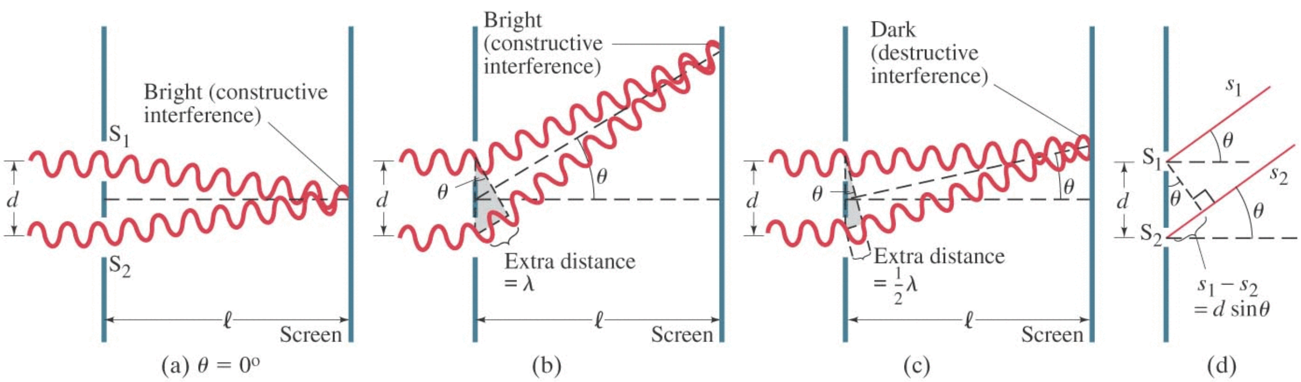
Physics 2233 : Chapter 34 Examples : Interference and Polarization

Two sources of waves (sound, EM, water, etc) of the same frequency can interfere with one another, ranging from complete cancellation (destructive interference) to complete enhancement (constructive interference).



Let s_1 be the distance from source S_1 to some point of interest, and s_2 be the distance from source S_2 to the point of interest. **If the two sources are in phase constructive** interference will occur if the **path difference** $\Delta s = s_2 - s_1$ is an exact multiple of the wavelength λ, and **destructive** interference will occur if the path difference is an integer number of wavelengths **plus** an additional half-wavelength.

$\boxed{\text{In General (figures a,b,c)}}$

- **Constructive Interference** : $s_2 - s_1 = m\lambda$ for $m = 0, \pm 1, \pm 2, etc$
- **Destructive Interference** : $s_2 - s_1 = (m + \frac{1}{2})\lambda$ for $m = 0, \pm 1, \pm 2, etc$
- **Intensity** : $I = I_o \cos^2(\frac{\pi \Delta s}{\lambda})$ where $\Delta s = s_2 - s_1$

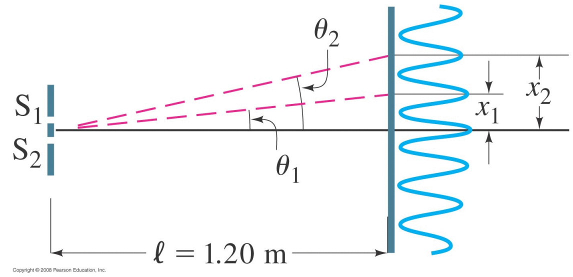
$\boxed{\text{Far Field Approximation (figure d) : \textbf{only when} } l >> d}$

Let the slit separation distance be d. If the observation point is **far away** $l >> d$, then the two paths are essentially parallel and we can write the path difference as $s_2 - s_1 = d \sin\theta$ and the constructive and destructive situations reduce to:

- **Constructive Interference** : $d \sin\theta = m\lambda$ for $m = 0, \pm 1, \pm 2, etc$
- **Destructive Interference** : $d \sin\theta = (m + \frac{1}{2})\lambda$ for $m = 0, \pm 1, \pm 2, etc$
- **Intensity** : $I = I_o \cos^2(\frac{\pi d \sin\theta}{\lambda})$

Small Angle Approximation : **only when** $d >> \lambda$

If the wavelength λ is much smaller than the source separation distance d there will be very many small angle solutions.
In the classic double-slit experiment, a screen is located a distance l from the slits, with $l >> d$. Defining a coordinate system on the screen with $x = 0$ at the location of the central spot, the other bright and dark spots are located at:

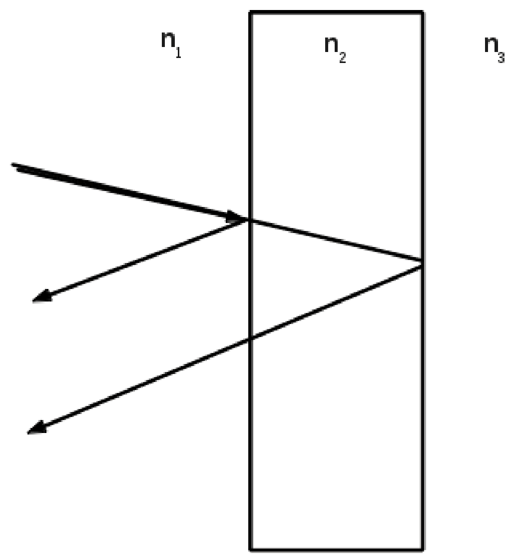


- **Constructive Interference** : $x_m = m\frac{l\lambda}{d}$ for $m = 0, \pm1, \pm2, etc$
- **Destructive Interference** : $x_m = (m + \frac{1}{2})\frac{l\lambda}{d}$ for $m = 0, \pm1, \pm2, etc$
- **Intensity** : $I = I_o \cos^2(\frac{\pi d}{\lambda l}x)$

Thin Film Interference

Some fraction of the incoming light will reflect off both the top and bottom surfaces of the thin film, so the 'signal' reaching the observer consists of two waves that may interfere with one another. The extra path length for the ray reflecting off the bottom surface will be **twice** the thickness of the film.
Things to watch for:

- EM waves reflecting off a material with a higher index of refraction undergo a 180^o phase shift.
- While in the film the wavelength changes to: $\lambda = \lambda_o / n_{film}$



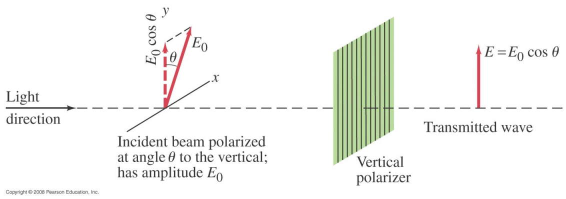
From earlier, when the path difference equals some multiple of the wavelength, $2t = m\lambda$ constructive interference occurs, but the 180^o phase shift noted above means that this may produce destructive interference instead.

Each case must be considered carefully.

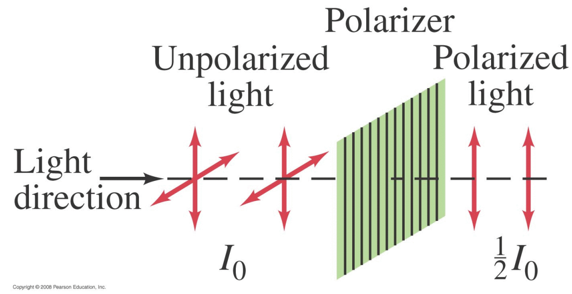
Polarization

Polarized light hitting a polarizing filter: $I = I_o \cos^2(\theta)$, where θ is the angle **between** the direction of the electric field in the incoming light and the angle of the filter.

Unpolarized light hitting a polarizing filter: $I = \frac{1}{2} I_o$

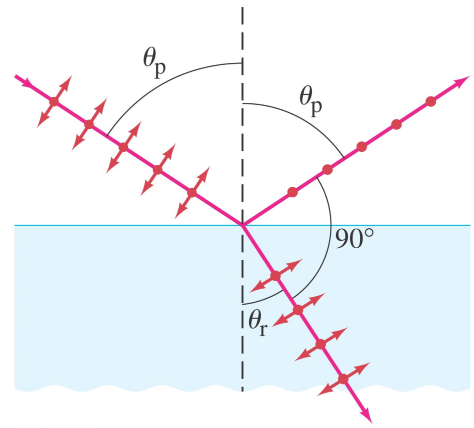


Polarizer and incoming polarized light

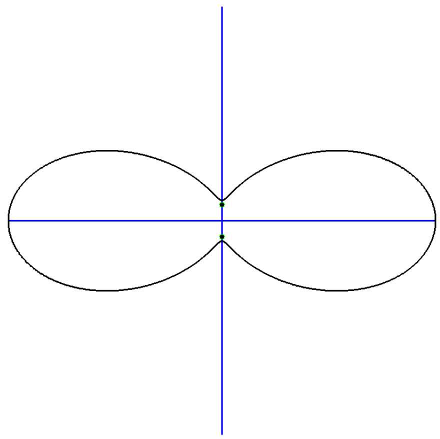


Polarizer and incoming Unpolarized Light

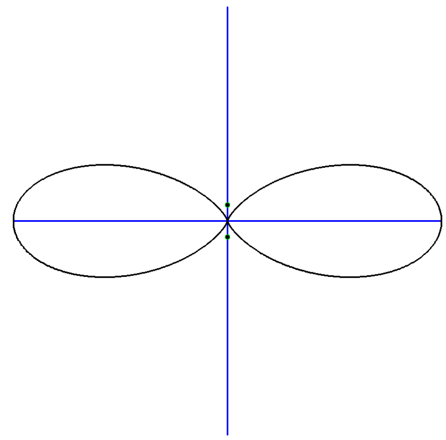
Brewster's angle: at this incoming angle, the reflected and refracted light beams are orthogonal and each is almost entirely polarized in opposite directions.
$\tan\theta_p = n_2/n_1$ (n_1 for the medium the EM wave is traveling through; n_2 is the medium it is reflecting off of).



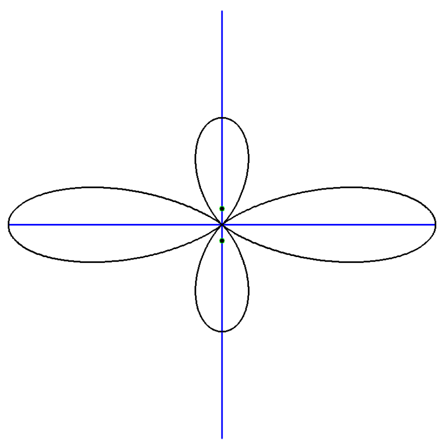
Two-source far-field intensity patterns. Note that in $I = I_o \cos^2(\frac{\pi d \sin\theta}{\lambda})$ we see that only the ratio of d/λ matters, so here we plot the intensity patterns for various such ratios. In all these intensity diagrams, the antennas are positioned along the Y axis, sometimes shown as little dots to represent the configuration, and $\theta = 0$ represents the $+X$ direction, which is to the right.



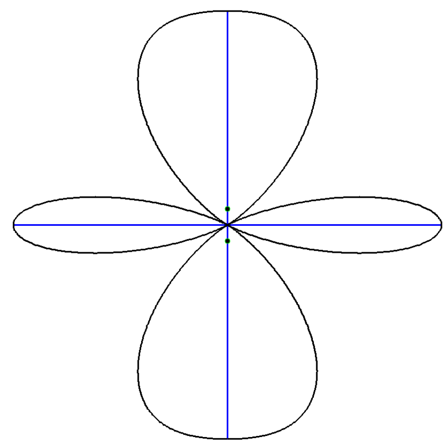
$d/\lambda = 0.40$



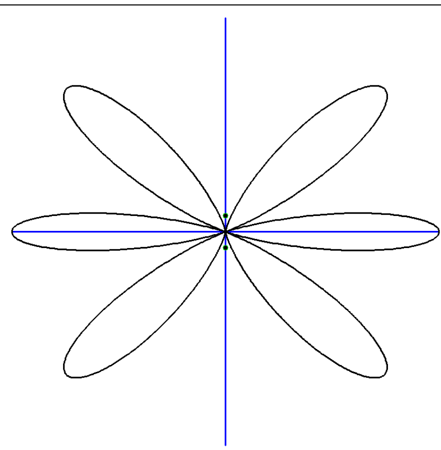
$d/\lambda = 0.50$



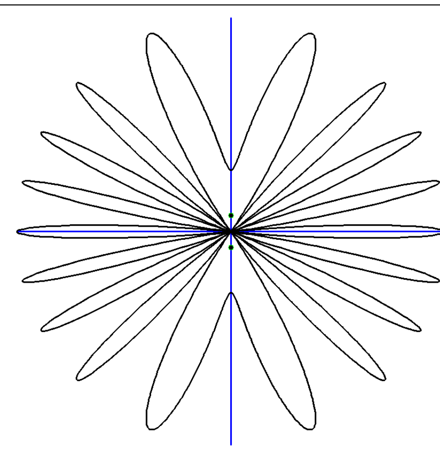
$d/\lambda = 0.75$



$d/\lambda = 1.00$



$d/\lambda = 1.50$

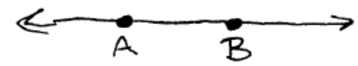


$d/\lambda = 4.32$ (from Example 22)

(See also examples 20 and 21.)

Example 1 Two coherent sources A and B of radio waves are 5.00 m apart. Each source emits waves with wavelength 6.00 m. Consider points along the line between the two sources. At what distances, if any, from A is the interference (a) constructive and (b) destructive?

Note: The wording of this problem can be confusing, but here the 'receiver' is to be located along the (infinitely long) line that passes through A and B. So draw a line that passes through A and B, extending off to each side past the points as well.



Constructive interference occurs when the difference in the path lengths is an exact multiple of the wavelength, and destructive interference when the path difference is an exact multiple **plus** half a wavelength.

(a) For constructive interference the path difference must be $m\lambda$ where $m = 0, \pm 1, \pm 2, ...$ Let's set up a coordinate system that starts at antenna A (at $x = 0$) and runs through B (at $x = 5$).

First, let's consider that the receiver is located somewhere **between** the two antennas. Then the distance from a receiver at x to antenna A will be just x. The distance from the receiver to antenna B will be $5 - x$. The difference in the path lengths then is $(x) - (5 - x)$ and that has to be some multiple m of 6 (meters) so $(x) - (5 - x) = 6m$ or $2x - 5 = 6m$ or finally $x = (6m + 5)/2$. Now, at $m = 0$, this yields $x = 2.5$ (a point that is exactly in the middle of the two antennas). For any larger values of m, the value of x will be outside of the line segment between the two antennas, so we have to derive a different equation.

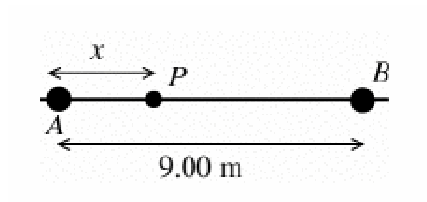
Now, let's assume that the receiver location is **outside** the line segment between A and B. The situation is completely symmetric here, so let's assume x is off to the right of B (still keeping our coordinate system where A was located at $x = 0$ and B was located at $x = 5$. Then now the distance from the receiver to A will be x and the distance from the receiver to B will be $x - 5$. (Remember 'distance' is a positive number, so we're arranging things so that this simple calculation produces a positive number.). The path difference then will be $(x) - (x - 5)$ which is just 5. Well 5 is not any multiple of 6 so there won't be any places there constructive interference occurs out here.

So in summary, the only place we will get constructive interference at this frequency is right at the midpoint between the two antennas.

(b) For destructive interference, the path difference has to be $(m + \frac{1}{2})\lambda$ for $m = 0, \pm 1, \pm 2, ...$ As argued above, the path difference cannot exceed 5 meters (which happens when the receiver is either to the left of A or the right of B). Here $\lambda = 6$ so our path difference equation becomes $6(m + \frac{1}{2})$ for $m = 0, \pm 1, \pm 2, ...$ For $m = 0$, the path difference would be 3 m. For $m = -1$, the path difference would be -3 m. For any other values of m, the difference exceeds 5 meters, so will not be possible. Both of our possible solutions have magnitudes less than 5, so we know the receiver must be somewhere between the two antenna. If x is the location of the receiver, then the distance to antenna A is just x. The distance to receiver B is $5 - x$. The path difference $r_b - r_a$ must be either 3 or -3, so $(5 - x) - x = 3$ from which $x = 1$, or $(5 - x) - x = -3$ from which $x = 4$. So the receiver is on the line between the two transmitters, either 1 meter to the right of A, or 1 meter to the left of B.

Example 3 A radio station operates at a frequency of 120 MHz and has two identical antennas that radiate in phase. Antenna B is 9.0 m to the right of antenna A. Consider point P between the antennas and along the line connecting them, a horizontal distance x to the right of antenna A. For what values of x will constructive interference occur at point P?

The figure here sketches out the situation. We were given that the point is somewhere between the two antennas. Putting an origin at A with the $+x$ axis going off the right through B, then the distance from the point P to A is just $r_A = x$ and the distance (a positive quantity) from P to B is $r_B = 9-x$. (Note that unlike the previous problem, we are only interested in the points on the line segment between the two antennas.)



The path difference is: $r_B - r_A = (9-x) - (x) = 9 - 2x$. For constructive interference, this must be a multiple of the wavelength, so $9 - 2x = m\lambda$.

Well we have the frequency here, not the wavelength, but $\lambda = c/f = (2.998 \times 10^8\ m)/(120 \times 10^6 Hz) = 2.50\ m$.

$9-2x = m\lambda$ becomes $9-2x = 2.5m$ or $x = \frac{9-2.5m}{2}$ or finally $x = 4.5-1.25m$. (Note we've dropped the m symbol for the units of meters to avoid confusion with the m symbol that is being used to count the number of wavelengths.) So we are now looking for solutions to this equation for which x is also somewhere between 0 and 9 (since we were told that the point is between the two towers). There are several such solutions:

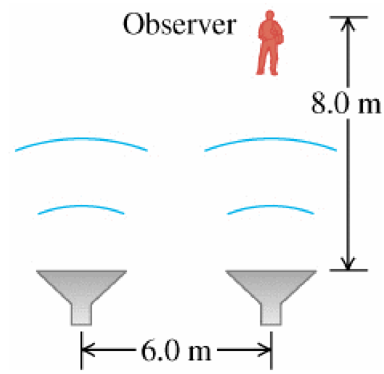
$m = 0$ gives $x = 4.5$
$m = +1$ gives $x = 4.5 - (1)1.25 = 3.25$
$m = +2$ gives $x = 4.5 - (2)1.25 = 2.00$
$m = +3$ gives $x = 4.5 - (3)1.25 = 0.75$
$m = -1$ gives $x = 4.5 - (-1)1.25 = 5.75$
$m = -2$ gives $x = 4.5 - (-2)1.25 = 7.00$
$m = -3$ gives $x = 4.5 - (-3)1.25 = 8.25$

All other values of m produce x values that are either negative or larger than 9, and are therefore NOT between the two antennas.

(Note: if we **were** interested in points along the line extending past A and B on either side, the path difference from the receiver to the two antennas will always be 9 meters which is not a multiple of the wavelength, so there will not be any locations out there where constructive interference can occur. For destructive interference to occur, we would need to find some multiple m of the wavelength such that $(m+1/2)\lambda = 9)$. Substituting in $\lambda = 2.5$ and multiplying both sides by 2: $(2m+1)(2.5) = 9$. Multiplying by two again: $(2m+1)(5) = 18$ from which $m = 1.3$ which is not an integer, so there are no points on the line extensions where complete destructive interference occurs either.)

Example 5 Two speakers, emitting identical sound waves of wavelength 2.0 m in phase with each other, and an observer are located as shown in the figure.

(a) At the observer's location, what is the path difference for waves from the two speakers. (b) Will the sound waves interfere constructively or destructively at the observer's location - or something in between constructive and destructive. (c) Suppose the observer now increases her distance from the speakers to 17 m staying directly in front of the same speaker as initially. Answer the questions for parts (a) and (b) for this new situation.



(a) Let r_1 be the distance from the right speaker to the listener and r_2 be the distance from the left speaker to the listener. Then $r_1 = 8$ and $r_2 = \sqrt{(6)^2 + (8)^2} = 10.0$ (meters). The path difference is $r_2 - r_1 = 10 - 8 = 2$ (meters).

(b) Since the path difference is an exact multiple of the wavelength (which was also 2 meters), then constructive interference will occur at this point.

(c) If we increase the distance of the listener to 17 meters from the right speaker, then $r_1 = 17$ and $r_2 = \sqrt{(6)^2 + (17)^2} = 18.03$ The path difference then is $r_2 - r_1 = 18.03 = 17 = 1.03$. This is almost exactly equal to half the wavelength, so nearly complete destructive interference will occur.

Example 6 Two speakers, emitting identical sound waves are six meters apart. Assume a listener is located 8 meters in front of the right speaker, perpendicular to a line connecting the two speakers. This is basically the **same figure** as in the previous problem, but this time we want to **determine all frequencies where constructive or destructive interference** will occur. I.e. standing at this position, the person will hear some frequencies with a doubled intensity, and some (other) frequencies will be completely missing.

We definitely can't use the 'far field' approximation here since the distance from the listener to the sources is not 'much greater than the source separation distance'. Sound wavelengths vary from centimeters to meters, so the speaker separation distance is not $d >> \lambda$ so we can't use the small angle approximation either. We'll just do this one using the general equations for constructive and destructive interference.

Since our interference equations involve a counter m, to avoid confusion with using m to denote meters, we'll make sure all our lengths are in meters and just skip writing the units.

Path Difference

Distance from listener to the right speaker: $s_1 = 8.000$

Distance from listener to the left speaker: $s_2 = \sqrt{(6)^2 + (8)^2} = \sqrt{100} = 10.0$

Path difference: $s_2 - s_1 = 10.0 - 8.0 = 2.000$ (meters).

Constructive Interference will occur when the path difference is a multiple of the wavelength, so $2.000 = m\lambda$. That would give us wavelengths, but we are interested in frequencies: $f = v/\lambda$ or $\lambda = v/f$ where $v = 343\ m/s$ is the speed of the waves (sound, in this case) so we can write this as:

$2.00 = m\lambda = m\frac{343}{f}$ or $f = m\frac{343}{2.00} = (m)(171.5\ Hz)$

For all these frequencies, the sound will have a doubled intensity. $m = 0$ yields $f = 0$ which isn't audible. $m = 1$ yields $171.5\ Hz$; $m = 2$ yields $343.0\ Hz$ and so on, up to $m = 116$ which yields $f = 19,894\ Hz$ and any higher m gives a frequency we can't hear.

Destructive Interference will occur when the path difference equals a multiple of the wavelength plus an extra half wavelength. After going through the same process above: $f = (m + \frac{1}{2})(171.5\ Hz)$ so we get complete elimination of frequencies $85.75\ Hz$, $257.3\ Hz$, $428.8\ Hz$, and so on.

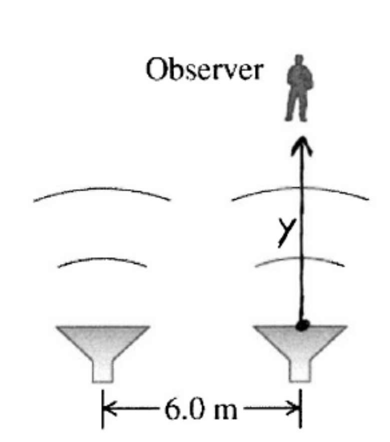
How real is this?

The results we found above sound pretty bad: listening to music when we are located 'off-center' from the speakers should result in major artifacts across the entire spectrum.

Sound from each speaker radiates outward, bouncing off furniture and anything else nearby, so there are **many** paths from each speaker to your ears, each with it's own different path length, making the actual constructive and destructive interference much more complex. If you were to sit in a special audiophile room designed to reduce all these other reflections and sat off-center from the speakers, this effect should be noticeable though.

Example 7 In problem 5, the speakers were located 6 m apart and were emitting sound with a wavelength of $\boxed{\lambda = 2\ m}$ (which corresponds to a frequency of $f = v/\lambda = (343\ m/s)/(2\ m) = 171.5\ Hz$). We found that when the listener was located 8 m from the speaker on the right, they encountered constructive interference (the sound would be louder), and when they were located 17 m from speaker on the right, they encountered almost perfect destructive interference (the sound would be nearly gone).

Suppose the speaker is playing just $f = 171.5\ Hz$ (so $\lambda = 2\ m$) and the listener changes their distance from the speaker on the right but continues to move along a line perpendicular to the line between the two speakers (see figure).



At what distances from the speaker on the right will they encounter destructive or constructive interference? Problem 5 gave us one solution for each case; let's find all of them. Let y be a coordinate starting at $y = 0$ at the speaker on the right and then heading off away from it. At what y values will constructive or destructive interference occur?

Let s_2 be the path from the left speaker to the listener. We can write this in terms of y as $s_2 = \sqrt{y^2 + (6)^2}$.

Let s_1 be the path from the right speaker to the listener; clearly $s_1 = y$. s_2 will always be larger than s_1 so the path difference $s_2 - s_1$ will always be positive, and so we only need to consider values of m that are zero or positive.

Constructive interference will occur when $s_2 - s_1 = m\lambda$ so $\sqrt{y^2 + 36} - y = 2m$. Careful: that m symbol is the counter $m = 0, 1, 2, ...$ We can rearrange this into the form: $\sqrt{y^2 + 36} = y + 2m$ and squaring both sides: $y^2 + 36 = y^2 + 4my + 4m^2$. Subtracting y^2 from both sides and rearranging terms, we find that $\boxed{y = \frac{36-4m^2}{4m}}$.

$m = 0$ leads to $y = \infty$ which I suppose is technically true since the path to each is the same huge number, but let's look for realistic solutions.

$m = 1$ leads to $y = 8$ (meters); $m = 2$ leads to $y = 2.5$ (meters); and $m = 3$ leads to $y = 0$. So there are two more locations in addition to the $y = 8$ we already knew about.

Destructive interference will occur when $s_2 - s_1 = (m + \frac{1}{2})\lambda$. so $\sqrt{y^2 + 36} - y = (m+0.5)(2)$. The right hand side we can write as $2m+1$. We can rearrange this into the form: $\sqrt{y^2 + 36} = y + (2m+1)$ and squaring both sides: $y^2 + 36 = y^2 + 2y(2m + 1) + (2m + 1)^2$. Subtracting y^2 from each side and rearranging terms: $\boxed{y = \frac{36-(2m+1)^2}{2(2m+1)}}$.

$m = 0$ gives $y = 17.5$ (meters), which is about where problem 5 had us check. $m = 1$ gives $y = 4.5$ (meters) and $m = 2$ gives $y = 1.1$ (meters).

(Note if you write out all these y values, the constructive and destructive points alternate.)

Example 10 Young's experiment is performed with light from excited helium atoms ($\lambda = 502\ nm$). Fringes are measured carefully on a screen 1.20 m away from the double slit, and the center of the 20^{th} fringe (not counting the central bright fringe) is found to be 10.6 mm from the center of the central bright fringe. What is the separation of the two slits?

The value of y_{20} here is MUCH smaller than the distance from the slits to the screen, so we have tiny angles and can use the approximation that $y_m = R\frac{m\lambda}{d}$. Rearranging to solve for d: $d = Rm\lambda/y_m$.

It's unfortunate that this field uses m to count the fringe number and we also use m to represent the units of meters. Safest thing to do is convert everything to standard metric units first so we can skip writing the units symbols, and know that the final answer will also come out in proper metric units (lengths in meters, for example).

So here, $R = 1.20$ (meters), $y_{20} = 10.6 \times 10^{-3}$ (meters), $\lambda = 502 \times 10^{-9}$ (meters) and $m = 20$ so $d = Rm\lambda/y_m = (1.20)\frac{(20)(502\times10^{-9})}{10.6\times10^{-3}} = 1.14 \times 10^{-3}$ (meters), or 1.14 mm.

(As a check, $\tan\theta_{20} = y_{20}/R = (10.6 \times 10^{-3})/1.20$ yields $\theta_{20} = 0.51^o$ or $8.9 \times 10^{-3}\ rad$ which is tiny enough that the approximations of $\tan\theta$ and $\sin\theta$ being approximately equal to θ (in radians) is true.)

Example 12 Two slits spaced 0.450 mm apart are placed 75.0 cm from a screen. What is the distance between the second and third dark lines of the interference pattern on the screen when the slits are illuminated with coherent light with a wavelength of 500 nm?

The dark lines correspond to destructive interference instead of constructive, so the path difference should be a half-wavelength off from an exact multiple or $d \sin\theta = (m + \frac{1}{2})\lambda$ from which $\sin\theta = (m + \frac{1}{2})\lambda/d$. The wavelength is $\lambda = 500 \times 10^{-9}$ m and the slit distance is $d = 0.45 \times 10^{-3}$ m so λ/d is extremely small. We can thus use the same 'small angle' approximations we used for the constructive interference case and assume that for all practical purposes: $y_m = R(m + \frac{1}{2})\lambda/d$ (i.e. the same equation the book derived for constructive interference, but now we have that extra half wavelength to produce destructive interference).

Each time we increase m by one in this equation, the value of y increases by a distance of $\Delta y = R\lambda/d$ so the distance between successive dark lines here is $\Delta y = (0.75\ m)(500 \times 10^{-9}\ m)/(0.450 \times 10^{-3}\ m) = 8.33 \times 10^{-4}\ m$ or 0.833 mm.

(Note that the bright fringes are located at $y_m = mR\lambda/d$ so the distance between successive bright fringes is also $\Delta y = R\lambda/d$.)

Example 14 Coherent light with wavelength 450 nm falls on a double slit. On a screen 1.80 m away, the distance between dark fringes is 4.20 mm. What is the separation of the slits?

The distance between the fringes (either light to light, or dark to dark) we showed in the previous problem to be $\Delta y = R\lambda/d$. Here we have the separation distance, the wavelength and the distance to the screen and desire the slit separation, so we rearrange this to solve for $d = R\lambda/\Delta y$. Here then: $d = (1.80\ m)(450 \times 10^{-9}\ m)/(4.20 \times 10^{-3}\ m) = 1.93 \times 10^{-4}\ m$ or $d = 0.193\ mm$.

Example 16 Coherent light from a sodium-vapor lamp is passed through a filter that blocks everything except light of a single wavelength. It then falls on two slits separated by 0.460 mm. In the resulting interference pattern on a screen 2.20 m away, adjacent bright fringes are separated by 2.82 mm. What is the wavelength?

Basically the same as the previous two problems. We found that the separation between adjacent dark fringes (or adjacent bright fringes) is $\Delta y = R\lambda/d$. Here, we have the distance to the screen, the separation distance between adjacent fringes of the same type, and the distance between the slits, so we rearrange this to solve for the wavelength: $\lambda = d\Delta y/R$. Here then: $\lambda = (0.460 \times 10^{-3}\ m)(2.82 \times 10^{-3}\ m)/(2.20\ m)$ or $\lambda = 5.9 \times 10^{-7}\ m = 590 \times 10^{-9}\ m = 590\ nm$.

Example 20 : Wireless Router (A)

A common design for wireless routers includes two antennae. Consider a wireless router that operates at 2.4 Ghz and has two antennae separated by 14.5 cm. At what angles will the wireless signal be particular strong and weak?

The signal has a wavelength of $\lambda = c/f = (3 \times 10^8\ m/s)/(2.4 \times 10^9\ /s) = 0.125\ m$ or 12.5 cm.

The antenna separation $d = 14.5\ cm$ and wavelength $\lambda = 12.5\ cm$ are similar, so we can't use the small angle approximation here.

Technically, if the router is in the same room, I can't use the far field approximation either, but for purposes of this problem let's assume it is located 'far away' (other end of the house maybe).

Constructive interference will occur when $d \sin\theta = m\lambda$ or here: $\sin\theta = m\lambda/d = (m)(12.5)/(14.5) = (m)(0.8621)$

$m = 0$ yields $\theta = 0$, so we have a strong signal along a line through the perpendicular bisector of the two antennae (as we always do).

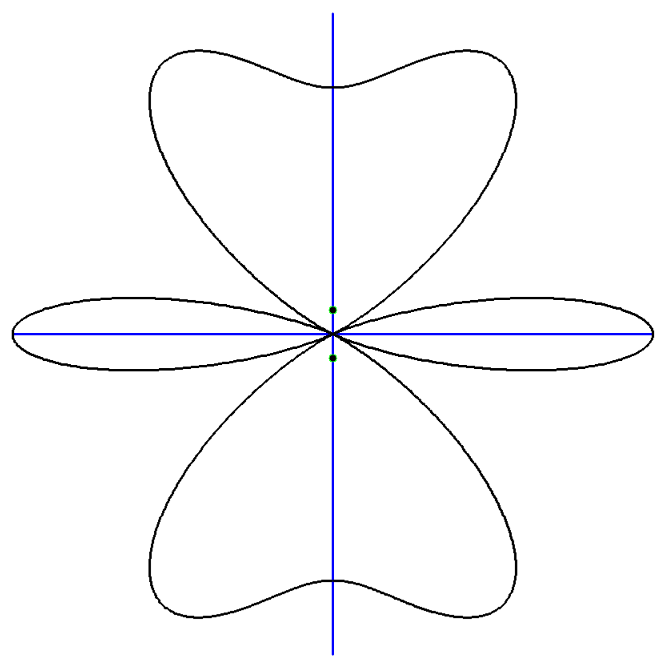
It looks like we'll have another strong angle though. When $m = \pm 1$ we have $\theta = \pm 59.55^o$.

Destructive interference will occur when $d \sin\theta = (m + \frac{1}{2})\lambda$ or going through the same steps as above: $\sin\theta = (m + \frac{1}{2})(0.8621)$.

This has two solutions. For $m = 0$ we have $\sin\theta = (0.5)(0.8621)$ or $\theta = 25.53^o$. For $m = -1$ we have $\sin\theta = (-0.5)(0.8621)$ or $\theta = -25.53^o$.

At those angles, we get no signal at all.

The complete intensity pattern for this situation is shown in the figure.



$\theta = 0$ is pointing to the right, and the antennas are located along the Y axis, so that direction represents someone sitting directly in front (or behind) the router: i.e. located along the line formed by the perpendicular bisector through the line segment connecting the two antennas. Note that this intensity drops off quickly as you move off that direct line though. At 0^o we have full strength, by 25.53^o we have none at all.

In the direction 'in-line' with the two antennas (i.e. in the Y direction), we get much wider coverage. (Not full intensity, but at least I can move around in that area quite a bit and still have a strong signal.)

Example 21 : Wireless Router (B)

The identical wireless router in the previous problem can transmit signals at two frequencies. It can use the 2.4 Ghz band which we used in the previous problem, but it can also transmit at 5.0 Ghz.

If we operate the router at this higher frequency, at what angles will the signal be particularly strong or weak?

The signal now has a wavelength of $\lambda = c/f = (3 \times 10^8\ m/s)/(5.0 \times 10^9\ /s) = 0.060\ m$ or 6 cm.

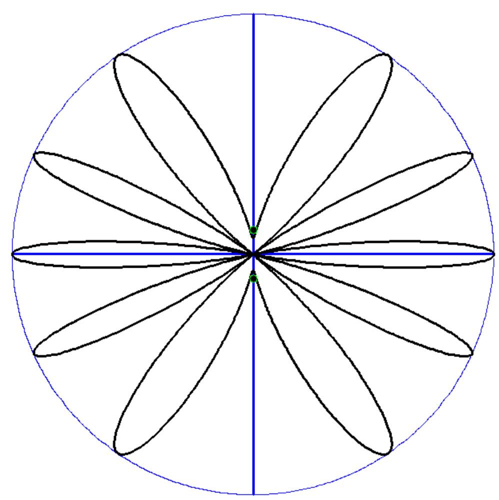
The antenna separation $d = 14.5\ cm$ and wavelength $\lambda = 6\ cm$ are still similar enough that we can't use the small angle approximation. We need $d >> \lambda$ to be able to do that and here d is only 2 or 3 times larger than λ.

Technically, if the router is in the same room, I can't use the far field approximation either, but for purposes of this problem let's assume it is located 'far away' (other end of the house maybe).

Constructive interference will occur when $d \sin\theta = m\lambda$ or here: $\sin\theta = m\lambda/d = (m)(6.0)/(14.5) = (m)(0.4138)$. Trying various values of m, $m = 0$ yields $\theta = 0$. $m = \pm 1$ yields $\theta = \pm 24.4^o$ and $m = \pm 2$ yields $\theta = \pm 55.85^o$. Operating at this frequency we have more angles where constructive interference is happening.

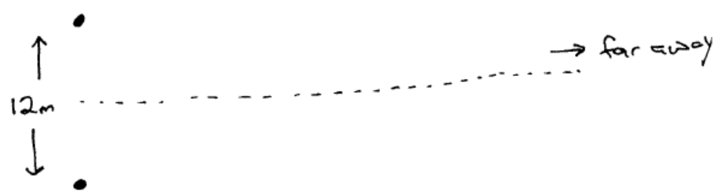
Destructive interference will occur when $d \sin\theta = (m + \frac{1}{2})\lambda$ or going through the same steps as above: $\sin\theta = (m + \frac{1}{2})(0.4138)$. Trying $m = 0$ and small positive and negative integers leads to angles of $\pm 11.94^o$ and $\pm 38.37^o$. At those angles, we get no signal at all.

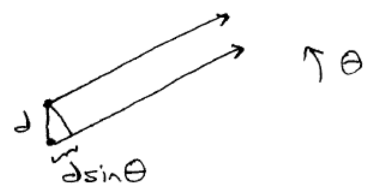
The complete intensity pattern for this situation is shown in the figure. Note that although we do not have complete cancellation at 90 degrees, it's pretty close. Compare this pattern to the same router operating at 2.4 Ghz (previous problem). At the lower frequency, we have a fairly broad high-ish intensity in a direction parallel with the two antenna. At this higher frequency, we get almost nothing.



This is a pretty undesirable intensity pattern since we have to be located along very specific directions to get a strong signal. My guess is that when the router is operating at this higher frequency, it will only use a single antenna so that it radiates uniformly in all directions.

Example 22 An FM radio station has a frequency of 107.9 MHz and uses two identical antennas mounted at the same elevation, 12.0 m apart. The antennas radiate in phase. The resulting radiation pattern has a maximum intensity along a horizontal line perpendicular to the line joining the antennas and midway between them. Assume that the intensity is observed at distances from the antennas that are much greater than 12.0 m. (a) At which other angles (measured from the line of maximum intensity) is the intensity maximum? (b) At which angles is it zero?





(a) At large distances from the sources, we have constructive interference (i.e. maximum intensity) where $d\sin\theta = m\lambda$ for $m = 0, \pm1, \pm2, ...$ We have the frequency, but need the wavelength: $\lambda = c/f = (2.998 \times 10^8\ m/s)/(107.9 \times 10^6 Hz) = 2.78\ m$. (Where that m represents the units of meters, not the integer m that is used to count the intensity peaks. To avoid confusion, we'll just make sure everything is measured in standard metric units and drop the m units...)

We might be tempted to assume that the angle is small so that $\sin\theta$ is about equal to θ in which case $\theta = m\lambda/d$. But here $\lambda/d = (2.78)/(12) = 0.23$ which would give us for $m = 1$ an angle of 0.23 rad or 13.3^o and the higher m values would be multiples of this. These are **not** small angles.

Using the exact (far-field) equation: $d\sin\theta = m\lambda$ for $m = 0, \pm1, \pm2, ...$ or $\sin\theta = m\lambda/d = (m)(2.78)/(12.0) = 0.232m$ so $\theta = \sin^{-1}(0.232m)$. (Again, watch out for potential confusion involving the m symbol. It doesn't represent meters here but the counter $m = 0, \pm1, \pm2, ...$)

At $m = 0$, we are located right along a line perpendicular to the two antennas. The path distance will be zero and that gives the first maximum intensity line. The question was interested in the other angles. The sine function is odd, so if we solve for θ for $m = 1$, we also have the solution for $m = -1$ (just flip the sign of the angle). Plugging in the first few positive and negative values for m we get $\theta = \pm13.4^o, \pm27.6^o, \pm44.1^o, \pm68.1^o$. Larger values for m give values of $0.232m$ that are larger than 1, so we can't take the inverse sine. Thus these eight angles (plus the one at $m = 0$ which was the line perpendicular to the line between the antennas) are where constructive interference (high intensity) will occur.

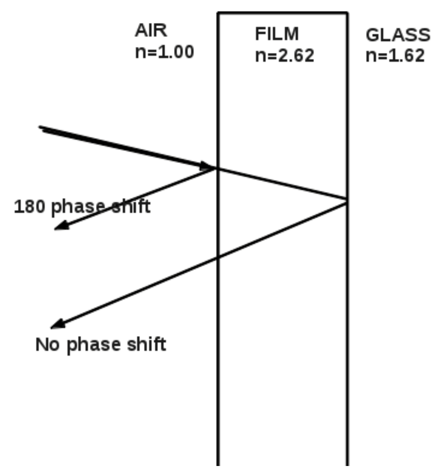
(b) At large distances from the sources, we have destructive interference (i.e. minimum intensity) where $d\sin\theta = (m + \frac{1}{2})\lambda$ for $m = 0, \pm1, \pm2, ...$ Following through the same arguments as above, we end up with the equation $\sin\theta = (m + \frac{1}{2})\lambda/d = (m + \frac{1}{2})(0.232)$. We can plug various values in for m (positive, negative, and zero) until the right hand side exceeds a magnitude of 1 again and find that: $\theta = \pm6.66^o, \pm20.4^o, \pm35.5^o, \pm54.3^o$. (Values of m larger than 3 or less than -4 give a right-hand side that exceeds the range of the arc-sine function.)

Example 30 When viewing a piece of art that is behind glass, one often is affected by the light that is reflected off the front of the glass (called glare) which can make it difficult to see the art clearly. One solution is to coat the outer surface of the glass with a film to cancel part of the glare. (a) If the glass has a refractive index of 1.62 and you use TiO_2, which has an index of refraction of 2.62, as the coating, what is the minimum film thickness that will cancel light of wavelength 505 nm? (b) If this coating is too thin to stand up to wear, what other thickness would also work? (Find only the three thinnest ones.)

Constructive interference occurs when waves are exactly in phase (the peak of one arriving at the same time as the peak of the other). For destructive interference, we need the two waves to arrive exactly half a wavelength out of phase.

The thing we have to be careful of when light is reflecting is that is can change its phase (like a transverse wave on a string when it reaches a fixed versus free end).

When electromagnetic waves reflect, the phase of the reflected waves depends on the indices of refraction of the two materials at the interface. When waves in one medium hit a medium with a higher index of refraction, the reflected waves pick up a 180^o phase shift.



At the front surface of the film, light in air ($n = 1$) reflects from the film ($n = 2.62$) **and** the reflected light picks up a 180^o phase shift. At the back surface of the film, light in the film ($n = 2.62$) reflects from glass ($n = 1.62$) and there is no phase shift due to this reflection. So we have light reflected from the front surface of the film (that now has a 180^o phase shift) combining with light that has passed through the film, reflected off the film-glass interface without any phase shift, passed back up through the film and exited into the air.

We want these two signals to combine destructively to eliminate the glare at the given wavelength. Since we already have a 180^o (i.e. half-wavelength) phase shift, that means that the light must travel an exact integer number of wavelengths while it is inside the film. That way when it combines with the (out of phase) light that was reflected directly from the surface of the film, they'll cancel each other out.

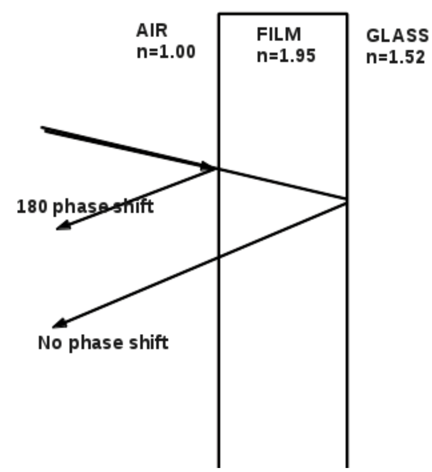
The path difference for the two rays is $2t$ (twice the thickness of the film). The wavelength of the light **in the film** is $\lambda = (505\ nm)/(2.62) = 192.7\ nm$. We need the thickness of the film to be such that the path difference ($2t$) is some multiple of the wavelength, or: $2t = m\lambda$. So: $t = m\lambda/2 = m\frac{192.7\ nm}{(2)} = (m)(96.4\ nm)$. (OK, that's a pretty horrible notation, but what we're meaning there is the integer m times 96.4 nanometers.) The minimum thickness (at $m = 1$) would be 96.4 nm.

(b) The next three thicknesses would be integer multiples of that thickness or 192 nm, 289 nm, 386 nm, and so on. (All these are incredibly thin. A typical metallic molecule might have a diameter of 1 nm so these films are only a few hundred atoms thick and are probably formed by spraying the material onto the glass.)

Example 31 : car window glass

There are thin films you can put on the outside of your car windows to reduce the amount of light getting into the car. What is the minimum thickness such that incoming light of wavelength 505 nm is mostly reflected back? The film has an index of refraction of 1.95 and the auto glass has an index of refraction of 1.52.

Note how this situation is different from the previous one. In the 'art' case, we wanted the reflected light to cancel at some wavelength, which means we wanted destructive interference to occur between the two reflected rays. In **this** case, we want light to **not** get into the car, which ultimately means we need it to mostly reflect back out: we want the two reflected signals (the ray reflecting off the air-film interface, and the ray reflecting off the film-glass interface) to **constructively** interfere and carry away as much of the incoming intensity as we can.



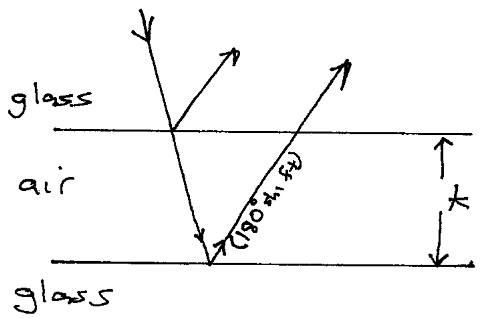
The ray bouncing off the air-film interface will pick up a 180^o phase shift. The ray that enters the film and reflects off the film-glass interface will not.

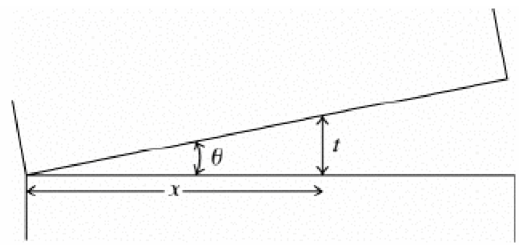
So if we want constructive interference to happen between these two rays, we need the waves to pick up an extra half-wavelength on their path through the film. Specifically, this time we're looking for: $2t = (m + \frac{1}{2})\lambda$ where λ is the wavelength of the light while in the film: $\lambda = \lambda_o/n_{film} = (505\ nm)/(1.95) = 259\ nm$, so our constructive interference equation for this situation becomes: $2t = (m + 0.5)(259\ nm)$.

The smallest thickness will occur when $m = 0$, leading to $t = 64.7\ nm$ which is probably much thinner than the actual film, unless it's done at the factory by spraying it on. The same constructive interference will occur at any higher integer value of m though as well, so eventually we'll arrive at a realistic, manufacturable thickness. Using higher values of m we arrive at thicknesses of 65 nm, 194 nm, 324 nm, 453 nm, and so on. Completely destructive interference for the reflected light would occur at thicknesses midway between each of these: at these other thicknesses, most of the light would pass into the car instead of being reflected back out.

Whatever thickness we use to cut down the light coming into the car, it must be very precisely maintained since apparently it only takes a few tens of nanometers to switch from constructive to destructive interference.

Example 32 Two rectangular pieces of plane glass are laid one upon the other on a table. A thin strip of paper is placed between them at one edge so that a very thin wedge of air is formed. The plates are illuminated at normal incidence by 546 nm light from a mercury vapor lamp. Interference fringes are formed, with 15.0 fringes per centimeter. Find the angle of the wedge.





Left figure: The fringes are produced by interference between light reflected from the top and bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift, but the waves reflected from the bottom surface of the air wedge do: they are reflected with a half-cycle phase shift. So in that air wedge, if exactly an integer number of waves 'fits', the two reflected rays will end up canceling each other out (destructive interference, dark bands). For constructive interference, in this case we'll need an integer number of waves **plus** a half a wavelength to fit. The condition for constructive interference when one of the paths has a half-wavelength phase shift then is $2t = (m + \frac{1}{2})\lambda$ or $t = (m + \frac{1}{2})\lambda/2$.

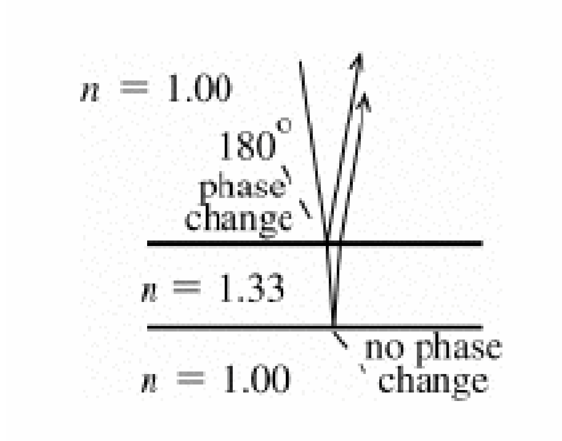
Looking at the geometry of the wedge now (right figure), let's say that the bright fringes (constructive interference) are located at positions x along the interface. These positions are related to the thickness of the air wedge t, and the angle between the glass slides by $\tan\theta = t/x$ so $t = x\tan\theta$. But the special thicknesses where constructive interference will occur are $t = (m + \frac{1}{2})\lambda/2$, so combining these two equations for t we see that: $(m + \frac{1}{2})\lambda/2 = x\tan\theta$ or $x_m = (m + \frac{1}{2})\frac{\lambda}{2\tan\theta}$. The distance **between** two adjacent fringes, x_{m+1} and x_m would be $\Delta x = \frac{\lambda}{2\tan\theta}$ or since we're trying to find the angle, we can rearrange this to be $\tan\theta = \frac{\lambda}{2\Delta x}$.

We have the wavelength of the light, but what is the spacing between the fringes? We know there are 15 fringes per centimeter, so they are $\frac{1}{15}$ centimeter apart or 0.0667 cm or $\Delta x = 6.7 \times 10^{-4}$ m. $\tan\theta = \lambda/(2\Delta x) = (546 \times 10{-}9\ m)/(2 \times 6.7 \times 10^{-4}\ m) = 4.09 \times 10^{-4}$. This is extremely small, and we can make the assumption that $\tan\theta = \theta$ so $\theta = 4.09 \times 10^{-4} rad$ (or 0.0234^o).

(This experiment is usually done with very small glass slides (like you would put under a microscope) since it is difficult to manufacture larger glass materials that remain very flat. The weight of the glass itself will cause it to bend. If the size of the glass slide is, say, 5 cm then we could estimate the thickness of the sliver of paper we stuck in on the right side: $\tan\theta$ will be the thickness of the paper divided by the width of the glass slide so $4.09 \times 10^{-4} = T_{paper}/(0.05\ m)$ which implies that $T_{paper} = 2 \times 10^{-5}\ m$. A typical stack of laser printer paper might have 500 sheets and be about 6 cm or 0.06 m thick which implies a thickness of $(0.06\ m)/(500) = 1.2 \times 10^{-4}\ m$ or $12 \times 10^{-5}\ m$ which is six times thicker than the paper apparently being used here. If we stuck one of these 'normal' pieces of paper between the slides, the angle would be six times larger. Still pretty tiny so the tangent of that angle is six times what we had before. That causes the spacing between the fringes to be six times closer together. That would cause there to be 90 fringes per centimeter instead of 15, which would be extremely difficult to perceive with the naked eye, but could still be resolved under a microscope. This method can be used to determine the sizes of very small objects.)

Example 34 The walls of a soap bubble have about the same index of refraction as that of plain water, $n = 1.33$. There is air both inside and outside the bubble. (a) What wavelength (in air) of visible light is most strongly reflected from a point on a soap bubble where its wall is 290 nm thick? To what color does this correspond? (b) Repeat part (a) for a wall thickness of 340 nm.

Consider the interference between rays reflected from the two surfaces of the soap film. Strongly reflected means we want constructive interference so the two rays should constructively interfere when they had out back towards our eyes. The ray that bounces off the outer surface of the film will have a 180^o phase shift. The ray that bounces off the inner surface will not. So while the ray is inside the film itself, it needs to pick up that extra half-wavelength in order to combine constructively with the ray that bounced directly off the outer surface of the film.



From the discussion above, we see that the path difference $2t$ must correspond to some integer number of waves **plus** an additional half wavelength to compensate for the phase shift. So for constructive interference we must have here: $2t = (m + \frac{1}{2})\lambda$, where $m = 0, 1, 2, ...$ and where λ is the wavelength of the light while it is inside the film. That wavelength is equal to λ_o/n where λ_o is the wavelength in air and n is the index of refraction of the soap film. So finally: $2t = (m + \frac{1}{2})\frac{\lambda_o}{n}$.

We're ultimately interested in the wavelengths for the case of varying the thickness of the film, so we can rearrange this into: $\lambda_o = \frac{2tn}{m+\frac{1}{2}}$.

(a) If the thickness of the film is 290 nm and the index of refraction is $n = 1.33$, we can write that last equation as: $\lambda_o = \frac{2tn}{m+\frac{1}{2}} = \frac{(2)(290\ nm)(1.33)}{m+0.5} = (771.4\ nm)/(m + 0.5)$.

Computing the wavelength for various values of m: $m = 0$ gives $\lambda = 1543\ nm$ (not visible), $m = 1$ gives $\lambda = 514\ nm$ (visible, green), and $m = 2$ gives $\lambda = 308\ nm$ (not visible). We don't need to go further since any higher m will just put the wavelength even further from what our eyes can perceive. So where the bubble has this thickness, green light is reflected with twice the amplitude of other colors and the bubble might look primarily green.

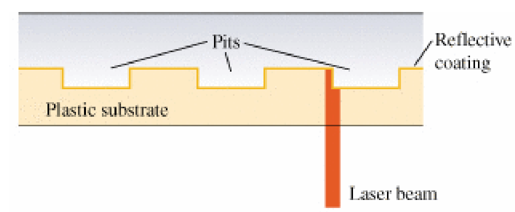
(b) Repeating the calculation for a part of the soap bubble that is 340 nm thick, $\lambda_o = \frac{2tn}{m+\frac{1}{2}} = \frac{(2)(340\ nm)(1.33)}{m+0.5} = (904.4\ nm)/(m + 0.5)$.

Computing the wavelength for various values of m: $m = 0$ gives $\lambda = 1809\ nm$ (not visible), $m = 1$ gives $\lambda = 603\ nm$ (visible, orange), and $m = 2$ gives $\lambda = 362\ nm$ (not visible). Again, we don't need to go further since any higher m will just put the wavelength even further from what our eyes can perceive. So where the bubble has this thickness, orange light is reflected with twice the amplitude of other colors and the bubble might look primarily orange.

In general, the thickness of the soap film fluctuates due to tiny air currents and undulations of the surface of the bubble, so we see various colors shifting around it's surface.

Example 36 A compact disc (CD) is read from the bottom by a semiconductor laser with wavelength 790 nm passing through a plastic substrate of refractive index 1.8.

When the beam encounters a pit, part of the beam is reflected from the pit and part from the flat region between the pits, so these two beams interfere with each other. What must the minimum pit depth be so that the part of the beam reflected from a pit cancels the part of the beam reflected from the flat region? (It is this cancellation that allows the player to recognize the beginning and end of a pit.)



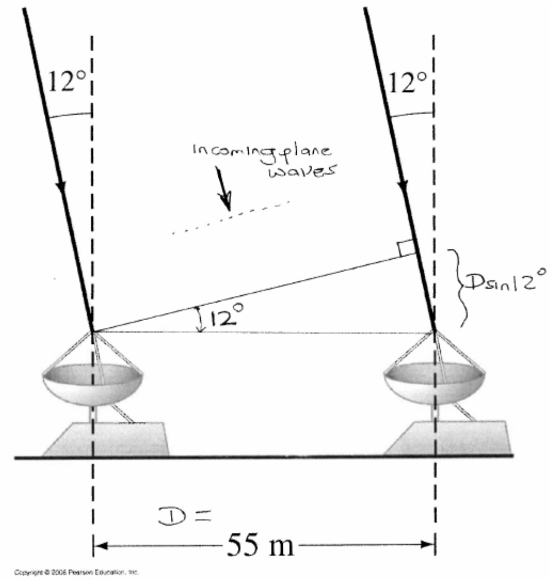
Both reflections occur for waves in the plastic substrate reflecting off the reflective coating, so whatever phase shift might be being induced, the same thing is happening to both of the rays and no net phase shift will occur. The condition for destructive interference then will be that the path difference (twice the thickness (well, depth) of a pit) should be $m + \frac{1}{2}$ wavelengths, so $2t = (m + \frac{1}{2})\lambda$ or $t = (m + \frac{1}{2})\frac{\lambda}{2}$.

We were given the wavelength of the light IN AIR, but here the path difference is being created by light while it is within the plastic substrate. The wavelength of the light in the substrate is not the same as in air, but we can compute it: $\lambda = \lambda_o/n$. Leaving things symbolically for now, this gives us: $t = (m + \frac{1}{2})\frac{\lambda_o}{2n}$.

The smallest thickness will occur where $m = 0$ at which point $t = \frac{1}{2}\frac{\lambda_o}{2n} = \frac{\lambda}{4n} = \frac{790\ nm}{(4)(1.8)} = 110\ nm$ or just 0.11 μm.

According to what is described in this problem, when information is written on a CD or DVD, the (writing) laser beam has to carve out a little pit that has this depth. Note that this is a physical change to the medium itself, which would be difficult to reverse. Pre-recorded CD's can be manufactured this way, and 'write-once' CD's (CD-R) might use this technology as well, but re-writable CD's use a different technology entirely that involves changing the reflectance of the substrate in a reversible way, rather than permanently burning little pits into it.

Example 38 A radio telescope, whose two antenna are separated by 55 m, is designed to receive 3.0 MHz radio waves produced by astronomical objects. The received radio waves create 3.0 MHz electronic signals in the telescopes left and right antennas. These signals then travel by equal-length cables to a centrally located amplifier, where they are added together. The telescope can be 'pointed' to a certain region of the sky by adding the instantaneous signal from the right antenna to a 'time-delayed' signal received by the left antenna, a time Δt earlier.



If a radio astronomer wishes to 'view' radio signals arriving from an object oriented at a 12^o angle to the vertical as shown in the figure, what time delay Δt is necessary?

The source is very far away (millions of kilometers, or even light years) so we are definitely in the 'far field' situation. From the figure we can see that the signals arriving at the antenna on the right have to travel an extra distance of $(55\ m)(\sin 12^o) = 11.435\ meters$, which means the signals from the two antennas will arrive slightly out of phase, and will partly cancel each other out when they're combined.

We'll have a sine wave from one antenna, plus a slightly shifted sine wave from the other. To maximize the overall signal, we'd like to rig things so that the two signals arrive in phase. We can do that by slightly delaying the signal from the left antenna. How much of a delay?

The signal is travelling at the speed of light c, so the signal is arriving at the left antenna earlier by $\Delta t = (distance)/(speed) = (11.435\ m)/(3 \times 10^8\ m/s) = 3.8 \times 10^{-8}\ s$ which we can write as $38 \times 10^{-8}\ s$ or 38 ns (nanoseconds).

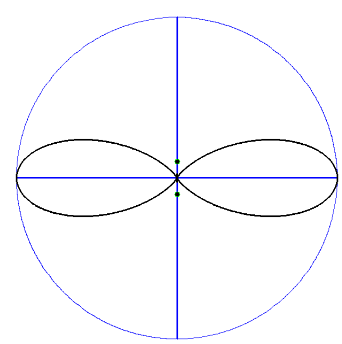
Different Question : if we remove this delay and just leave the antennas in sync, at what angles will they 'hear' radio signals strongly?

Constructive interference will occur at any angle such that $d \sin\theta = m\lambda$ or $\sin\theta = m\frac{\lambda}{d}$.

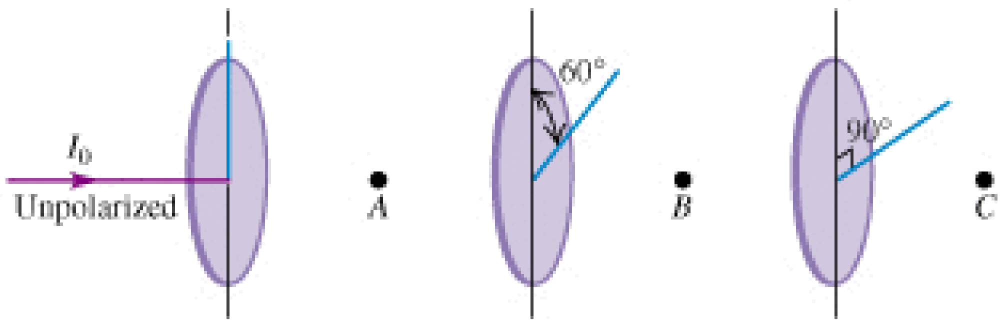
The receiver is apparently tuned to receive signals of $f = 3\ MHz$, so what wavelength is this? $\lambda = v/f = (3 \times 10^8\ m/s)/(3 \times 10^6\ /s) = 100\ m$.

We'll have constructive interference at $\sin\theta = m\frac{100\ meters}{55\ meters} = (1.818)(m)$

Note this has only one solution: $\theta = 0$ (well, and the symmetric point at $\theta = \pi$). So this pair of antennas (without the delay from the first part) will receive signals strongly along a line through their perpendicular bisector. The complete antenna pattern at this frequency is shown here. (Note: at 90^o there is almost complete cancellation, but not quite.)



Example 40 A beam of unpolarized light of intensity I_o passes through a series of ideal polarizing filters with their polarizing directions turned to various angles as shown. (a) What is the light intensity (in terms of I_o at points A, B, and C? (b) If we remove the middle filter, what will be the intensity at point C?



When unpolarized light passes through a polarizer, its intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light will now be polarized along the axis of the polarizer. When (already) polarized light is incident on a polarizer, the transmitted intensity is reduced by a factor $\cos^2\phi$ where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

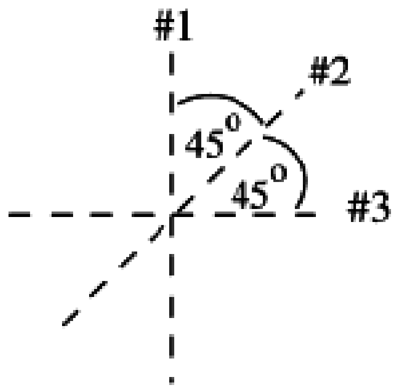
(a) Here we have unpolarized light entering, so the intensity that is transmitted through to point A is $I_A = \frac{1}{2}I_o$ and this light is now polarized in the vertical direction.

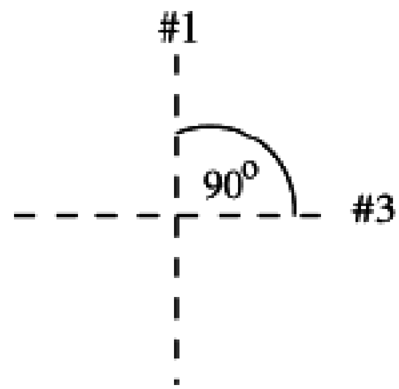
Moving from A to B, we have incident light that is polarized vertically which now encounters a second filter that is tilted 60^o with respect to the incoming polarization, so the intensity will be reduced here by a factor of $\cos^2 60^o$ or 0.25. In terms of the original intensity, then, we now have $I_B = (0.25)I_A = (0.25)(0.50)I_o = 0.125 \times I_o$

From B to C, we have incident light that is polarized in the direction of that middle filter and it is now hitting a third filter. That third filter is making an angle of 30^o with respect to the incoming light so the intensity at C will be reduced by a factor of $\cos^2 30^o$ or 0.75 with respect to the intensity entering that filter. So in terms of the original intensity: $I_C = 0.75I_B = (0.75)(0.125)I_o = 0.0938I_o$.

(b) If we remove the middle filter then light coming out of the first filter will have an intensity of $\frac{1}{2}I_o$ as argued above and this light will be polarized vertically (per the polarization direction of the first filter). When this light hits the third filter directly, we see that the angle of the light is exactly 90^o off from the angle of the filter, so $\phi = 90^o$ and the intensity that gets through will be $\cos^2 90^o$ or zero.

Example 42 Three polarizing filters are stacked with the polarizing axes of the second and third at 45^o and 90^o respectively with that of the first. (a) If unpolarized light of intensity I_o is incident on the stack, find the intensity and state of polarization of light emerging from each filter. (b) If the second filter is removed, what is the intensity of the light emerging from each remaining filter?





When unpolarized light hits any polarizing filter, the intensity transmitted is reduced by half. When already polarized light hits a polarizing filter, its intensity is reduced by $\cos^2\phi$ where ϕ is the angle between the light and the filter.

(a) Here, then, the intensity after passing through filter 1 will be $0.5I_o$ and the light is now polarized in the direction of filter 1. This light now hits filter 2 which is oriented at a 45^o angle with respect to the light, so the amplitude will be reduced by a factor of $\cos^2 45$ or 0.5. This light now hits filter 3, which is oriented at a 45^o angle with respect to the incoming light, so the amplitude is cut down by another factor of $\cos^2 45$ or 0.5. Overall then the final amplitude is $(0.5)(0.5)(0.5)I_o$ or $0.125I_o$.

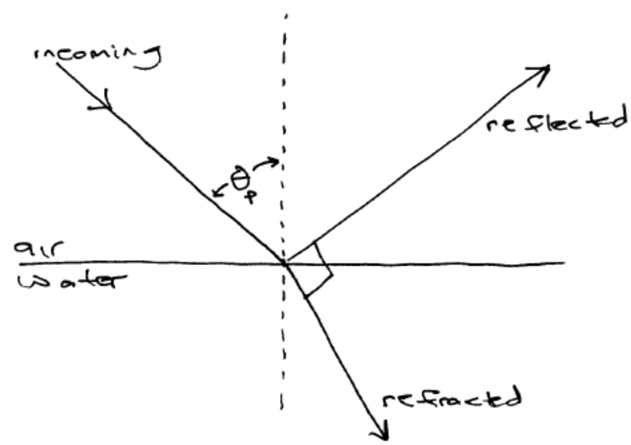
(b) Now we remove the middle filter. The light leaving the first filter is the same as before: intensity reduced by half, and this light is now polarized in the direction of that filter. When this light hits the remaining filter, we see that the angle between the light and the polarizing filter is 90^o so the intensity will be multiplied by a factor of $\cos^2 90$ which is zero. So no light passes through to the other side of that filter now.

Note what's happening here since it seems counterintuitive. Filters 1 and 3 alone would cancel all the light passing through them, but by adding another polarizing filter in between them the result is some light coming through.

It's important in these problems to take each filter one at a time and see what each does to the light falling on it, and be especially careful about what the angle ϕ means in the intensity equation. (I.e. that it the angle between the orientation of the incoming waves and the orientation of the filter it's about to interact with.)

Example 44 Consider sunlight reflecting off a water surface (such as a pool or lake). Polarizing sunglasses can reduce this glare. Where does the sun have to be in the sky for these sunglasses to be most effective?

Polarizing glasses are basically polarizing filters with their axis oriented vertically. They will be most effective when the glare represents light that has a horizontal polarization. This is exactly the situation that occurs at the Brewster angle where the angle of incidence is such that $\tan\theta = n_2/n_1$ where n_1 is the index of refraction of the material the signal is traveling through (in this case air) and n_2 is the index of refraction of the medium the waves are reflecting from.



For an air-water interface, we have $n_1 = 1$ and $n_2 = 1.33$ so $\tan\theta = n_2/n_1 = (1.33)/(1.00) = 1.33$ which yields $\theta = 53.1^o$.

From the figure, we see that angle is the usual Snell convention of measuring angles relative to the normal of the surfaces, which means here it's an angle relative to the vertical direction. The angle relative to the horizon then would be $90 - 53$ or 37^o. So, in general, when the sun is 37^o above the horizon, the reflected glare is all polarized horizontally and the vertically-oriented polarizing filter will cancel it out entirely.

Note: the time of day when that happens will vary quite a bit since it depends on the tilt of the earth, your latitude, and where the earth is in its orbit around the sun: all those affect the maximum angle the sun reaches in the sky at a given location on a given day of the year...