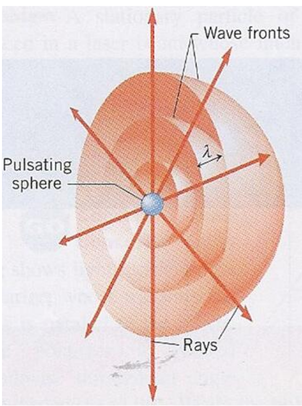
PH2233 Fox : Lecture 16

Chapter 34 : The Wave Nature of Light; Interference and Polarization

As already noted, EM waves like light can be modelled two ways. The previous chapters focused on the 'particle' model where the energy is carried by photons travelling in straight lines (as long as the wave speed remains the same, otherwise refraction can occur). This approach (generally called **geometric optics**) was successful explaining how mirrors and lenses work.

Light (and other EM waves) can also display very wave-like behavior too, and the next two chapters will focus on those wave-like aspects, with the wave fronts (points along the wave at the same phase) being perpendicular to the 'rays' (the photon paths).



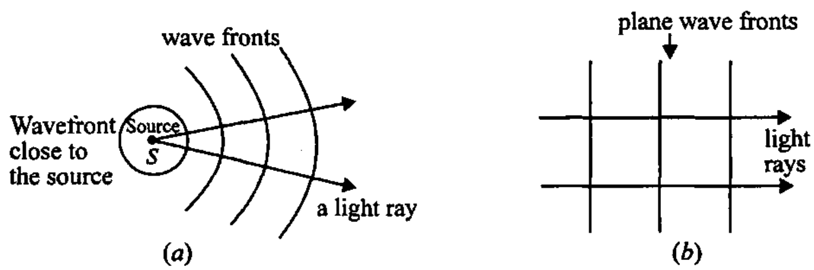
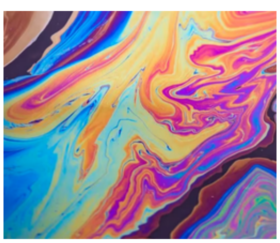


Fig. 20.1

The wave approach is needed to explain several behaviors of light and other EM waves:

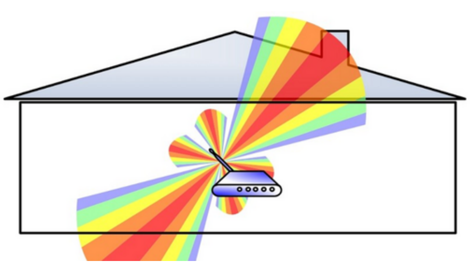


Oil Slick

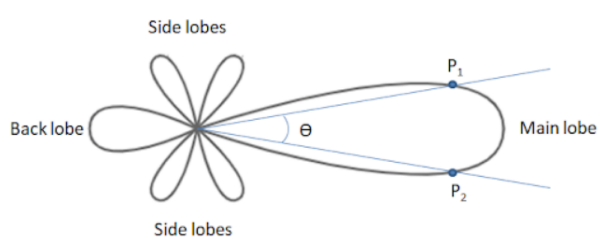
Soap Bubble

Window Film

The wave nature of EM waves is also the foundation of **antenna theory**, designing collections of sources that will 'focus' a signal in a particular direction (or multiple directions, like a multi-antenna wireless router).



Wireless Antenna

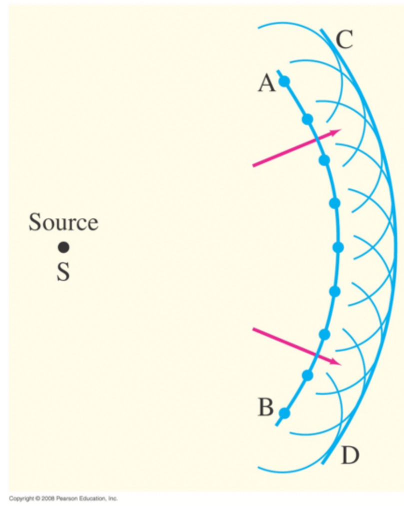


Wireless Antenna

(34-1) : Huygens' Principle and Diffraction

Back in the mid 1600's, Isaac Newton focused on the 'ray' approach (geometric optics) but in parallel Dutch scientist Christian Huygens focused on using a wave-based approach.

The big idea here is to treat each point on the wave front as a point-source radiating outward, allowed us to predict where the wave front will be some Δt in the future. Huygens did this geometrically. If the wave speed at a certain point in space is v, then at a time Δt later, each point-source will spread out a distance $v\Delta t$. By drawing lots of these spreading point-sources, a 'nexus' forms showing where the overall wavefront will be Δt in the future. (Typically Δt is chosen to be one period of the wave, so given the position of a wave front at some time, this geometric construction shows where that wave front will be one period (one wavelength) in the future.)

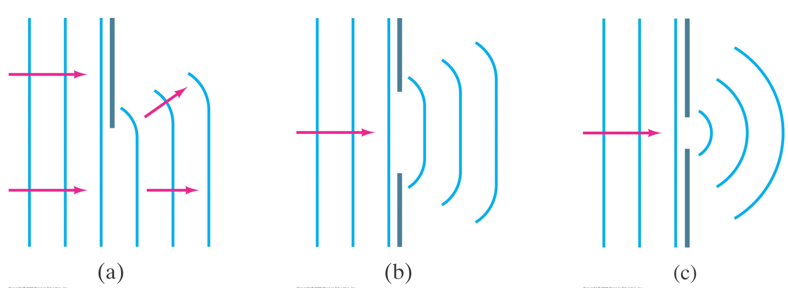


It can also be done mathematically where every point in a snapshot of the 'wave field' is treated as a point source (with an amplitude equal to whatever its value is at that point in the field), expanding outward with whatever the wave speed is at that point, and then integrating over all those to yield the wave field at some time in the future. We did that for a few simple cases in a graduate optics class and I remember it being pretty gruesome mathematically, involving complex variables, so we won't go down that path here. We don't need to either since Huygens' simple geometric idea helps us understand quite a few wave phenomena.

One of the wave features Huygens' principle helps explain is **diffraction**. In the figures below, we see a simple plane wave propagating to the right and encountering a barrier with a hole in it. In the left figure, only half the wave is blocked by the barrier and as the wave passes through, the Huygens wavelets yield nice strong wave fronts on the other side of the opening, but also weaker wave fronts 'diffracting around' the edge of the barrier.

In the middle figure, we have a barrier with a fairly large hole in it (a hole that's larger than the λ of the wave).

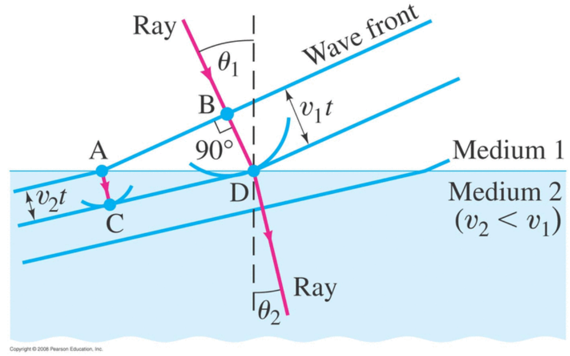
In the figure on the right, we have a very small hole (smaller than λ) where the wave field on the other side of the barrier looks more like what we'd get from a point source.

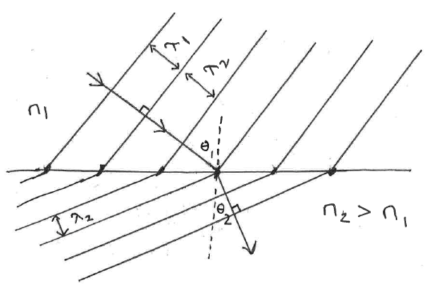


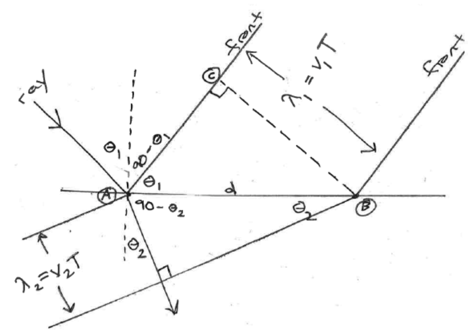
(We'll focus much more on this diffraction effect in the next chapter.)

(34-2) : Huygen's principle and the law of refraction

We already used Huygens' ideas to derive Snell's Law of refraction back in chapter 32 (see **ch32-lecture13.pdf**) and I won't repeat that derivation here.
Essentially each wave front is advanced one period in time by drawing the little Huygens wavelets, each with a radius of vT but since v is different in the two media, the wave fronts in the lower-speed medium are closer together.







This process led directly to the generic form of Snell's Law:

$$\boxed{\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}}$$

When EM waves are involved (light, radio/tv, infrared, etc) we define the **index of refraction** as $n = c/v$ so $v = c/n$ and making that substitution provides the version more often used with EM waves:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

Important Concept : waves don't disappear or pile up endlessly anywhere. Waves leave one medium at a certain rate (frequency) and enter the other medium at the same rate (frequency). **The frequency doesn't change.**

Impact on wavelength

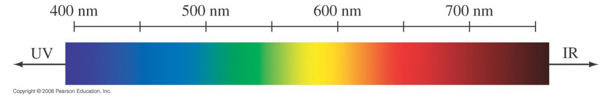
$v = \lambda/T = \lambda f$, so if the frequency doesn't change but the wave speed does change, then it's the wavelength that ends up changing as the wave moves into a different medium.

$\lambda_2 = v_2 T \rightarrow \lambda_2 = cT/n_2$
$\lambda_1 = v_1 T \rightarrow \lambda_1 = cT/n_1$

Dividing these two equations:

$\lambda_2/\lambda_1 = v_2/v_1 = n_1/n_2$

Example : the Sun emits photons spanning the visible spectrum and beyond. We see something as 'blue' because the material preferentially reflects that light and/or absorbs other colors.



What wavelength would this 'blue' light have underwater? In air, let's suppose this 'blue' has $\lambda_1 = 480\ nm$.

I'll use subscript 1 to represent parameters while the beam is in air, and subscript 2 to represent when under the water, so here $n_1 = 1.000$ and $n_2 = 1.33 \approx 4/3$.

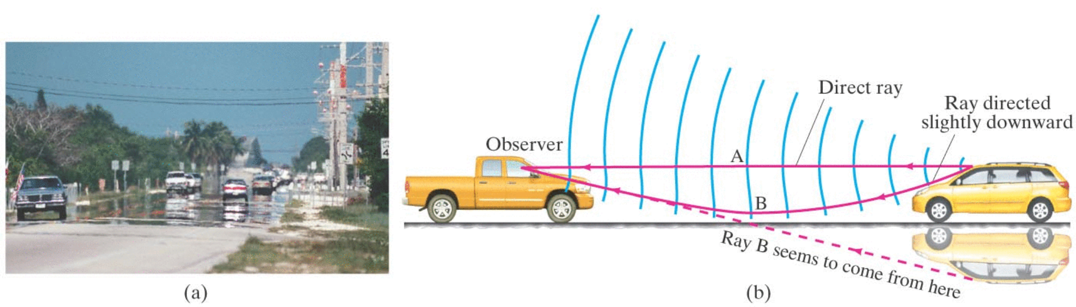
Visible Spectrum	
ultraviolet	$< 380\ nm$
violet	$380 - 450\ nm$
blue	$450 - 495\ nm$
green	$495 - 570\ nm$
yellow	$570 - 590\ nm$
orange	$590 - 620\ nm$
red	$620 - 750\ nm$
infrared	$> 750\ nm$

What wavelength will the light have underwater? $\lambda_2 = (\frac{n_1}{n_2})\lambda_1 = (\frac{1}{1.33..})(480\ nm) = (0.75)(650) = 360\ nm$ (which is NOT VISIBLE).

How about a different color like red, with a wavelength of about 650 nm?

$\lambda_2 = (\frac{n_1}{n_2})\lambda_1 = \frac{1}{1.333...} \times (650\ nm) = 488\ nm$, which is BLUE.

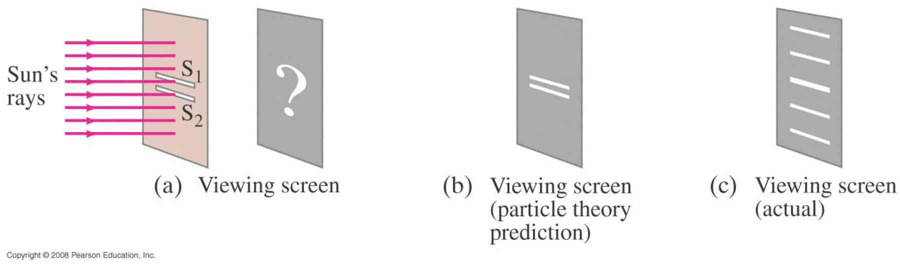
Highway Mirages : Huygens' wavelets help explain the phenomenon seen in the figure below. This happens much more frequently in the desert areas of the southwest but I've seen it here too on very hot days. Basically the air near the road surface is so hot that its density is lower, resulting in a slightly lower n right near the road than the n a bit higher up. The wave fronts travel farther $\lambda = vT$ so we 'see' rays taking two different paths from the object to our eye.



(a) (b)

(34-3) : Interference - Young's double-slit experiment

Suppose we have an opaque sheet with two thin parallel 'slits' cut into the sheet. If we send waves (with a single frequency or wavelength) towards this sheet, what will happen?

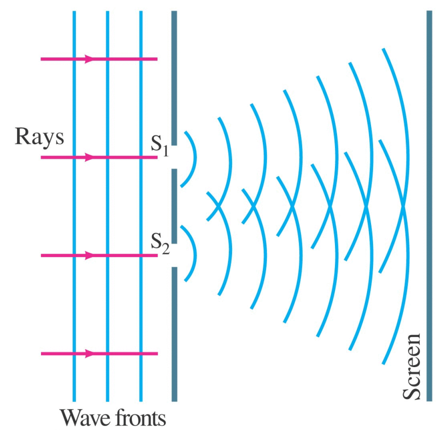


The **ray model** of light would say that we should see two bright lines on the other side of the sheet. Photons either go through the little openings, or they run into the opaque part of the screen.

In fact, what we see is an image like the one shown in the bottom figure.

What's going on?

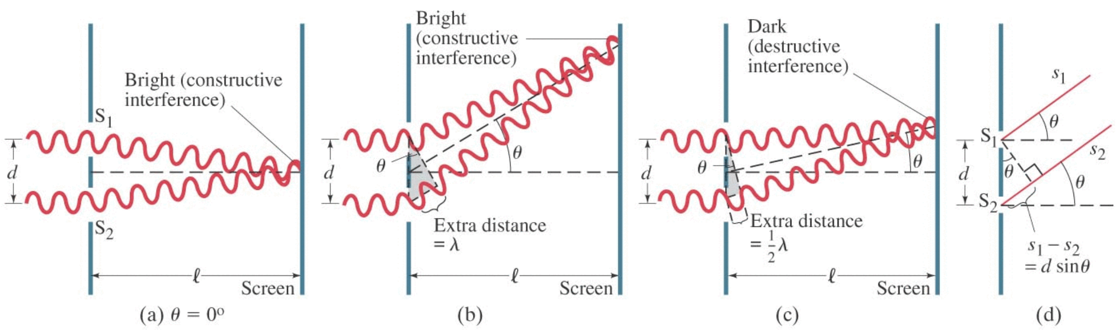
Well, light is also an electromagnetic **wave**. If sound waves or water waves ran into a barrier with two small openings in it, the wave shape passes through each opening and spreads out like we see here.
This geometry looks like what we did back in chapter 16 when we had two speakers each emitting sound of the same frequency.



If we place a screen on the other side from the source, what signal will we record on the screen at various locations?

The first pictures below show the general situation, but normally the distance to the screen is vastly larger than the distance between the two slits. As a result, the paths from each slit to the screen are essentially parallel.

This geometry, where $l >> d$ is called the **FAR FIELD** approximation.



If we're right along the mid-line between the two slits (figure (a) above), the waves from each 'source' travel the same distance to reach that point, so they arrive 'in phase'. The two (say) sine waves combine and create a higher amplitude (i.e. higher intensity, or 'brighter') signal.

If figure (b) above, the sine waves from the lower slit had to travel exactly one wavelength farther than the waves from the upper slit. When they arrive at the screen, one sine wave is shifted by exactly one wavelength relative to the other but that means they're still 'in phase' with each other and still yield a strong signal.

The upper set of three figures here on the right shows that situation: the two sine waves are in sync with each other and yield **constructive interference**.

In figure (c) above though, one path was exactly $\frac{1}{2}\lambda$ longer than the other. When these reach the screen, one of the waves is delayed by a half-wavelength relative to the other. That situation is shown in the bottom set of three figures on the right here. The two sine waves essentially cancel each other out, yielding **destructive interference**.

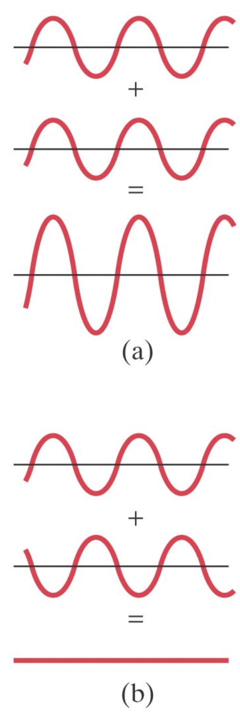
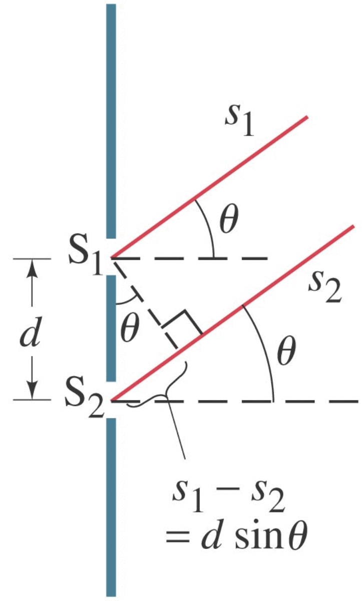


Figure (d) above is reproduced and magnified here. If the distance to the screen is much larger than the distance between the two slits, the two paths are very nearly parallel, so the 'extra path length' is seen to be $d \sin\theta$.



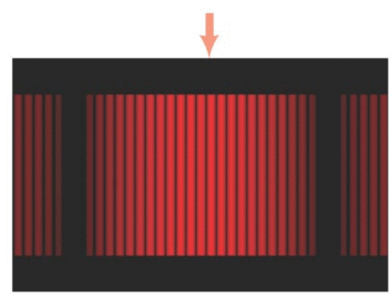
(d)

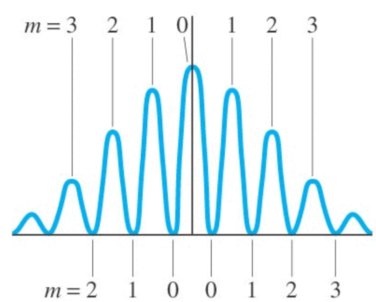
If the path difference is exactly a multiple of λ, the two waves arrive in phase and yield a strong signal (a bright spot). Mathematically we can write this as:
CONSTRUCTIVE interference

$$d \sin\theta = m\lambda \text{ for } m = 0, 1, 2, \cdots$$

If the path difference is exactly a multiple of λ PLUS AN ADDITIONAL HALF WAVELENGTH, the two waves arrive perfectly OUT of phase and cancel each other (a dark spot). Mathematically we can write this as:
DESTRUCTIVE interference

$$d \sin\theta = (m + \tfrac{1}{2})\lambda \text{ for } m = 0, 1, 2, \cdots$$


Constructive interference
(a)



Destructive interference
(b)

What about angles in between? For any other angle, the two sine waves will combine and yield a somewhat reduced amplitude. Not a double-strong signal when they're perfectly in phase, but not a zero amplitude when they arrive perfectly out of phase, so something in between (we'll look at this in more detail in the next section).

Typical Lab Design

A typical double-slit lab experiment involves two closely cut slits with the interference pattern forming on a screen some distance away (a distance much larger than the slit separation).

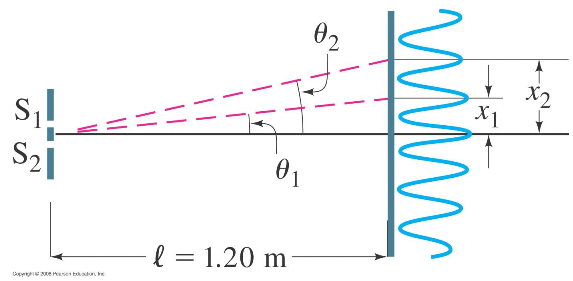
Consider a double-slit experiment where the two slits are separated by $d = 0.1\ mm$, with a screen placed 1.20 m away. We shine a laser with $\lambda = 500\ nm$ on the slits. Determine where the bright (constructive) peaks will be located?

Constructive interference occurs when $d \sin\theta = m\lambda$ for $m = 0, \pm 1, \pm 2, ...$

The first peak will be at $m = 0$ for which $\theta_0 = 0^o$.

The next peak will be at $m = 1$ for which $\sin\theta_1 = \frac{m\lambda}{d} = \frac{(1)(500\times 10^{-9}\ m)}{1\times 10^{-4}\ m} = 5 \times 10^{-3}$ from which $\theta_1 = 0.2865^o$ or $\theta_1 = 5.000.. \times 10^{-3}\ rad$.

Note that for angles this small, $\sin\theta \approx \theta$, when the angle is measured in radians. Since $\sin\theta = m\frac{\lambda}{d}$, the angles will be tiny when $\boxed{d >> \lambda}$. This is called the **SMALL ANGLE** regime.
How far are these two bright spots from each other on the screen 1.2 m away?
From the figure, $x_m = (l)\tan\theta_m$ but again these angles are so small that as long as we stick with radians, we can use $\tan\theta \approx \theta$ and $x_1 = (1.2\ m)(5 \times 10^{-3}\ rad) = 6 \times 10^{-3}\ m = 6\ mm$.



As long as $l >> d$ (the 'far field' situation) AND $d >> \lambda$ (the 'small angle' case) then:
$d \sin\theta_m = m\lambda$ becomes $\boxed{\theta_m \approx \frac{m\lambda}{d}}$.

The bright spots on the screen will be located at $x_m = l \tan\theta_m$ or $x_m \approx (l)\theta_m$ or finally $\boxed{x_m \approx m\frac{l\lambda}{d}}$

The distance from each bright spot to the next is $\boxed{\Delta x = \frac{l\lambda}{d}}$.

Rearranging that last equation: $\lambda = \frac{d\Delta x}{l}$ which means for a given slit spacing (d) and distance away (l), the separation distance between the bright lines (Δx) directly maps into the wavelength involved.

This gives us a tool to determine the wavelength of a light source (or potentially any EM wave): a **spectrometer**. As long as we know l and d and can measure how far apart the bright (high intensity) spots are, we can determine the wavelength of the source. Note we've made no other assumptions about this wave, other than $l >> d >> \lambda$, so this process can sometimes be applied to other waves too, including sound, water waves, and so on.

(Most spectrometers use either prisms or 'gratings' which we'll see in the next chapter and we'll see there why these are better options than trying to use the double-slit geometry as a spectrometer.)

Example: Spectrum

Suppose we direct light from the Sun through a double-slit arrangement with $d = 0.50\ mm$ with the screen located at $l = 2.5\ m$ away from the slits. What will we see? The Sun includes all visible wavelengths (and beyond), sometimes called 'white' light.

The peaks are at $x_m \approx m\frac{l\lambda}{d}$ so the first peak ($m = 0$) will be at $x_0 = 0.0\ m$ regardless of the incoming wavelength of the light.

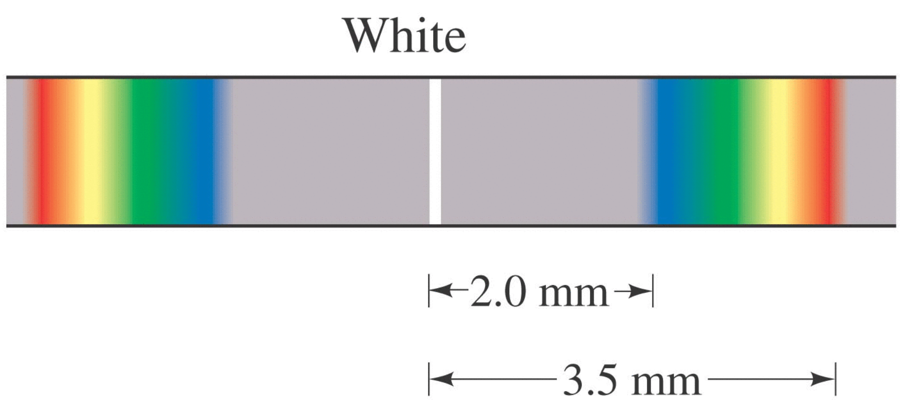
The next 'peak' at $m = 1$ will occur at $x_1 = (1)\frac{l\lambda}{d}$ which we could write as $x_1 = \frac{l}{d}\lambda$. Each different wavelength λ in the light source will cause a bright line at that physical location x on the screen.

Here then: $x_1 = (1)\frac{(2.5\ m)(5\times10^{-4}\ m)}{\lambda}$.

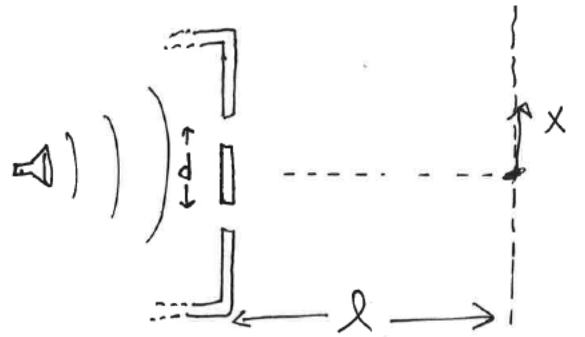
Looking at the range of visible wavelengths, violet light ($\lambda = 400\ nm$) has its peak at $x_1 = 2\ mm$ but $700\ nm$ red light has its peak at $x_1 = 3.5\ mm$.

The 'white' light from the Sun has been spread out so we can see its entire (visible) spectrum.

A sodium vapor lamp or LED light has only a few wavelengths instead of this continuous spectrum, so we'd see lines with just those colors instead of a complete spectrum.



Example: Single Speaker and Two Open Windows



Consider a 500 Hz tone in a room with two (narrow) open windows separated by 1 m. What pattern will be have? Assume FAR FIELD but NOT small angle. (Why can't we use the 'small angle' approximation here?)
$v_{sound} = 343\ m/s$ so 500 hz corresponds to $\lambda = v/f = 344/500 = 0.688\ m$.
(We need $d >> \lambda$ to be able to use the 'small angle' approximation but here $d = 1\ m$ and $\lambda = 0.688\ m$ so the $d >> \lambda$ test fails.)

Constructive interference (a loud sound) will occur where:

$d \sin\theta_m = m\lambda$ so $\sin\theta_m = m\frac{\lambda}{d} = (m)\frac{0.688\ m}{1\ m} = 0.688m$, for $m = 0, \pm1, \pm2, etc.$

At $m = 0$ we have $\sin\theta = 0$ so $\theta = 0$ is one direction where we get a strong signal.

At $m = \pm1$ we have $\sin\theta = \pm0.688$ or $\theta = \pm43.4^o$

For any higher m values, the RHS exceeds 1, so we have no solutions.

Destructive interference ('no' sound) will occur where:

$d \sin\theta_m = (m + \frac{1}{2})\lambda$ so $\sin\theta_m = (m + 0.5)(0.688)$, for $m = 0, \pm1, \pm2, etc.$

For $m = 0$, we get $\theta = +20.1^o$ and for $m = -1$ we get $\theta = -20.1^o$.

Any other m values yield a RHS larger than 1 and we can't do the inverse sine any more, so we have no other solutions for the 'dead spots'.

Example: wireless router

Consider a wireless router operating at 2.4 GHz. This particular router has two antennas separated by 6.25 cm.

The wireless signals are radio (EM) waves travelling at the speed of light, so what will the wavelength of these waves be? $v = \lambda/T = \lambda f$ so $\lambda = v/f = c/f = ((3 \times 10^8\ m/s)/2.4 \times 10^9 s^{-2}) =$ 0.125 m or 12.5 cm.

The wavelength is comparable to the separation between the two antenna, so we can't use the 'small angle' approximation here, but we're probably at least several meters away from this router, so the 'far field' approximation should be ok.

Constructive interference occurs where $d \sin\theta = m\lambda$ for $m = 0, \pm1, \pm2...$ so: $6.25 \sin\theta = (m)(12.5)$ or $\sin\theta = (2)(m)$. The ONLY solution will be with $m = 0$ in which case $\theta = 0$ (and $\theta = 180^o$) so we'll have a nice strong single along a line perpendicular to the antennae.

Destructive interference occurs where: $d \sin\theta = (m + \frac{1}{2})\lambda$ for $m = 0, \pm1, \pm2...$ so: $6.25 \sin\theta = (m + 0.5)(12.5)$ or $\sin\theta = (2)(m + 0.5)$.

$m = 0$ yields $\sin\theta = (2)(0 + 0.5) = 1$ which occurs at $\theta = 90^o$

$m = -1$ yields $\sin\theta = (2)(-1 + 0.5) = -1$ which occurs at $\theta = -90^o$.

There are no other solutions, so this pair of antennae has a strong single in front and behind, but basically zero signal off to the sides.

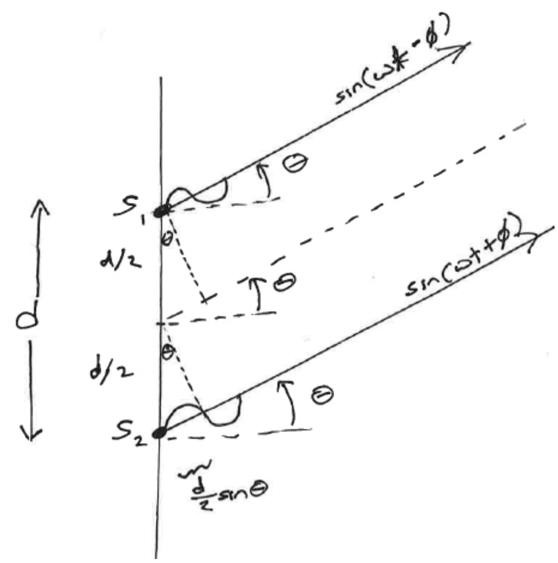
What about other angles?

(34-4) : Intensity in the double-slit interference pattern

Consider two sources, each emitting the same frequency signal in phase with one another, and suppose we're 'far away' from these sources so we can use the far-field approximation.

We're interested in how the intensity behaves as a function of angle θ as shown in the figure.

Each source is putting out a signal proportional to $\sin\omega t$ but at the angle shown, the signal from source 1 has a shorter distance to travel, so arrives earlier than the signal from source 2. The signal from source 2 had to travel an additional $d \sin\theta$ (meters), so it's wave shape is shifted by that much relative to the signal from source 1. It's easier to combine these two sine waves if we draw an imaginary point at their midpoint and look at how each is shifted relative to that point. In that case, source 1 travels a distance that is shorter by $\frac{d}{2}\sin\theta$, source 2 travels a distance that is longer by $\frac{d}{2}\sin\theta$.



What **phase shift** does that physical distance represent? A physical shift of one full wavelength λ represents a phase shift of 2π radians so a distance shift of $\frac{d}{2}\sin\theta$ represents a phase shift of: $\phi = (\frac{d}{2}\sin\theta) \times \frac{2\pi}{\lambda}$.

We can write the combined signal now as: $E(t) = \sin(\omega t + \phi) + \sin(\omega t - \phi)$ and now we can use a trig identify to simplify this.

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ which turns our combined signal into: $E(t) = 2\sin(\omega t)\cos\phi$.

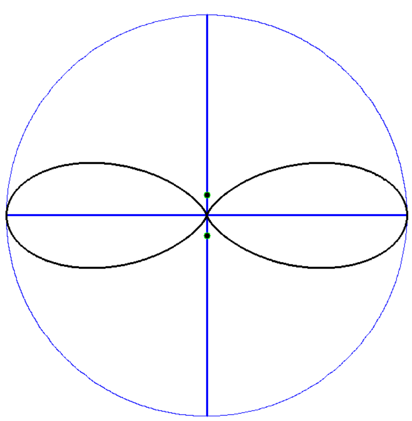
This gives us the **amplitude** of the combined signal, but as with any wave the intensity (brightness in the case of light, loudness in the case of sound, etc) is the time-average of the amplitude squared, ultimately leading to the intensity being proportional to $\cos^2\phi$.

We can write this as: $I = I_o \cos^2(\frac{\pi d \sin\theta}{\lambda})$ and now we can draw a (polar) graph showing the intensity of our wireless router at any angle. If you want to know the intensity at $\theta = 30^o$ for example, draw a line starting at the origin radiating out making an angle of $+30^o$ relative to the $+X$ axis and where it intersects this graph, that's the intensity at that angle.

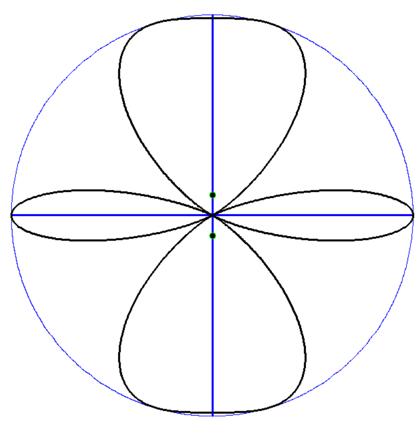
The left figure shows our 'antenna pattern' for the pictured router operating at $f = 2.4\ GHz$, showing a strong signal at $\theta = 0$ and $\theta = 180^o$ and no signal at $\theta = \pm 90^o$.

This router also uses the $f = 5.0\ GHz$ band and the figure on the right shows the pattern at that frequency. Now we have narrower peaks at $\theta = 0$ and $\theta = 180^o$ but very broad **strong** signals at $\theta = \pm 90^o$.

Both frequencies have low intensity right between the four axis directions though ($\theta = \pm 45^o$ and $\theta = \pm 135^o$), so people trying to use this router in those directions would have difficulty getting a signal.



$f = 2.4\ GHz$ with $d = 6.25\ cm$



$f = 5.0\ GHz$ with $d = 6.25\ cm$

(At the end of the lecture I talked a little about adding a delay to an antenna to effectively shift the angles where the peaks occur, but that's getting a bit deeper into antenna theory than we can go in this class. Lots of neat stuff in this field though, so stop by if you want more information on that topic.)