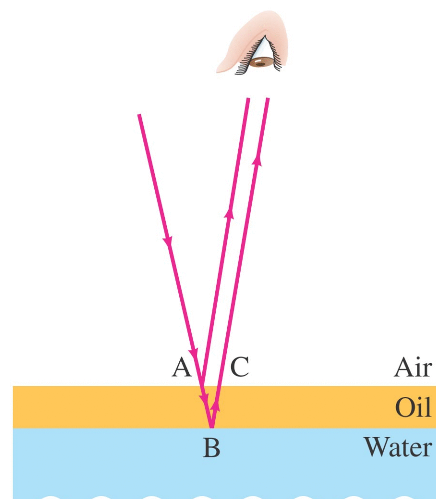
PH2233 Fox : Lecture 17
Chapter 34 : The Wave Nature of Light; Interference and Polarization

(34-5) : Interference in thin films

Glare due to bright lights can be reduced by adding a thin film coating to the glass covering artwork, a computer monitor, phone/tablet screens, etc. Thin films of oil on water can produce shiny swirls of various colors. Soap bubbles (which are very thin) will show swirls of color. All these are examples of interference.

Consider the scenario of light bouncing off a calm water surface that has a very thin layer of oil floating on it.



The oil is semi-transparent, so some of the light reflects off the top of the oil, and the rest passes into the oil (refracting slightly as it does). When that light then encounters the bottom of the oil layer, again some will be reflected back up and the remainder will pass into the water.

In the figure, we'll focus on just the two paths the light will take that lead towards our eye. (Actually, every time the ray encounters an interface between two different materials it will split into reflected and transmitted parts, but the first two rays will generally represent much more intensity than the others, so we usually just focus on the first two.)

The effect here is that two (or more) copies of the original signal are making it into our eye: one that reflected directly off the top of the oil film, and one (which took a slightly longer path) reflecting off the bottom of the oil film. Since the second signal travelled farther (basically twice the thickness of the oil), it's slightly **delayed** relative to the first path. Focusing on a particular wavelength, our eyes are simultaneously receiving a sine wave and a delayed sine wave. **That's a perfect recipe for interference.** We've already seen a couple of forms this interference can take. Back in chapter 15 we had mechanical waves travelling along wires, bouncing back and forth creating standing waves (stringed musical instruments, for example). In chapter 16 (and again in this chapter), we saw this when we listen to two speakers: at certain locations, a given frequency of sound would be extra loud or soft. The same process occurs with light (and any wave, really), and we can exploit this to reduce glare off glass screens or keep some of the Sun's heat from getting into a room.

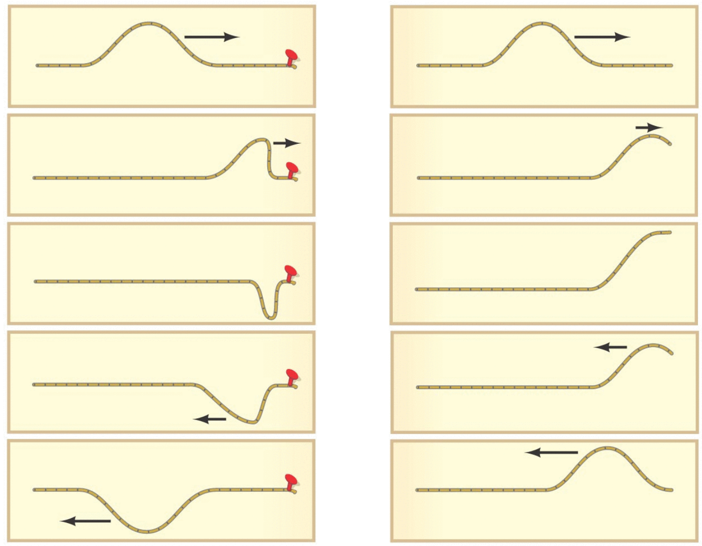
If we adjust the thickness just right, we could cause the two rays to be perfectly out of phase and thus cancel one another. Or we could cause the delay to be such that the rays are in phase (identical sine waves, one just delayed by an exact integer number of wavelengths), in which case they'll enhance each other.

You might think is should be simple. For nearly vertical rays, the extra path length is $2t$ (twice the film thickness), so constructive interference <u>should</u> occur if $2t = m\lambda$ and destructive if $2t = (m+\frac{1}{2})\lambda$, but unfortunately there are some annoying details that mess this up.

Thin film fine print

When we're dealing with **electromagnetic waves** (and sound waves for that matter), there are a couple of extra bits we need to deal with.

Detail 1 : When the 'sine wave' that represents the wave **reflects** off a surface, it **MAY change it's sign**. We saw this with mechanical waves too (figure from chapter 15, section 7). In the case of EM waves, this sign flip occurs when the wave **reflects** off a material that has a **higher** index of refraction than the one it was travelling in already. Note that a higher n means a slower wave speed $v = c/n$, so the more general way of describing this process is that waves **reflecting** off a lower wave-speed medium will have their sign flipped. (Since a negative sine wave is really just a positive sine wave that's been shifted by half a wavelength, this is also referred to as the wave receiving a 180^o phase shift.



(a) (b)

Note that this effect ONLY applies to the reflected signal. The part of the wave that passes INTO the new material retains whatever sign it had already.

- A ray REFLECTED off a HIGHER n has its SIGN FLIPPED
- A ray TRANSMITTED into a different n shows no change in sign

Detail 2 : The part of the ray that passes into the film does change in another way though. The **frequency** of the light remains constant, but $v = \lambda/T = \lambda f$ and for EM waves, we have $v = c/n$ so $c/n = \lambda f$ or ultimately $n\lambda = cf = constant$. If the ray moves into a material with a higher index of refraction, its wavelength will decrease. If it moves into a material with a lower index of refraction, its wavelength must increase.

Destructive vs constructive interference involves counting the number of waves (or half waves) that fit into the extra path length, so this affects our final equation(s) also.

In this case of our oil film on water, we'd have $n_{air}\lambda_{air} = n_{film}\lambda_{film}$ or (since $n_{air} = 1$):

$$\boxed{\lambda_{film} = \lambda_{air}/n_{film}}.$$

Putting it all together

Applying these steps can be tricky. Normally, we'd say constructive interference would occur if the extra path length is an integer number of wavelengths (using the wavelength in the film), **but that's not the whole story now.** Either of the reflected waves (and maybe both of them) **might** have picked up an extra 180^o phase shift, and each half-wavelength shift like that switches the situation between constructive to destructive.

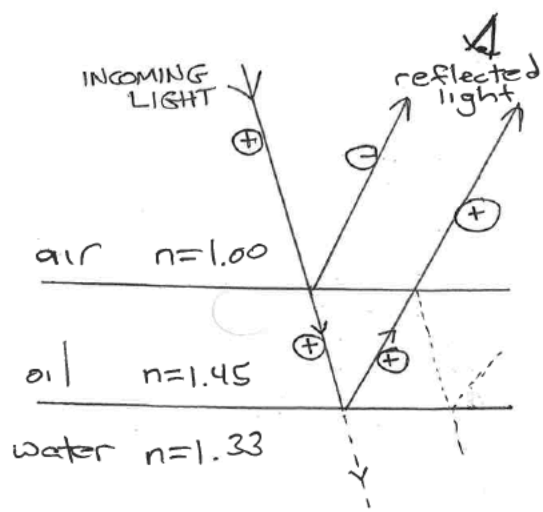
In effect, we'll need to **derive** the proper equation to use based on the situation.

Examples of going through these steps can be found in the **examples34.pdf** file on Canvas, specifically examples 30, 31, and 34. Here, let's look at the thin film of oil on water in gory detail.

Example : Thin Film of Oil on Water

Consider a very thin film of oil (with $n = 1.45$) floating on water (with $n = 1.33$). A ray from a light source (the Sun maybe) comes down through the air (with $n = 1$). Part of its energy reflects off the air-oil interface, and some continues into the oil and partially reflects off the oil-water interface.

- A ray coming in from above strikes the air-film interface. The oil has a higher index of refraction than the air, so the **reflected** ray will have its **SIGN FLIPPED**. I've denoted that by putting a ⊕ sign on the incoming ray, and a ⊖ sign on the reflected ray.
- Some of the energy continues into the oil (the 'transmitted' part of the ray) and no sign change occurs for transmission, so the incoming ⊕ sine wave remains a ⊕ sine wave as it passed into the oil.



- That transmitted ray now reflects off the oil-water interface. This time, the ray is reflecting off a material with a LOWER index of refraction, so NO SIGN CHANGE occurs. The ray reflected off the water retains a ⊕ sign.
- That reflected ray continues through the oil and now passes out into the air. Transmitted rays don't change signs, so when it passes back into the air, it's still a ⊕ sine wave.

What we **see** then is the ray reflected from the air-oil interface (which has picked up a sign flip along the way, turning it into a ⊖ sine wave) and the ray that passed through the oil and reflected off the oil-water interface and that ray never switched signs, so it's still a ⊕ sine wave.

Deriving the Equation(s) for this situation

The ray that travelled through the oil ended up travelling an extra distance though of $2t$, where t is the thickness of the oil.

If that extra distance happens to be exactly an integer multiple of the wavelength of the light, then the two reflected rays heading towards out eye will be perfectly out of phase with one another (one is a $+sine$ wave and the other is a $-sine$ wave here) and will tend to cancel each other out.

The wave has a different wavelength while it's in the oil film though so our equations become:

- If $2t = m\lambda_{film}$, the two rays remain out of phase, leading to DESTRUCTIVE interference.
- If $2t = (m + \frac{1}{2})\lambda_{film}$, that extra half-wavelength puts the two rays back in sync, leading to CONSTRUCTIVE interference.
- In each case, $\lambda_{film} = \frac{1}{n_{film}}\lambda_{air}$, and $m = 0, 1, 2, 3, ...$

What happens depends on the thickness of the film, the wavelength of the light, and the various indices of refraction involved. You have to consider the possible sign flips affect these two rays and also consider what you need to happen in a given scenario to figure out which equation does what!

$\boxed{\text{Suppose the oil film thickness is } t = 1\ micron = 1\mu m = 1 \times 10^{-6}\ m = 1000\ nm.}$

For what wavelengths of visible light will CONSTRUCTIVE interference occur? These colors will have their intensity increased so will be brighter: the oil film will look like this color in effect. How about DESTRUCTIVE? These wavelengths (colors) will be mostly cancelled out, so they're colors we won't see reflected from the oil.

Constructive: Based on the analysis we did above, constructive interference in this case will occur when $2t = (m + \frac{1}{2})\lambda_{film}$ so: $(2)(1000\ nm) = (m + 0.5)(\lambda_{air})/1.45$ or $\lambda_{air} = (1450\ nm)/(m + 0.5)$. Trying various values for m, $m = 0$ yields 2900 nm which is not visible, $m = 1$ yields 967 nm (again, not visible). $m = 2$ yields 580 nm which is yellow. $m = 3$ yields 414 nm which is a dark violet color. The rest yield colors we can't see anyway. So the reflected light will have extra yellow and violet colors in it.

Destructive: Again, we found above the destructive interference in the light reflected off the oil towards out eyes will occur when $2t = m\lambda_{film}$ so: $(2)(1000\ nm) = (m)(\lambda_{air})/1.45$ or $\lambda_{air} = (1450\ nm)/(m)$. Trying various values for m, $m = 1$ yields 1450 nm which isn't visible. $m = 2$ yields 725 nm which is a deep red color. $m = 3$ yields 483 nm which is blue. $m = 3$ yields 362.5 nm which is not visible. So basically the blue and deep red colors will be missing when we look at the oil.

$\boxed{\text{Suppose the oil film looks very GREEN. How thick is it?}}$

If it looks green (about $\lambda = 540\ nm$) that means constructive interference must be happening at that wavelength of incoming light.

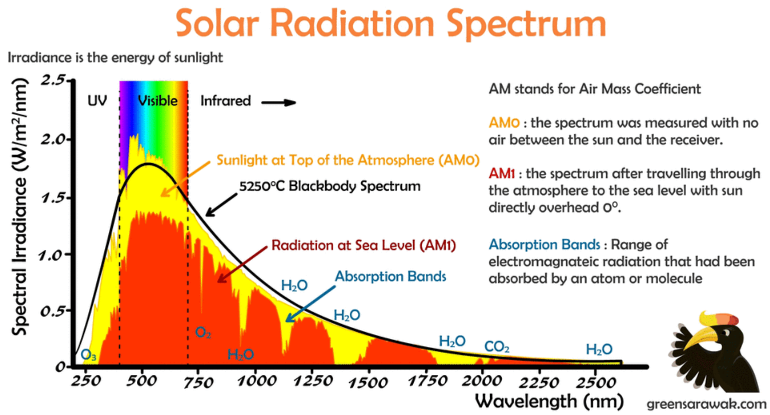
$(2)(t) = (m + 0.5)(540\ nm)/1.45$ or $t = (m + 0.5)(186.21\ nm)$.

$m = 0$ yields $t = 98.1\ nm$, $m = 1$ yields $t = 279\ nm$, $m = 2$ yields $t = 465.5\ nm$ and so on.

(The actual film will have varying thicknesses, leading to different colors being enhanced or dimished at different points on the oil slick.)

Example : Window tint to reduce heat getting into the house

The Sun puts out energy that extends well beyond the visible spectrum, and includes infrared energy (i.e. **heat**) that we would like to keep from getting into the house. The figure at the right is a good example I found on the internet that also highlights the narrow window represented by the visible part of the spectrum.



Several versions of the 'solar spectrum' are shown: what arrives at Earth (at the top of the atmosphere), what arrives down at the surface where we are, and there's a black line that represents the 'blackbody' spectrum you may have heard about if you took thermodynamics. (Basically if you take an ideal material and heat it to a given temperature, this is theoretically the spectrum it would emit. The black line is a best-fit to the actual data, indicating that the Sun acts like an object heated up to 5250^oC.)

GOAL : Suppose we want to completely eliminate energy at $\lambda = 1000\ nm$ from getting into our house.

NOTE: These films are often installed on the inside of the window, but let's assume we put this film on the **outside** of the window so our 'thin film' picture would look like the figure below. Note that EVERY TIME a ray encounters an interface, some of it will be reflected (picking up a possible sign change) and some will pass into the next material. Technically this happens many times, but we'll focus on the first two (we haven't proved it, but those will account for most of the energy).

ADD (to addendum maybe?) to this topic:
Show that putting the film on the INSIDE or OUTSIDE doesn't change the math. As long as n_{film} is between n_{air} and n_{glass} then in each case the rays reflected back to the outside are in phase with one another (either both + or both -) and we want to maintain that for this wavelength to be reflected back and NOT allowed into the house, so the equation generated is ultimately $2t = m\lambda_{film}$ in each case.

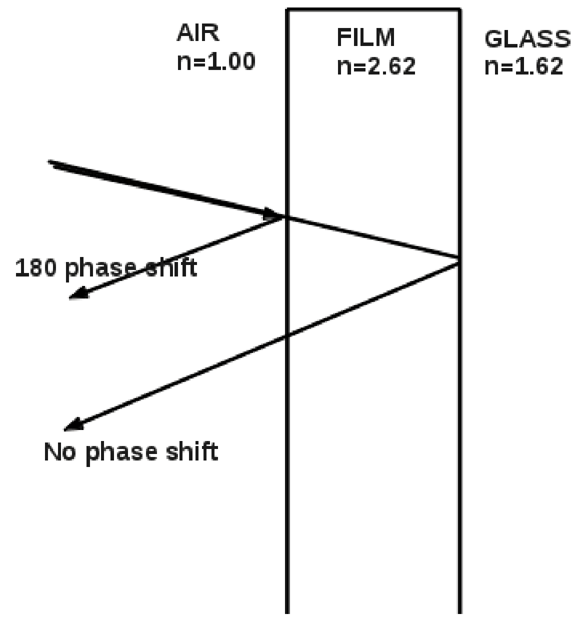
$\boxed{\text{Step 1 : figure}}$

Suppose our geometry will be air (with $n = 1$), film (with $n = 2.62$), and glass (with $n = 1.62$). The outside (i.e. the Sun) will be to the left, and the inside of the house will be over on the right of the figure.

The light hits the film. Part of that energy will be reflected back outside right away, and since the ray is going from a lower to a higher index of refraction material (from a higher wave speed, reflecting off a material with a lower wave speed), that ray will have it's sign flipped. If we think of the incoming ray as a + sine wave, the reflected ray will be a − sine wave.

The part of the light that passes into the film doesn't get shifted: it retains the same 'polarity' (sign) the incoming ray had. That ray now partly reflects off the film-glass interface but this time since it's reflecting off a material with a lower n than it was travelling in (the film), it will NOT pick up any sign flip. The + sine wave that hits that second interface remains a + sine wave when it reflects back to the left.

When that reflected ray passes through the air-film interface and heads back into the air, it retains that + sign since this sign-change bit only applies to reflections.



$\boxed{\text{Step 2 : Goal}}$ **What do we want to happen here?** We want waves with $\lambda = 1000\ nm$ to NOT get into the room: instead, we want the two rays in the figure to undergo **constructive** interference, carrying most of this 'heat' wavelength away (back outside) with them.

$\boxed{\text{Step 3 : Design Equation}}$

Note here that the ray taking the shorter path has picked up a sign change, but the ray reflected off the bottom of the film (the film-glass interface) didn't. That means the two rays are, in effect, set up to cancel one another. That's NOT what we want to happen though. We need the ray that took the longer path to spend an extra 1/2 wavelength in the film so that it comes out IN PHASE with the first ray. Remember, we want these two rays to show constructive interference so the heat will be reflected back outside.

Finally then we have our equation: $2t = (m + \frac{1}{2})\lambda_{film}$, where $\lambda_{film} = \lambda_{air}/n_{film}$. We had to add that extra 1/2 because the rays are set up to cancel, but we want them to NOT cancel and we can make that happen by having the extra path length include that extra 1/2 wavelength.

For the numbers we have here then: $2t = (m + \frac{1}{2})\frac{1000\ nm}{2.62}$ or $t = (m + \frac{1}{2})(190.084\ nm)$.

There are many thicknesses that will work. The thinnest will be when $m = 0$, resulting in $t = 95.42\ nm$. This is pretty thin and may be difficult to manufacture and handle, so higher m values might be used in practice. $m = 1$ would yield a thickness of $286.3\ nm$, $m = 2$ would yield $477.1\ nm$, and so on.

We'll go ahead and use $t = 95.42\ nm$ for the rest of this problem.

$\boxed{\text{Impact on Visible Light}}$ (CHECK ALL THIS : something not right)

Visible light wavelengths will also be hitting the window and being affected by the film. Are there any wavelengths that will be extra bright or dim?

First, let's ask what visible wavelengths (if any) will be strongly reflected back outside. Any such wavelengths will have most of their energy reflected back outside, meaning they won't be getting into the house: if we're inside looking out, we'll be (somewhat) missing that color.

'Strongly reflected back outside' means we're in the same situation we had during the derivation of the film thickness. We want to know what other λ's will undergo the same sort of **constructive** interference. Again, given that one of the reflected rays has picked up a sign change (180 deg phase shift), we need the ray travelling through the film to take an extra 1/2 wavelength to get back in phase with the other ray:

$2t = (m + \frac{1}{2})\lambda_{film}$, where we now know that $t = 190.84\ nm$ and $\lambda_{film} = \lambda_{air}/2.62$ so we can write this as:

$(2)(95.42\ nm) = (m + \frac{1}{2})\frac{\lambda_{air}}{2.62}$ or rearranging to solve for the wavelength:

$\lambda_{air} = \frac{(2.62)(2)(95.42\ nm)}{m+\frac{1}{2}} = \frac{500\ nm}{m+0.5}$

Trying various values for m: $m = 0$ yields $\lambda = 1000\ nm$ (the infrared heat we're trying to keep out so we definitely want that wavelength to be reflected back outside); $m = 1$ yields $\lambda = 333\ nm$ (which is just outside the visible range down in the violet end of the spectrum, so violet would be somewhat reflected and NOT make it into the house. Higher m values yield even smaller wavelengths (even more outside the range we can see).

So this film would likely cut out some violet light from entering the house.

(Since this wavelength is strongly reflected back outside, that also means that someone looking at our window (from outside) would see a slightly violet tint.

Next, let's look at the opposite case: what colors coming in from outside landing on this film would be extra bright coming into the house?

Extra bright coming into the house means that the light being reflected back outside must be undergoing destructive interference (instead of the constructive interference we had before).

Going back to our 'Goal' section, now we want those two rays in the figure to cancel each other out (meaning all the light at that wavelength would go into the house instead). Given the sign changes in the rays, they're already set up to do that so we'd maintain that by having the extra path being an integer multiple of the wavelength this time: $2t = m\lambda_{film}$ where $\lambda_{film} = \lambda_{air}/n_{film}$.

Using the same thickness and other parameters as before, the wavelengths involved would be:

$(2)(95.42\ nm) = (m)\frac{\lambda_{air}}{2.62}$ or rearranging to solve for λ:

$\lambda_{air} = \frac{(2.62)(2)(95.42\ nm)}{m} = \frac{500\ nm}{m}$

$m = 1$ yields $500\ nm$, which is 'cyan' (a blue-ish-green). $m = 2$ yields $250\ nm$ (not visible), and any higher m values would be even less visible.

Looks like this time just a single wavelength (a greenish-blue color) will have the effect we're looking for. The film won't reflect back outside much of this color at all: more of that color will get into the house than any other color, so from inside we might think everything outside has a sort of blue-green tint to it.