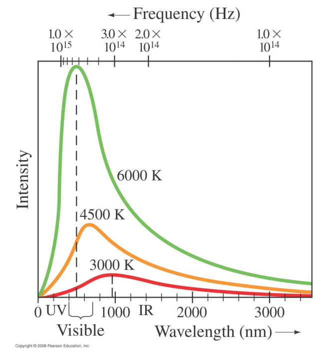
Physics 2233 : Chapter 37 Examples : Early Atomic Theory

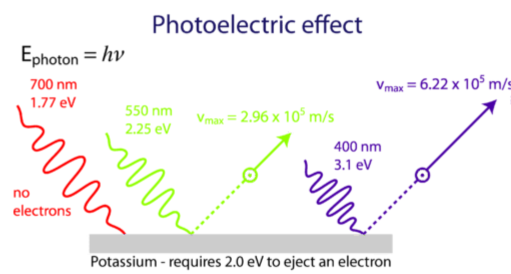
Blackbody radiation



- Photon energy: $E = hf = hc/\lambda$
- Shortcut: $E = (1240\ eV\ nm)/\lambda$ with λ in nm and E in eV
- Planck's constant: $h = 6.6261 \times 10^{-34}\ J\ s$
- Reduced Planck's constant: $\hbar = h/(2\pi) = 1.055 \times 10^{-34}\ J\ s$
- Intensity spectrum: $I(\lambda, T) = \frac{2\pi hc^2\lambda^{-5}}{e^{hc/(\lambda kT)}-1}$

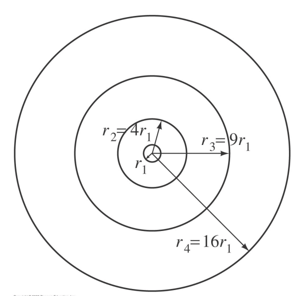
Photoelectric effect

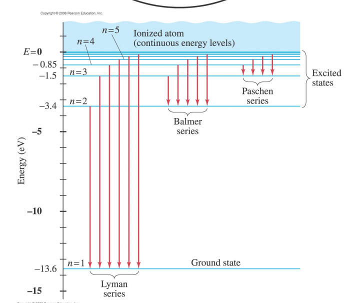
- work function W_o is how much energy is needed to remove outermost electron from the atom
- CoE: $E_\gamma = W_o + K$ where K is the kinetic energy of the ejected electron
- V_s is the 'stopping voltage' : the voltage needed to 'stop' the electron, so $eV_s = K$
- Results in: $hf = W_o + eV_s$ or $V_s = (\frac{h}{e})f - (\frac{1}{e})W_o$
- Typical W_o for most materials: couple of eV



Bohr Model of the Atom

- single electron with $q = -1e$ 'orbiting' dense nucleus with $q = +Ze$
- electron angular momentum quantized: $mvr = nh/(2\pi) = n\hbar$
- 'allowed' orbits: $r_n = \frac{n^2h^2\epsilon_o}{\pi mZe^2} = \frac{n^2}{Z}r_1$ where $r_1 = \frac{h^2\epsilon_o}{\pi me^2}$ (called the Bohr radius)
- r_1 for hydroden is $0.529 \times 10^{-10}\ m = 0.0529\ nm$
- hydrogen electron energy levels: $E_n = -(\frac{Z^2e^4m}{8\epsilon_o^2h^2})\frac{1}{n^2} = -(13.6\ eV)\frac{Z^2}{n^2}$





Example: photon energy (1)

What is the energy range (in joules and eV) of photons in the visible spectrum (i.e. between wavelengths of 390 nm and 750 nm?

$E = hc/\lambda$ so:

- Violet: $\lambda = 390\ nm$ so $E = hc/\lambda = \frac{(6.63\times10^{-34}\ J\ s)(3\times10^{8}\ m/s)}{390\times10^{-9}\ m} = 5.1 \times 10^{-19}\ J$ Converting to electron volts: $E = (5.1 \times 10^{-19}\ J) \times \frac{1\ eV}{1.602\times10^{-19}\ J} = 3.18\ eV$.
- Red: $\lambda = 750\ nm$ so $E = hc/\lambda = \frac{(6.63\times10^{-34}\ J\ s)(3\times10^{8}\ m/s)}{750\times10^{-9}\ m} = 2.652 \times 10^{-19}\ J$ Converting to electron volts: $E = (2.652 \times 10^{-19}\ J) \times \frac{1\ eV}{1.602\times10^{-19}\ J} = 1.655\ eV$.

Shortcut: with λ in nm and E in eV : $E = (1240\ eV\ nm)/\lambda$ so:

- Violet: $\lambda = 390\ nm$ so $E = (1240\ eV\ nm)/(390\ nm) = 3.18\ eV$
- Red: $\lambda = 750\ nm$ so $E = (1240\ eV\ nm)/(750\ nm) = 1.653\ eV$

(Same result to 3 significant figures anyway. The actual factor is about 1239.84 $eV\ nm$ and not just 1240 so that explains part of the slight mismatch.)

- Each photon of **visible light** then carries energy between 1.65 eV and 3.18 eV.
- X-ray photons carry energies roughly between 100 eV and 100 keV
- Gamma rays carry energies into the MeV range and beyond

Example: photon energy (2)

What wavelength photon would have the same energy as a 145 *gram* baseball moving at 27.0 m/s?

Baseball: $K = \frac{1}{2}mv^2 = (0.5)(0.145\ kg)(27\ m/s)^2 = 52.8525\ J$

$E = hf = hc/\lambda$ so $\lambda = (hc)/E = \frac{(6.6261\times10^{-34}\ J\ s)(3\times10^{8}\ m/s)}{52.8525\ J} = 3.76 \times 10^{-27}\ m$

Pretty unrealistic. Converting the energy to electron volts: $E = (52.8525\ J) \times \frac{1\ eV}{1.602\times10^{-19}\ J} = 3.3 \times 10^{20}\ eV$. The highest energy gamma ray ever detected (so far) had an energy of about 20 TeV (tera-electron-volts) which is $20 \times 10^{12}\ eV$, so the baseball has an energy about 16.5 *million* times larger.

Example: Photo-electric Effect (1)

In a photoelectric-effect experiment it is observed that no current flows unless the wavelength is less than 540 nm. (a) What is the work function of this material? (b) What is the stopping voltage required if light of wavelength 480 nm is used?

(a) No current flows until the photon energy is just enough to break an electron free from the material, that energy being called the 'work function' for the material. Here that occurs at $\lambda =$ 540 nm which represents an energy of $E = hc/\lambda = (1240\ eV\ nm)/(540\ nm) = 2.296\ eV$, so $W_o = 2.296\ eV$.

(b) The stopping voltage is related to the frequency of the light and the work function: $V_s = (\frac{h}{e})f - \frac{1}{e}W_o$. We could do this 'directly' (converting the W_o we found into joules first, converting the new wavelength into f, and so on), but let's think about this.

The incoming photon energy E_γ gets converted into the work function of the material plus the kinetic energy of the electron: $E_\gamma = W_o + K$.

The 480 nm photon has an energy of $E = (1240\ eV\ nm)/(480\ nm) = 2.5833\ eV$. When it strikes the material, the first 2.296 eV (the 'work function' for the material) is being 'used up' just to break the electron free from the material, leaving $2.5833 - 2.296 = 0.2873\ eV$ as the kinetic energy of the electron. The stopping voltage represents the electric potential energy $U = (e)(V_s)$ being just enough to cancel out the kinetic energy of the electron before it can turn into current in the circuit so here V_s must be 0.2873 $volts$.

Example: Photo-electric Effect (2)

What is the maximum kinetic energy of electrons ejected from barium ($W_o = 2.48\ eV$) when illuminated by white light with wavelengths running from $\lambda = 410\ nm$ to $\lambda = 700\ nm$?

The photons of light carry energy of $E_\gamma = hf = hc/\lambda$ and when a photon strikes the barium, the first 2.48 eV goes into breaking the electron free from a barium atom, with the remaining energy going into the kinetic energy of the electron: $E_\gamma = W_o + K$.

Note that $E \propto 1/\lambda$ so the photon with the smallest wavelength will have the most incoming energy and (once we subtract the work function from that) will yield the highest kinetic energy.

At the 410 nm end of the range, $E = (1240\ eV\ nm)/(410\ nm) = 3.024\ eV$. The first 2.48 eV is 'used up' by the work function, leaving $K = 3.024 - 2.48 = 0.5444\ eV$ as the maximum kinetic energy an electron will have.

At the 700 nm end of the range, the photons have $E = (1240\ eV\ nm)/(700\ nm) = 1.77\ eV$ which isn't enough to release an electron. Those photons will just be absorbed or reflected.

What wavelength of light in the provided range will be the first to actually cause an electron to be released? That will occur when the photon energy is just enough to overcome the work function: $E = 2.48\ eV = (1240\ eV\ nm)/\lambda$ yielding $\lambda = 500\ nm$.

Thus only photons in the range from 410 nm to 500 nm will cause electrons to be released. The rest, from 500 nm to 700 nm will just be absorbed or reflected by the material.

Example: Bohr Model (1)

How much energy is needed to ionize a hydrogen atom in the $n = 3$ state?

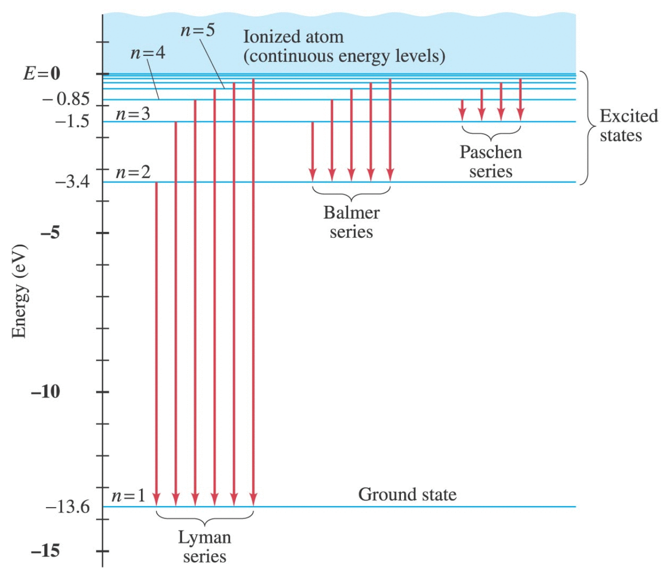
An atom with Z protons but just ONE electron in orbit has energy levels of $E_n = -(13.6\ eV)\frac{Z^2}{n^2}$

Here we're dealing with hydrogen, so $Z = 1$ and the electron is apparently in the third possible orbit so $n = 3$ and $E_3 = -(13.6\ eV)\frac{(1)^2}{(3)^2} = -1.511\ eV$. A photon will need to add 1.511 eV to break the electron from this orbit, leaving an ionized hydrogen atom (a naked proton) behind. This represents a wavelength of $E = (1240\ eV\ nm)/\lambda$ so $\lambda = (1240\ eV\ nm)/(1.511\ eV) = 820.6\ nm$ which would be in the infrared range. (We didn't cover this, but heating the hydrogen up sufficiently can also provide enough energy to release the electron. We could also fire a beam of electrons at the hydrogen and hope for a collision to occur that might transfer enough energy to break the electron free. Basically anything that could somehow transfer that much energy to the orbiting electron could do it.)

Example: Bohr Model (2)

(a) Determine the wavelength of the second Balmer line ($n = 4$ to $n = 2$ transition). (b) Determine the wavelength of the third Lyman line ($n = 4$ to $n = 1$ transition).

From the figure, $E_1 = -13.6\ eV$, $E_2 = -3.40\ eV$, $E_3 = -1.51\ eV$ and $E_4 = -0.85\ eV$.



(a) Here the electron is moving from an orbit where $E = -0.85\ eV$ to one where $E = -3.40\ eV$, so the electron energy is changing by: $\Delta E = E_{final} - E_{initial} = (-3.40\ eV) - (-0.85\ eV) = -2.55\ eV$. It does so by releasing a photon with $E = +2.55\ eV$. $E = (1240\ eV\ nm)/\lambda$ so $\lambda = (1240\ eV\ nm)/(2.55\ eV) = 486\ nm$ (blue-green).

(b) Here the electron is moving from an orbit where $E = -0.85\ eV$ to one where $E = -13.6\ eV$, so the electron energy is changing by: $\Delta E = E_{final} - E_{initial} = (-13.6\ eV) - (-0.85\ eV) = -12.75\ eV$. It does so by releasing a photon with $E = +12.75\ eV$. $E = (1240\ eV\ nm)/\lambda$ so $\lambda = (1240\ eV\ nm)/(12.755\ eV) = 97.25\ nm$ (ultraviolet; not visible).