that parallel rays coming in from the left will get spread out in such a way that they appear to be coming from F (which is over on the incoming ray side now).

Again, let's start with an object farther away than F from the lens.

First, note again we've marked two focal point here: the one labelled F (over to the left, on the 'incoming' ray side) which is where parallel rays coming from the left appear to be coming from after they pass through the lens. These lenses are 'symmetric' too, so parallel rays of light coming in from the right would pass through the lens and spread out on the left, but they'll appear to be coming from the point labelled F'.



- **Ray 1** : rays (photons) from a point on the object that travel parallel to the axis will hit the lens and head off to the right, but appear to have come from F.
- **Ray 2** : If we draw a ray from the point that is aimed at F' over on the right, it will refract into a ray parallel to the axis (because a ray coming in from the right at just the right distance from the axis will refract as shown, appearing to come from F' and passing through that point on the object.
- **Ray 3** : A ray (photon) that passes through the center (vertex) of the lens will ultimately exit at the same angle it came in with, appearing to just keep going in a straight line.

In this case, notice that the image is forming over on the left (that's where it looks like it is) but the photons aren't actually coming from there to our eye, so this is a virtual image. An image sensor placed there would record nothing.

Example : Suppose we have a diverging lens with a 20 cm focal length and place an object 10 cm from the lens. Use a ray diagram to find where the image will form. Again, is it upright? Inverted? Larger or smaller? Real or virtual?

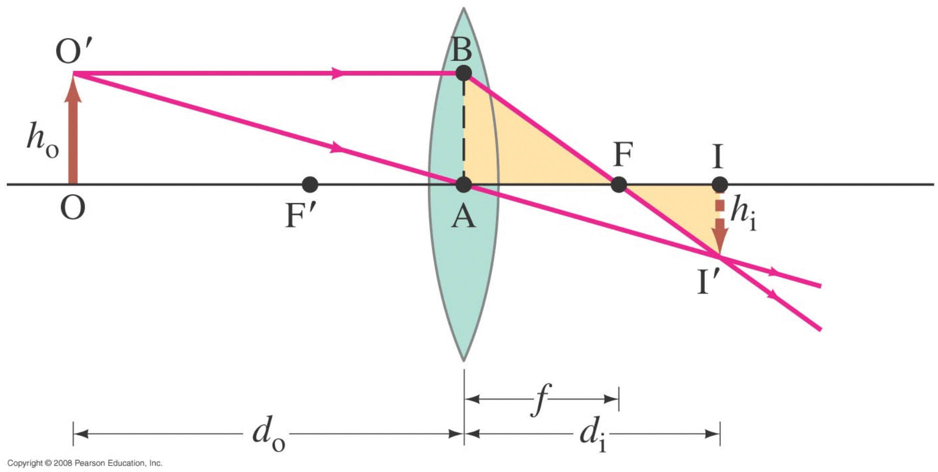
Example : Repeat with an object 30 cm from the lens.

(See the last page of this pdf for the figure, along with the numerical solution via the 'lens equation'.)

(33.2) The Thin Lens Equation; Magnification

I'll leave this derivation here, but we won't actually do it in class.

Let's start with a converging lens, with an object outside the focal point. We've drawn two rays here: one parallel to the axis that refracts through the focal point F, and one that passes straight through the vertex.



Triangles $FI'I$ and FBA are similar so we can do ratios of various pairs of sides.

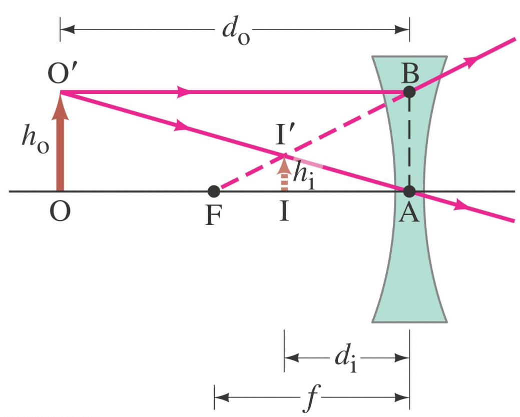
Note that the image is upside down here and will be a negative number (like we encountered with mirrors), so unlike the book I'm going to call that **length** $-h_i$. The length AB is just h_o so $\frac{-h_i}{h_o} = \frac{d_i - f}{f}$

Triangles OAO' and IAI' are also similar, so: $\frac{-h_i}{h_o} = \frac{d_i}{d_o}$. (Giving us $\boxed{m \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}}$).

Both these equations have the same LHS so their RHS's must be equal too:
$\frac{1}{f} - \frac{1}{d_i} = \frac{1}{d_o}$ or $\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$.

Conveniently, these equations are identical to the equations we used for mirrors!

We can do the same thing for a diverging lens. This time the image is upright so the 'length' of that side of the triangle is just h_i and I don't have to throw in a negative sigh this time.
Triangle IAI' is similar to triangle OAO', and triangle IFI' is similar to triangle AFB so comparing the matching sides of those pairs we have: $\frac{h_i}{h_o} = \frac{d_i}{d_o}$ and $\frac{h_i}{h_o} = \frac{f - d_i}{f}$. As before the LHS's are the same, so the RHS's must be too.
This leads us (unfortunately) to the equation $\boxed{\frac{1}{d_o} - \frac{1}{d_i} = -\frac{1}{f}}$, which is NOT the same as what we have now for both mirrors and converging lenses, so let's see if we can fix that.



It would be nice if we could use a single equation for mirrors and **both** types of lens, and we can do that by making some **sign convention** choices.

In the converging lens, the actual rays (photons) travelled from the object on the left, creating a (real) image on the right. With this diverging lens, the image is being created on the 'wrong' side in a sense. It's a virtual image since the rays only appear to be coming from it. Let's call this a **negative** image distance.

Same with the focal length. Since F is on the 'wrong' side compared to a converging lens, let's say that f is negative for a diverging lens.

This is summarized in the book and I'll repeat those **sign rules** (conventions, really) here:

SIGN CONVENTIONS

1. The focal length f is positive for converging lenses and negative for diverging lenses.

2. The object distance is positive if the object is on the side of the lens from which the light is coming (you'd think this would always be the case, but when we start to deal with multiple lenses, like we did with multiple mirrors, we can end up with a negative object distance).

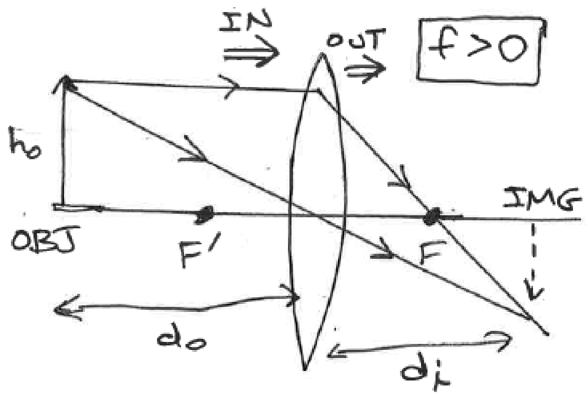
3. The image distance is positive if the image is on the opposite side of the lens where the light is coming; if it is on the same side, d_i is negative. Equivalently, $d_i > 0$ for a real image and $d_i < 0$ for a virtual image.

4. The height of the image h_i is positive if the image is upright and negative if the image is inverted (relative to the object; h_o is always taken as upright and positive).

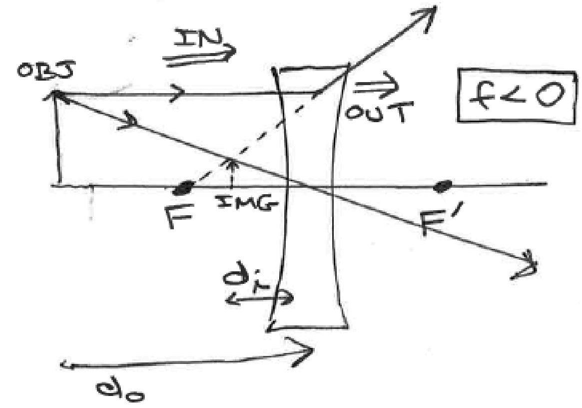
The magnification m of a lens is defined as the ratio of the image height to the object height: $m \equiv h_i/h_o$ and using the sign conventions we have now, we can also write this as $m = -\frac{d_i}{d_o}$

At the end of the day, we basically end up with a single equation that we can use for both mirrors and lenses. (Some other textbooks don't use the same sign rules and end up with alternate equations, so be careful if you use other books for lens examples.)

Sign Rules : Just summarizing them again with a picture to help follow the arguments.



Converging Lens



Diverging Lens

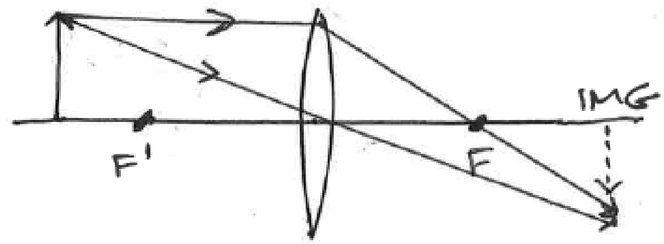
The easiest way to deal with this is to consider the actual photon paths. Photons leave the object and enter the lens on the left side in these figures, making that the 'incoming ray' or 'incoming photon' side of the lens. The lens is clear, so those photons exit on the other side, defining the 'outgoing ray' or 'outgoing photon' side of the lens.

If the **object** is on the 'correct' side (the incoming ray side) then d_o is positive. (If we only have a single lens, this will always be the case, but sometimes not when more than one lens is involved.)

If the **image** is on the 'correct' side (the outgoing ray side) then d_i is positive. (Otherwise, d_i is negative.) the lens

Let's place a lens between an object (over to the left in these examples) and our eye (somewhere over to the right of the lens).

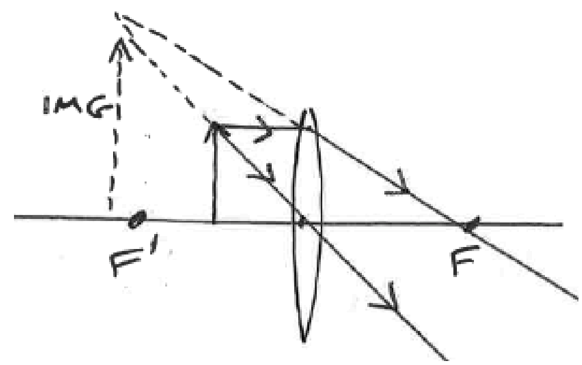
$\boxed{\text{EXAMPLE 1}}$: Suppose we have a converging lens with $f = 20$ and place an object 30 cm to the left of this lens. Use a ray diagram to determine where the image will form. Is it real or virtual? Upright or inverted? Larger or smaller than the object? Use the lens equations, which should yield numerical values that are consistent with the ray diagram.



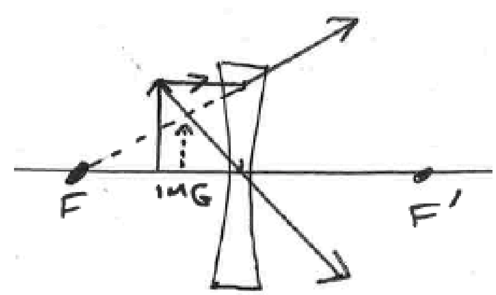
The object is on the 'incoming ray' side, so $d_o = +30\ cm$. This is converging lens, so $f = +20\ cm$.

$d_i = \frac{d_o f}{d_o - f} = \frac{(30)(20)}{30-20} = \frac{600}{10} = +60\ cm$. ($d_i > 0$, so this is a REAL image.)
$m = -d_i/d_o = -(60\ cm)/(30\ cm) = -2.0$ so the image is larger, but INVERTED.

$\boxed{\text{EXAMPLE 2}}$: Suppose we use the same lens but place an object just 10 cm from the lens.
The object is on the 'incoming ray' side, so $d_o = +10\ cm$. This is converging lens, so $f = +20\ cm$.
$d_i = \frac{d_o f}{d_o - f} = \frac{(10)(20)}{10-20} = \frac{200}{-10} = -20\ cm$.
$m = -d_i/d_o = -(-20\ cm)/(10\ cm) = +2.0$
$d_i < 0$ so we have a VIRTUAL, magnifed and UPRIGHT image.
(NOTE: this is the geometry present when you use a **magnifying glass** to look at something.)



$\boxed{\text{EXAMPLE 3}}$: Suppose we have a **diverging** lens with a 20 cm focal length and place an object 10 cm from the lens.
The object is on the 'incoming ray' side, so $d_o = +10\ cm$. This is diverging lens, so $f = -20\ cm$.
$d_i = \frac{d_o f}{d_o - f} = \frac{(10)(-20)}{(10)-(-20)} = \frac{-200}{30} = -6.67\ cm$.
$m = -d_i/d_o = -(-6.67\ cm)/(10\ cm) = +0.667$
$d_i < 0$ so we have a VIRTUAL image that is UPRIGHT but smaller than the object.



$\boxed{\text{EXAMPLE 4}}$: Repeat with an object 30 cm from the lens.
The object is on the 'incoming ray' side, so $d_o = +30\ cm$. This is diverging lens, so $f = -20\ cm$.
$d_i = \frac{d_o f}{d_o - f} = \frac{(30)(-20)}{(30)-(-20)} = \frac{-600}{50} = -12.5\ cm$.
$m = -d_i/d_o = -(-12.5\ cm)/(30\ cm) = +0.417$
$d_i < 0$ so we have a VIRTUAL image that is UPRIGHT and even a little smaller than in the previous example.