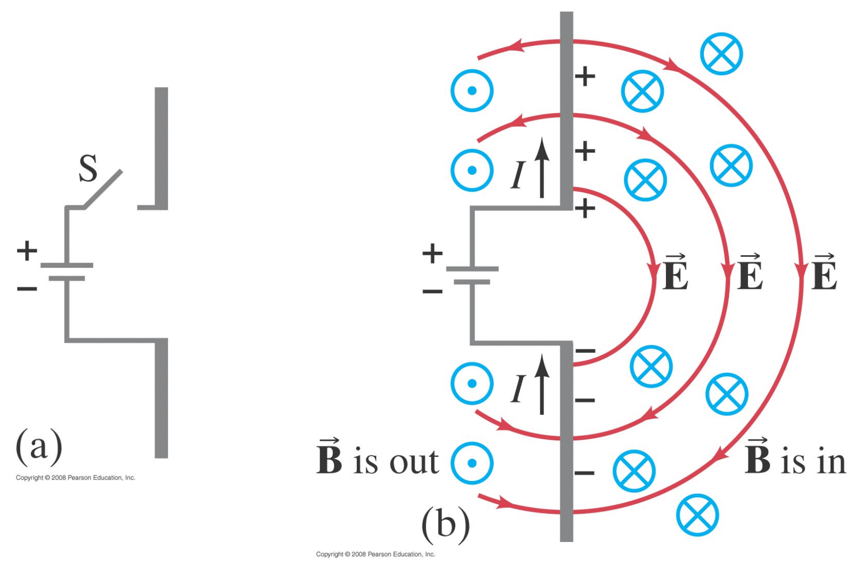
PH2233 Fox : Lecture 18
Chapter 34 : The Wave Nature of Light; Interference and Polarization

(Just the polarization topic from chapter 34.)

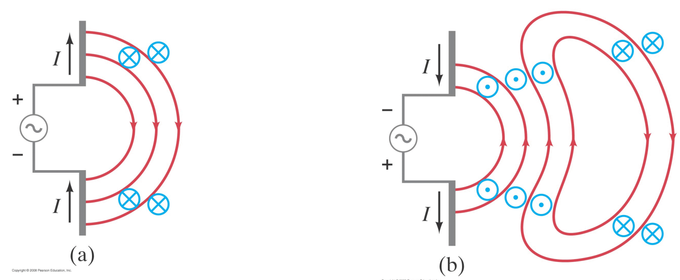
(34-7) : Polarization

Chapter 31 (we'll get to parts of that later) describes how the electric $\vec{E}$ and magnetic $\vec{B}$ field equations from PH2223 can be combined into a single second order differential equation that represents waves propagating at the speed of light. We can hand-wave a bit using what we covered in PH2223 to illustrate how this works.

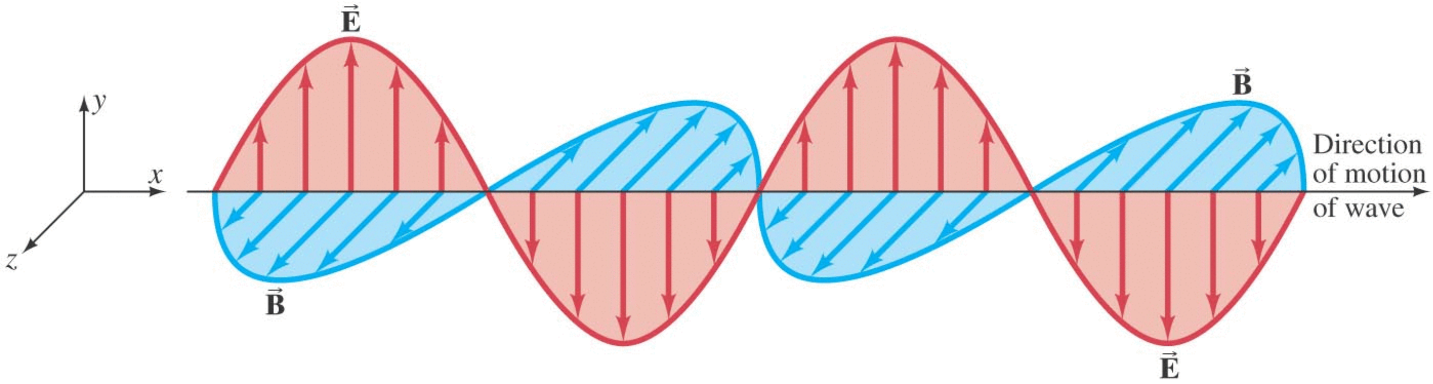
Consider wires connected to a battery as shown. When the switch is closed, current will start flowing (briefly) building up a positive charge on the upper wire, leaving a negative charge on the lower wire, leading to both electric and magnetic fields in the space around this 'antenna'.



If we send an AC signal into this antenna, the resulting fields also vary and still propagate away at $v = c$. If we measure these fields at some location away from the antenna, we see that the fields behave as shown below.

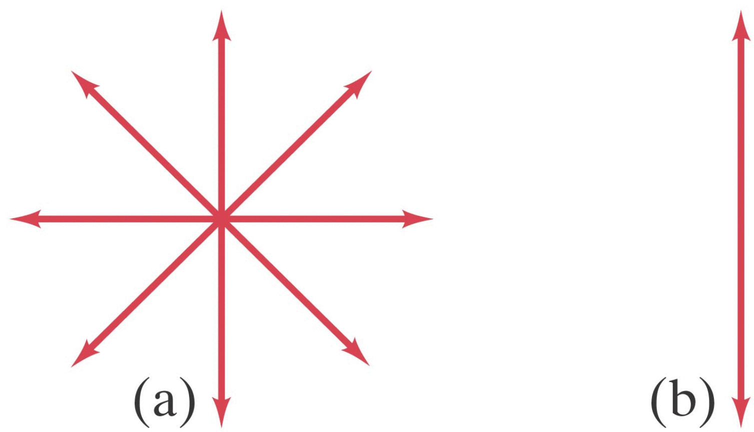


If we're some distance away from the source, what we 'see' is oscillating $\vec{E}$ and $\vec{B}$ fields propagating away from the source, with the directions of $\vec{E}$ and $\vec{B}$ being perpendicular to $\vec{v}$.



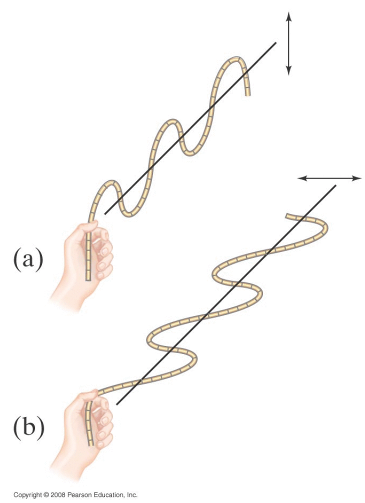
Each component of this wave, $\vec{E}$ and $\vec{B}$, is a vector with a particular direction, and the direction of $\vec{E}$ is taken to be the **polarization** of the wave.

A source of EM waves might naturally generate waves in a particular orientation (radio antennas, for example) but some sources of EM waves (the Sun or a hot filament in an old incandescent light bulb, for example) don't have a preferred direction and emit EM waves (heat and light, in the case of the old light bulb) in all directions. Randomly oriented waves are considered **unpolarized**. (Only a few sample directions are shown here, but there would probably be a continuum covering all possible orientations.)

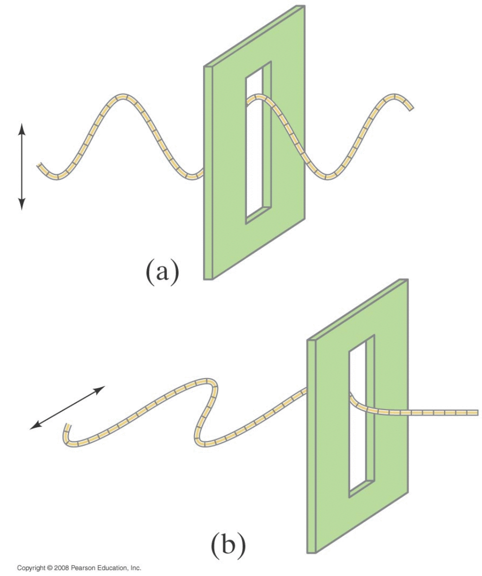


The word 'direction' can be confusing here. The wave (ray, photon) is travelling in some direction defined by $\vec{v}$ (which would be in the $+X$ direction in the figure at the bottom of the previous page) but the POLARIZATION is taken to be the direction the $\vec{E}$ is oscillating in, which in the case of that figure would be in the Y direction.

Consider transverse mechanical waves passing through a slit:



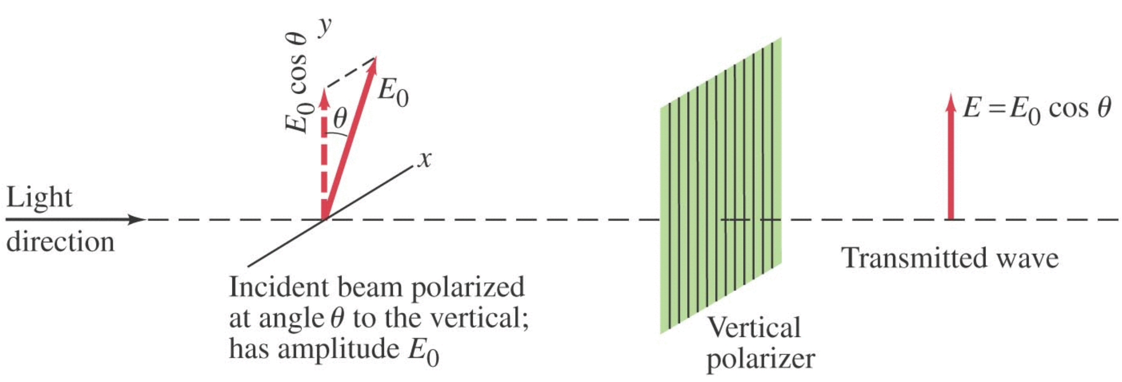
Transverse waves on a rope polarized (a) in a vertical plane and (b) in a horizontal plane.



(a) Vertically polarized wave passes through a vertical slit, but (b) a horizontally polarized wave will not.

Just as with mechanical transverse waves, **the orientation (polarization) of EM waves can affect how they interact with matter.** Back in 1929, Edwin Land experimented with thin films of materials where the long molecular chains making up the material were oriented in a particular direction. If an EM wave with it's $\vec{E}$ oriented in the same direction as these molecules tries to pass through this film, much of the energy in the wave gets **absorbed** by these long molecules. If the $\vec{E}$ is oriented perpendicular to these long molecules, it tends to pass through the material. These materials were called **polaroid** sheets.

Consider what happens when an EM wave with it's $\vec{E}$ interacts with such a material. If the $\vec{E}$ and molecules are oriented relative to each other at some angle θ, only the **component** of the wave perpendicular to the molecular orientation passes through.

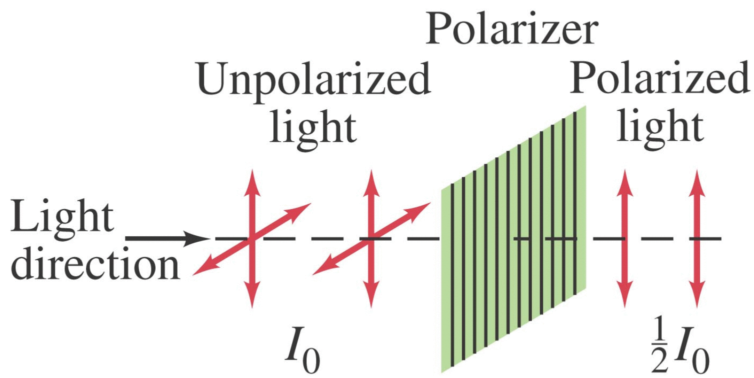


Warning : by convention, the angle is traditionally defined to be 0 when $\vec{E}$ is perpendicular to the direction of the long molecules so it can pass through unaffected. Drawing the lines the way they do in this figure should be taken to mean that an $\vec{E}$ in **that** direction will pass through. (So the long molecular chains are really oriented perpendicular to the lines shown.)

If a beam of plane polarized light strikes a Polaroid filter whose **transmission axis** (perpendicular to the actual molecular orientation) is at an angle θ relative to the incident $\vec{E}$ polarization angle, only the component matching the filter will pass through, so $E = E_o \cos\theta$.

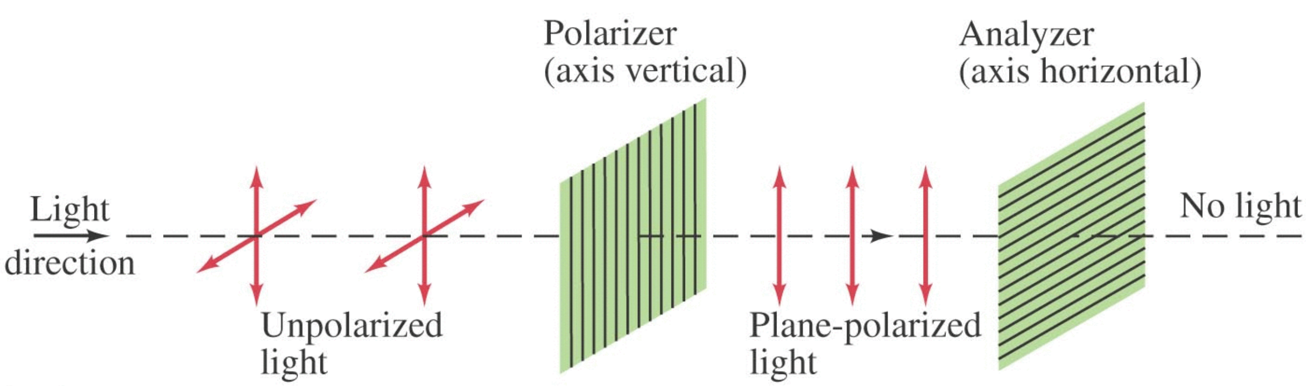
Since the intensity is proportional to the square of the amplitude of the field, that means the transmitted intensity will be $\boxed{I = I_o \cos^2\theta}$ (where I_o is the intensity of the EM wave that enters the filter). **In addition** all the light that makes it through the filter will now be oriented at the same angle as the filter.

What if completely unpolarized light strikes a polaroid filter? Unpolarized light means we have light that has $\vec{E}$ oriented in all possible directions, and each of these will be reduced by the squared cosine of it's angle relative to the filter, so the net effect if we sum over all possible orientations will be the average of cos^2, which is 1/2.



Unpolarized light hitting a polaroid filter will come out with HALF the amplitude, and the light will be completely polarized in the filter direction.

This has an interesting side effect. If we put two polaroid filters oriented at right angles, we can prevent any light from getting through. The first filter here takes the unpolarized light and reduces it's intensity by half AND turns it entirely into light polarized at the angle of the first filter. The second filter is oriented at 90 degrees relative to the first, so the light leaving filter 2 will have it's amplitude multiplied by a factor of $(cos(90))^2 = 0$, eliminating it completely.

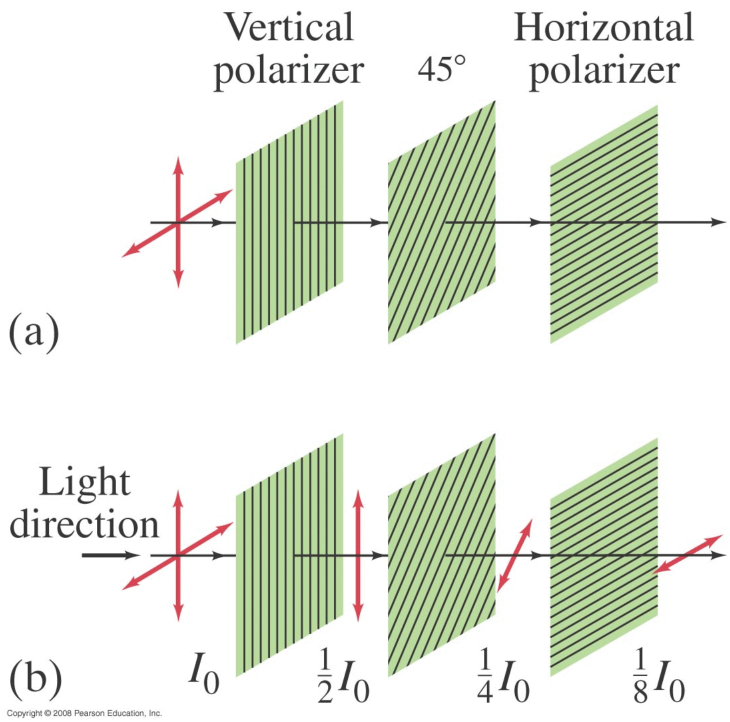


Example 35-14 : Three Polaroids (extended) Suppose we have three polaroid filters lined up as shown in the figure, with unpolarized light (say from the Sun) entering from the left.

- Filter 1 : Incoming unpolarized light becomes outgoing polarized light with it's intensity reduced to half and now all the light passing through 1 is polarized at 0^o.

- Filter 2 : it sees light of intensity $\frac{1}{2}I_o$ that is polarized at 0^o. Only the vector component in the same direction as the filter is allowed through. The filter is at a 45 degree angle relative to the light polarization, so is reduced by a factor of $(cos(45))^2$ which is another factor of 1/2. The light leaving filter 2 is now reduced to $\frac{1}{2}(\frac{1}{2}I_o) = \frac{1}{4}I_o$ and is now polarized at 45 degrees.

- Filter 3 : light polarized at 45 degrees now hits a filter whose polarization angle is 90 degrees, which means it's oriented by 45 degrees relative to the incoming light. It's this relative orientation that matters here, so this filter reduces the intensity by another factor of $(cos(45))^2$ or another factor of 1/2. The light finally leaving this set of filters will have it's amplitude reduced to $\frac{1}{8}I_o$ and it's now polarized at 90^o (horizontally polarized).



WHAT IF we swap the positions of the 2nd and 3rd filters? The light coming out from filter 1 is still oriented at 0^o (vertically polarized in the context of this figure) and still has it's intensity reduced by half, but now it encounters a filter oriented at 90^o relative to the first, so the intensity of the light passing through it will be reduced by a factor of $(cos(90))^2 = 0$. Now nothing gets through these filters.

Multiple polaroid filters can be a bit tricky, but just look at what each is doing in turn:

- Unpolarized light falling on a polaroid filter produces polarized light (at the angle of the filter) and with an amplitude reduced by half.

- Polarized light falling on a polaroid filter produces light polarized at the angle of the second filter and with an amplitude reduced by $(cos(\theta))^2$ where θ is the angle between the filter and the light.

- Keep in mind that this has nothing to do with the angle the ray may be travelling in - it's not a Snell's law type of angle. All the angles involved here are referring to the orientation of the electric field and the transmission axis of the filter.

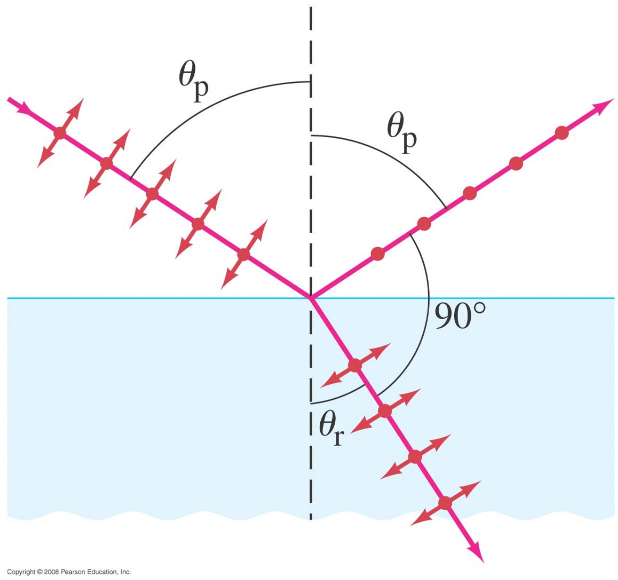
Polarization by Reflection

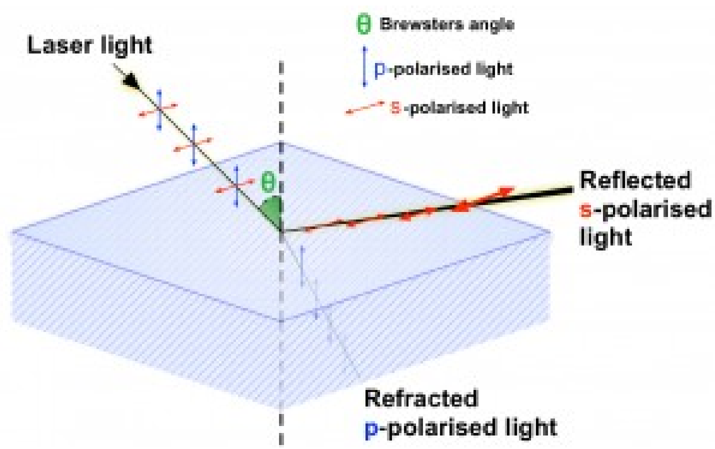
Polaroid filters let us create polarized light from unpolarized light but another mechanism involves just letting the light reflect off a material.

Light reflecting off an interface **tends** to be polarized. The figure shows unpolarized light striking an interface. The reflected light tends to have it's $\vec{E}$ oriented in and out of the page, and the light transmitted into the material has it's $\vec{E}$ oriented in the plane of the page.

This effect is maximized at a particular angle called the Brewster Angle, which occurs when the reflected and refracted rays make a 90 degree angle with each other.

(This is hard to describe in words - perhaps the figure on the right will illustrate these directions more clearly.)





If we apply Snell's law to the left figure: $n_1 \sin\theta_p = n_2 \sin\theta_r$ but minimally propagating angles, we see that $\theta_p + \theta_r = 90$ so $\theta_r = 90 - \theta_p$ and using $\sin(90 - x) = \cos x$ we can write this as: $n_1 \sin\theta_p = n_2 \cos\theta_p$ which we can rearrange into:

$\boxed{\tan\theta_p = n_2/n_1 \text{ : Brewster, or Polarizing Angle}}$

Example: Sunlight

Light from the Sun is unpolarized, so what would be the polarizing angle for sunlight reflecting off water? Here, $n_2 = 1.33$ and $n_1 = 1$ which yields $\theta_p = 53.1^o$, which would occur when the Sun is $90 - 53.1 = 36.9^o$ above the horizon. At our latitude, the Sun at noon gets about 60^o above the horizon, so this would be mid-morning and mid-afternoon. At **all** times, reflections of the Sun's light off water will be polarized, but it will be particularly so around those times of day. Wearing glasses with a polarizing angle perpendicular to this will help cancel out these reflections (i.e. cut down on the glare).