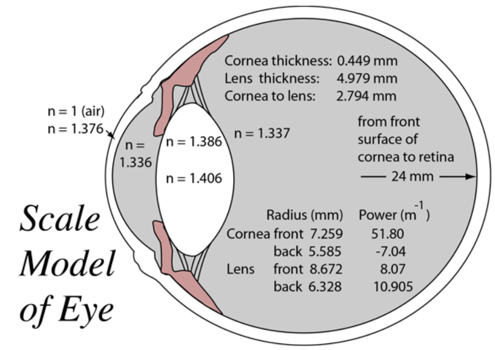
PH2233 Fox : Lecture 15
Chapter 33 : Lenses and Optical Instruments

(33-6) : Corrective Lenses

This diagram shows an eye for a typical human (or any mammal really). Refraction happens at each interface, but most of it happens at the air-cornea interface since that's where (by far) the largest contrast in n occurs.
In a 'relaxed' eye, light coming in from far away ($d_o = \infty$) focuses on the retina, which is about 24 mm from the front surface of the cornea.

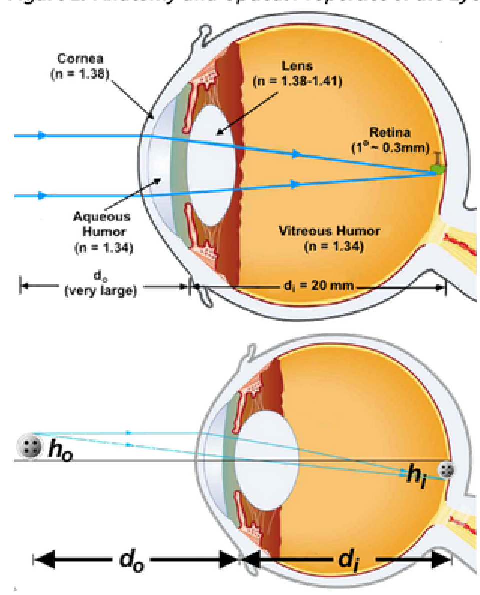


From our lens equation: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ so $\frac{1}{f} = \frac{1}{24\ mm}$ so $f = 24\ mm$. That's $f = 0.024\ m$, so $P = 1/f = 41.7\ D$, and about 2/3 of that is achieved by the air-cornea interface alone.

We don't always look at things far away though, so the eye has to be able to change its focal length so we can look at things closer to us. Muscles can change the 'length' of the eye only very slightly. Most of this **accomodation** is handled by **altering the shape of the lens itself** as seen here.
Definitions:

- **Near Point** : the distance to the closest object you can still focus on. For a 'normal eye', this is about $d_o = 25\ cm$.
- **Far Point** : the farthest object you can focus on. For a 'normal eye' this is $d_o = \infty$ but for near-sighted people that can be much smaller. Before I had my lenses replaced a few years ago, my far point was only about 30 cm away from my eye, and my near point was about 10 cm away.

Figure 1. Anatomy and Optical Properties of the Eye



(n = index of refraction, d_o= object distance, d_i = image distance)
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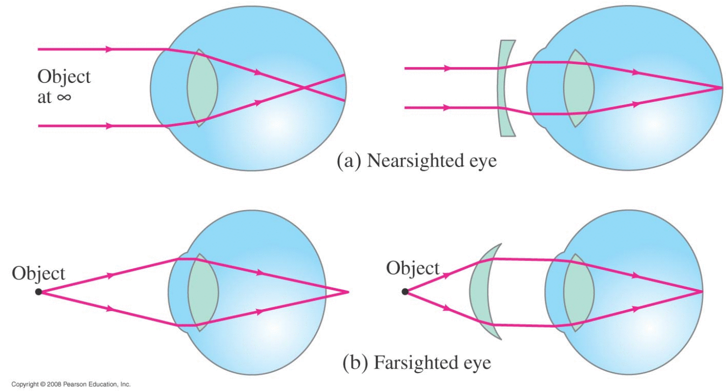
What overall focal length and power does a 'normal' eye have when focusing on something 25 cm away? With everything in millimeters: $f = \frac{d_o d_i}{d_o + d_i} = \frac{(250)(24)}{250+24} = 21.9\ mm$ or $f = 0.0219\ m$ from which $P = 1/f = 45.7\ D$.

(Compare that to looking at something very far away: the focal length doesn't have to change very much at all, and squishing or relaxing the lens is sufficient to provide this adjustment.)

Near- and Far-Sightedness

(Upper-Left Figure) : For a near-sighted person, their lens isn't able to adjust enough to correctly focus objects far away onto their retina. The 'rays' basically focus too soon, resulting in a blurred image forming on the retina.

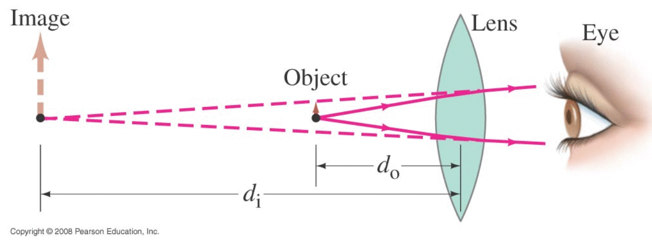
(Lower-Left Figure) : For a far-sighted person, their lens isn't able to adjust enough to correctly focus objects close up onto their retina. The 'rays' basically focus behind their retina, again resulting in a blurred image forming on the retina.



Fortunately we can fix this by adding another lens to the scenario - either with eyeglasses or contact lenses.

Farsighted Eye Correction (Contacts)

A far-sighted person can focus on things far away just fine, so what if we add a lens that converts a near object into an image that's farther away? Suppose this person's near point is at 100 cm (a very far-sighted person) and they want to read a book held just 25 cm away from their eye (where this allegedly 'normal eye' can). What lens power (contact) would be needed to achieve this?



Initially, let's assume the lens is right up against the person's eye (true for a contact lens, but obviously not true for eyeglasses).

Then we need a lens that take an object at $d_o = +25\ cm$ and creates an image that's 100 cm away but still upright(!). Looking at the figure, this image will be NOT on the outgoing ray side, so that means $d_i = -100\ cm$.

Applying our lens equation: $f = \frac{d_o d_i}{d_o + d_i} = \frac{(25)(-100)}{25+(-100)} = 33.33..\ cm$ or $0.3333..\ m$ from which $P = 1/f = +3.0\ D$. (These are commonly called 'reading glasses' and racks of them with various lens powers can be found in most pharmacies.)

Note that far-sighted folks need $f > 0$ lenses meaning they use **converging** lenses (thicker in the middle).

Magnification: $m = -d_i/d_o = -(-100\ cm)/(25\ cm) = +4.0$.

Apparent Magnification: $\theta_{obj} = (size)/(distance) = h_o/25$ and $\theta_{img} = (size)/(distance) = (4h_o)/100$ so $M_{apparent} = \theta_{img}/\theta_{obj} = (\frac{h_o}{25}) \times (\frac{100}{4h_o}) = 1.00$.

The presence of this lens (this contact lens specifically) doesn't change the **apparent** size of the object.

Farsighted Eye Correction (Glasses)

What focal length should we use if this person wants to wear glasses instead of contacts?

Suppose the glasses are located 2 cm away from the person's eye.

The book is 25 cm from their eye, so it's only 23 cm from the lens: $d_o = 23\ cm$ now.

Their eye needs to focus on an image 100 cm **from their eye**, which means that the image is now just 98 cm from the lens: $d_i = -98\ cm$.

We get a slightly different focal length lens now:

Applying our lens equation: $f = \frac{d_o d_i}{d_o + d_i} = \frac{(23)(-98)}{23+(-98)} = 30.05\ cm$ or $0.3005\ m$ from which $P = 1/f = +3.3\ D$. (The rack of 'reading glasses' in the pharmacy usually jumps from $P = 3.0\ D$ to $P = 3.5\ D$ or even $P = 4.0\ D$ so they won't be able to buy exactly the right lenses off the rack, but they'll be 'almost' right.

Magnification: $m = -d_i/d_o = -(-98\ cm)/(23\ cm) = +4.261$.

Apparent Magnification: $\theta_{obj} = (size)/(distance) = h_o/25$ and $\theta_{img} = (size)/(distance) = (4.261 h_o)/100$ so $M_{apparent} = \theta_{img}/\theta_{obj} = (\frac{4.261 h_o}{100}) \times (\frac{25}{h_o}) = 1.06525$.

When they put these glasses on, the object they couldn't focus on becomes an image they can focus on, but it **looks** a few percent slightly larger now.

Nearsighted Eye Correction (Contacts)

Nearsighted people typically want contacts so they can see distance objects clearly (road signs, movie theater screens, etc). We basically need to convert a far-away object into an image at their **far point** (the farthest distance away they can still focus on something).

Suppose a person has a far point of 17 cm and wants to see things several meters away (far enough that we can approximate $d_o = \infty$), and we're using contacts so the corrective lens will be right on top of their eye. The image created by the lens is 'not on the outgoing ray side' again, so $d_i = -17\ cm$.

What focal length contacts do they need?

$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = 0 + \frac{1}{-17\ cm}$ so $f = -17\ cm = -0.17\ m$ and $P = -5.88\ D$.

Near-sighted folks need $f < 0$ lenses meaning they use **diverging** lenses (thinner in the middle).

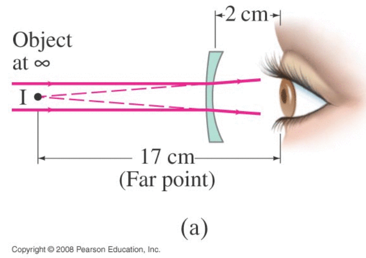
Magnification: we can't have infinities here so let's say I'm this nearsighted person and want to watch a movie screen 20 m away that is 5 m tall. The image will be at $d_i = -17\ cm$ and $m = -d_i/d_o = -(-17\ cm)/(2000\ cm) = +0.0085$ so $h_i = m h_o = (0.0085)(500\ cm) = +4.25\ cm$. The contacts are converting that large screen far away into a small screen very close to my eyes.

Apparent Magnification: $\theta_{obj} = (size)/(distance) = (5\ m)/(20\ m) = 0.25\ rad$ and $\theta_{img} = (size)/(distance) = (4.25\ cm)/(17\ cm) = 0.25\ rad$ so $M_{apparent} = \theta_{img}/\theta_{obj} = (0.25\ rad)/(0.25\ rad) = 1.00$.

When they put these contacts on, the object they couldn't focus on before becomes an image they can focus on and it appears to be the same size (takes up the same amount of their field-of-view).

Nearsighted Eye Correction (Glasses)

I used glasses instead of contacts, so let's do that case with the glasses 2 cm from my eyes.
Most of this doesn't change. I need glasses that will convert a 'far away' object into something 17 cm from my eye, so $d_o = \infty$ and $d_i = -17\ cm$ still, so we have the same $f = -17\ cm$ focal length.



What does this do to the movie screen? It's 20 m = 2000 cm from my eye, so 1998 cm from the glasses, and the image is 17 cm from my eye but just 15 cm from the glasses. $m = -d_i/d_o = -(-15\ cm)/(1998\ cm) = 0.0075075$ so $h_i = mh_o = (0.0075075..)(500\ cm) = +3.754\ cm$.

Apparent Magnification: $\theta_{obj} = (size)/(distance) = (5\ m)/(20\ m) = 0.25\ rad$ and $\theta_{img} = (size)/(distance) = (3.754\ cm)/(17\ cm) = 0.2208\ rad$ so $M_{apparent} = \theta_{img}/\theta_{obj} = (0.2208\ rad)/(0.25\ rad) = 0.883$.

When we put these glasses on, the things we look at **appear** smaller than they actually are. In addition, this is a much more obvious change than what happens with farsighted people when they put their eyeglasses on.

Surgical Replacement of the Lens of the Eye

In my case, cataracts caused me to be very near-sighted and the solution was to replace the lenses in my eyes with artificial ones. These lenses don't adjust like 'real' lenses do in the eye, so you have to pick a distance you want to be able to focus on and I decided I wanted to be able to look at my gaming monitors about 80 cm from my eyes without needing glasses. That makes me slightly far-sighted now, so what reading glasses do I need to buy to be able to read a book 25 cm from my eyes?

We need a lens that will take an object 25 cm from the eye and create an image 80 cm from the eye, and here the lens will be 2 cm from the eye so we have $d_o = 23\ cm$ and $d_i = -78\ cm$.

$f = \frac{d_o d_i}{d_o + d_i} = \frac{(-78\ cm)(+23\ cm)}{(+23\ cm)+(-78\ cm)} = +32.6\ cm$ so $f = +0.326\ m$ and $P = 1/f = +3.06\ D$.

A fairly recent option for lens replacement is called a **multifocal intraocular lens** which is kind of like a Fresnel lens except that each 'ring' has a different focal length! One ring lets you see things far away, another ring works for things a bit closer, and so on. The net results is that for a given object distance d_o, most of the light coming from a point on that object doesn't focus correctly on the retina but **some of it does** and somehow our brains are able to pick out just the part that's in focus. Apparently they don't work for everyone though and I didn't feel like taking a chance so stuck with the traditional fixed-focal-length replacement lenses.