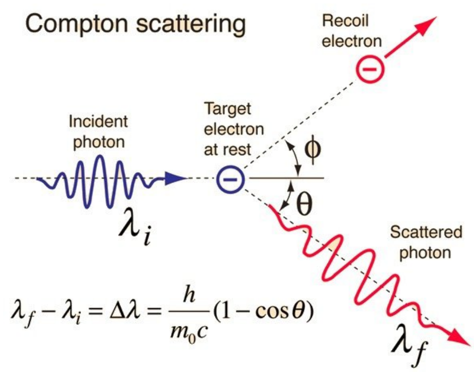
PH2233 Fox : Lecture 24
Chapter XX : (final bits) - updated 5 Dec
Photon momentum, Radiation pressure, Matter Waves

37.4 : Photon Momentum

Interestingly, EM waves (photons) also carry momentum. In a 1923 experiment, Arthur Compton shot X-rays at various materials. The X-rays carry enough energy to ignore any electron binding energy and basically scatter off the electrons, sending them in one direction and a (reduced energy) photon in another. By analyzing the energy and directions, he found that for momentum to be conserved, photons needed to carry not only an energy of $E = hf = hc/\lambda$ but also a **momentum** of $p = E/c = hf/c = h/\lambda$, **even though they have no mass**.



Example : suppose we have a ^1_1H hydrogen atom with its lone electron in the $n = 3$ orbit. The electron drops back down to the $n = 1$ orbit, releasing a photon that carries off some energy (and some momentum). The atom will need to **recoil** as a result in order to conserve momentum. But in doing so, it also now has some $K = \frac{1}{2}mv^2$ of energy: energy we assumed was all going into the photon. Let's see if this messes anything up.

We found earlier that the energy levels will be $E_n = -(13.60569\ eV)\frac{Z^2}{n^2}$ (as long as we only have a single electron in an atom), and here we have hydrogen with $Z = 1$, so: $E_n = -(13.60569\ eV)/n^2$.

In the $n = 3$ orbit, the electron has $E_3 = -13.60569/9 = -1.51219\ eV$ of energy, but when it drops down to the $n-1$ orbit, it has $E_1 = -13.60569\ eV$. The atom has lost $12.0975\ eV$ of energy, which is carried off by the photon.

(The photon will have a wavelength of $\lambda = (1239.84\ eV\ nm)/(12.0975\ eV) = 102.5\ nm$, which is in the ultraviolet part of the spectrum and not visible.)

The photon will have a momentum of $p = E/c$ with $E = (12.0975\ eV) \times \frac{1.602\times10^{-19}\ J)}{1\ eV} = 1.938 \times 10^{-18}\ J$ and then $p = E/c = 6.46 \times 10^{-27}\ kg\ m/s$.

The hydrogen atom will then **recoil** with the same momentum in order to conserve momentum. The atom's momentum will be $p = mv$ so we'll need the mass of a single hydrogen atom.

From the table of isotopes, the ^1_2H isotope of hydrogen has an atomic mass of **1.007825 u** so converting: $m = (1.007825\ u) \times \frac{1.6605\times10^{-27}\ kg}{1\ u} = 1.6735 \times 10^{-27}\ kg$.

Finally, $v = p/m = (6.46 \times 10^{-27}\ kg\ m/s)/(1.6735 \times 10^{-27}\ kg) = 3.86\ m/s$.

And what kinetic energy would the hydrogen atom have?

$K = \frac{1}{2}mv^2 = (0.5)(1.6735\times10^{-27}\ kg)(3.86\ m/s)^2 = 1.25\times10^{-26}\ J$ which is about $7.9\times10^{-8}\ eV$.

We assumed the photon carried off $12.0975\ eV$ of energy but actually it will be this tiny amount less: a number small enough we can ignore this effect entirely (whew).

31.9 : Radiation Pressure

The fact that photons carry momentum also means that momentum has to be transferred when a photon is absorbed (or reflected) by a material. **This interaction implies a force is present on the object.**

A given intensity of light (or other EM waves) represents some (large) number of photons, each carrying some momentum. If these waves (photons) are absorbed by (or reflected from) some object, a momentum transfer will occur and we can relate the intensity directly to a force (more specifically a pressure) acting on the object.

Intensity (I) is energy per area, per time. At the Earth's distance from the Sun, the intensity of sunlight is about $I = 1350\ W/m^2$ meaning that $1350\ J$ of energy fall in each square meter, each second. Multiplying the intensity by the area ΔS of our solar panels gives the total joules we're capturing each second (i.e. the watts of power the solar panels are generating). Further multiplying by a time interval Δt would yield the total amount of energy in Joules that we've collected over that time interval.

Derivation of radiation pressure:

- If we have an intensity I of light (any EM wave will do here though), then the total energy falling on an area ΔS in a time interval Δt will be $E = (I)(\Delta S)(\Delta t)$.
- How many photons does this imply? Each photon has an energy of $E = hf$ so:
- Number of photons falling on our area ΔS during the time interval Δt : $\frac{I\Delta S\Delta t}{hf}$
- Each of those photons is carrying a **momentum** of hf/c, so how much momentum did all those photons transfer to our object?
- Momentum transferred: $(\frac{I\Delta S\Delta t}{hf}) \times (\frac{hf}{c}) = \frac{I\Delta S\Delta t}{c}$
- FORCE is $\Delta p/\Delta t$ so the FORCE being applied to the target is $F = \frac{I\Delta S}{c}$
- PRESSURE is force per area, so ultimately the PRESSURE this light (EM wave) represents is $P = I/c$.

If these waves are totally REFLECTED by the target, the momentum transfer will be twice that.

- $P = \frac{F}{A} = \frac{I}{c}$ if the waves (photons) are absorbed
- $P = \frac{F}{A} = \mathbf{2}\frac{I}{c}$ if the waves (photons) are reflected

This **radiation pressure** has been used by actual space probes on purpose a few times, but **it's there whether we want it to be or not**. (See: Japan's 2010 IKAROS probe to Venus, NASA's 2010 NANOSEL-D2 mission, and the 2019 Planetary Society LightSail-2.)

Example: pressure on us due to room lights

In a typical well-lit room, the light intensity will be about 200 W/m^2. Compare the radiation pressure the lights are exerting on us to the usual atmospheric pressure we're already feeling: $1\ ATM = 14.7\ lb/in^2 = 101325\ N/m^2$. Can we 'feel' the additional pressure that will be present when the lights are on?

We have the intensity in standard metric units, so no conversions needed here.

If we completely absorbed all the photons: $P = I/c = (200)/(3 \times 10^8) = (6.7 \times 10^{-7}\ N/m^2) \times \frac{1\ ATM}{101325\ N/m^2} = 6.6 \times 10^{-12}\ ATM$

If we're covered in shiny foil and reflect all of them: $P = 2I/c = (2)(200)/(3 \times 10^8) = (13.3 \times 10^{-7}\ N/m^2) \times \frac{1\ ATM}{101325\ N/m^2} = 13.2 \times 10^{-12}\ ATM$.

We absorb some and reflect others, so we'll be somewhere in between those numbers, but either way it's completely imperceptible.

Example: highest energy gamma ray detected

As of now, the highest energy single photon ever detected had an energy of 20 TeV. That's **tera**-electron volts, or $20 \times 10^{12}\ eV$ or $(20 \times 10^{12}\ eV) \times \frac{1.602\times10^{-19}\ J}{1\ eV} = 3.204 \times 10^{-6}\ J$.

- Wavelength: $E = hf = \frac{hc}{\lambda}$ so $\lambda = \frac{hc}{E} = \frac{(6.62\times10^{-34})(3\times10^8)}{3.204\times10^{-6}} = 6.2 \times 10^{-20}\ m$
- Momentum: $p = E/c = h/\lambda = \frac{6.62\times10^{-34}}{6.2\times10^{-20}} = 1.07 \times 10^{-14}\ kg\ m/s$

For comparison, a tiny piece of dust floating in the air might have a mass on the order of $m \approx 8 \times 10^{-10}\ kg$ so how fast would that dust have to be moving to have the same momentum as this record-making photon? $p = mv$ so $v = p/m = (1.07 \times 10^{-14})/(8 \times 10^{-10}) = 1.3 \times 10^{-5}\ m/s$ (about $0.01\ mm/s$).

You wouldn't 'feel' that photon hitting you, but it would certainly leave quite a trail of destruction behind. Basically every atom (and even every nucleus) that it encounters would be split apart. Still, it's an infinitesimal fraction of the molecules in our bodies, so 'probably' nothing bad would happen.

Computers are a different story since high energy gamma and other cosmic rays can flip bits in computer memories and disks (or SSD's). One article claimed that for every gigabyte of RAM (memory) in your computer, about one bit per week will have its value flipped. Mission-critical computers need to allow for this (error-correcting memory is one option; doing every calculation in parallel twice or more is another).

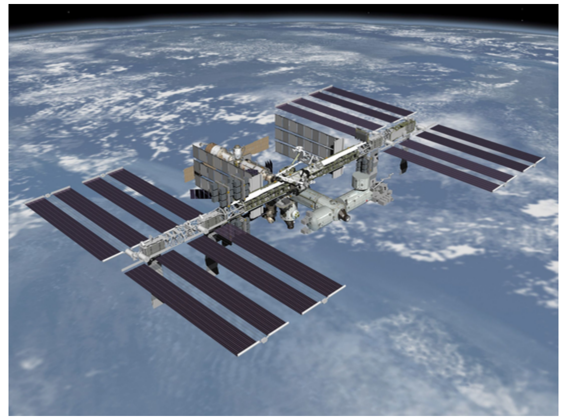
See: `https://bigthink.com/hard-science/cosmic-rays-computer-crash/`

Impact on International Space Station

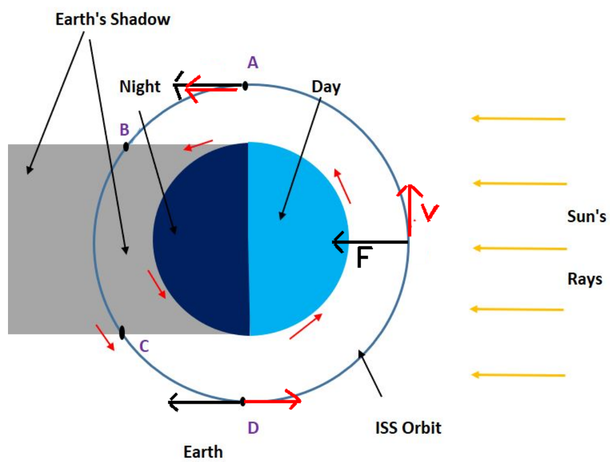
At the Earth's distance from the Sun, we have an intensity of $I = 1350\ W/m^2$ so if this light is absorbed by something it falls on, the resulting pressure will be:

$$P = \frac{1350}{3\times10^8} = 4.5 \times 10^{-6}\ N/m^2$$

If the light is reflected by that object, the pressure will be twice that.

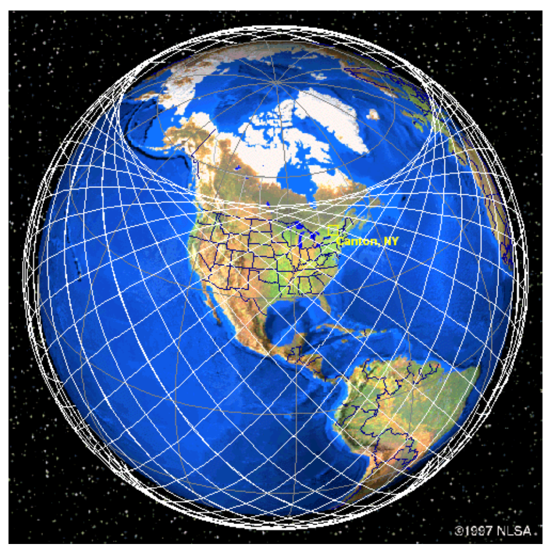


That doesn't sound like much, but the ISS presents a very large cross section to the Sun, with the solar panels alone totalling $\boxed{3500\ m^2}$ which means the Sun is exerting a force on it of just under $0.02\ N$. Given the mass of the ISS, that doesn't sound like much, but over long periods of time it's enough to require small course correction thrusters to be used to compensate.



The ISS has a mass of about $450,000\ kg$ so from $F = ma$ this would represent an acceleration of $a = F/m = (0.02\ N)/(450,000\ kg) = 4.4 \times 10^{-8}\ m/s^2$.

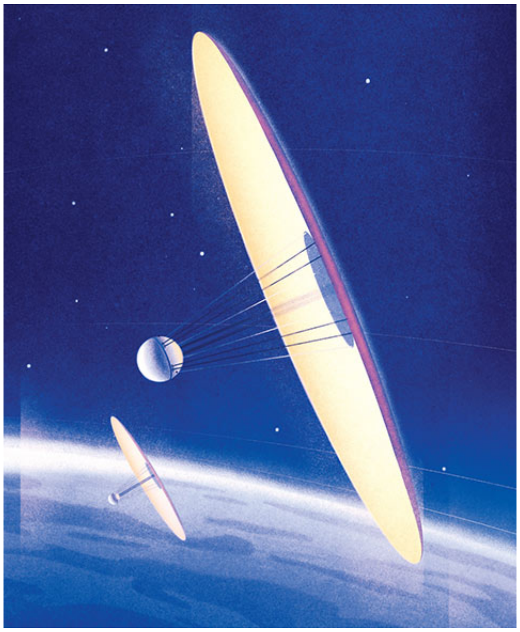
The ISS is obviously going around the Earth in a circular-ish orbit, but what distance would this tiny acceleration add up to in a single day?
$s = \frac{1}{2}at^2$ and we have $(24\ hours) \times (3600\ sec/hour) = 86,400\ s$ in a day, resulting in $\boxed{s = 164\ meters}$.



As the ISS orbits the Earth, the solar panels (usually) rotate to face the Sun so the force is constantly changing direction (and obviously this force will be gone when the Earth blocks the sunlight). It's also being affected by the rarified remnants of the atmosphere at that altitude, and as it swings around the Earth this force is pushing the ISS in different directions, but at the end of the day, radiation pressure can easily alter the location of the ISS by meters relative to where it's supposed to be, requiring regular small thrusters to fire to compensate.

Crazy(?) Idea

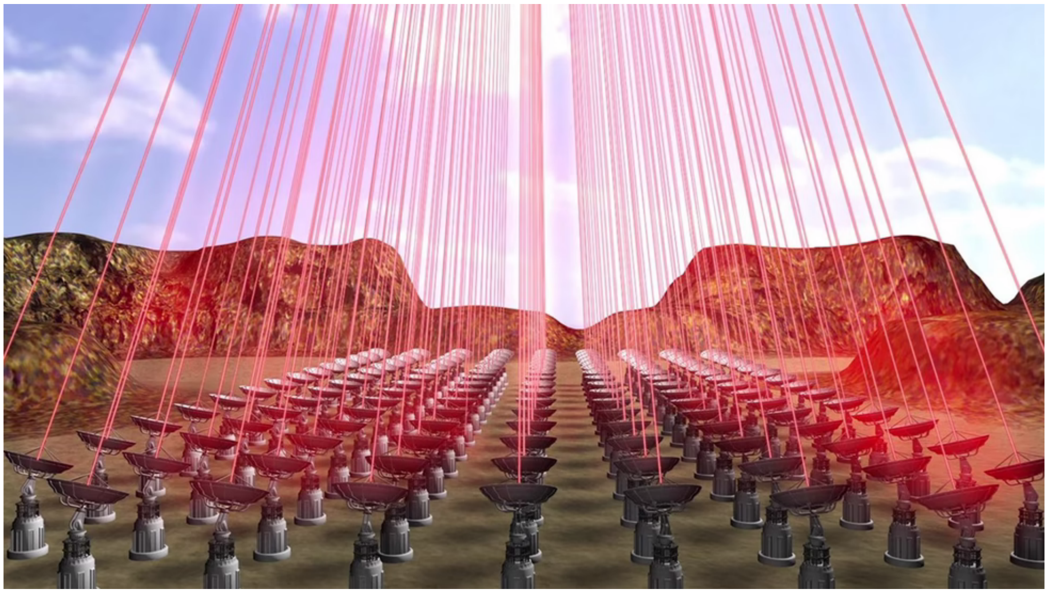
Recently, it has been proposed (Project Breakthrough-Starshot) that we launch thousands of tiny satellites with large reflective 'solar sails' and then fire a powerful laser at them to accelerate them to a significant fraction of the speed of light so they could reach a nearby star such as α Centauri in our lifetimes.



The 'light' would be produced via several powerful lasers orbiting the Earth and capable of putting out about 100 GW of power.

Such lasers could be turned towards Earth and vaporize cities, which has caused some 'concern' so this project as proposed seems unlikely, to say the least!
Placing the lasers on the ground solves one problem but creates another: firing light that intense up through the atmosphere could ionize the air and lose quite a bit of its intensity. (Presumably planes and birds wouldn't try to fly through the intense beams!)

For additional information, see the Wikipedia article on Solar Sails.



Idea: each probe would have a mass of 'a few grams' and the lasers would accelerate each to 0.15 c in 10 minutes, implying an acceleration of $a = 100\ km/s^2$ (note that's **km**, not m) or about $a \approx 10,000\ g's$. Each probe's 'sail' would be a disk about 5 m across. Are these values consistent with what we know about radiation pressure now? (If you use $m = 3\ grams$, the numbers do appear consistent.)

PH2233 : Chapter 37-7 : Matter Waves

In chapter 31 and the other bits of chapter 37 we did, we saw where **photons** carried energy of $E = hf = hc/\lambda$, but oddly they also carry momentum even though they are massless.

We're used to energy appearing in many forms, and apparently momentum can as well.

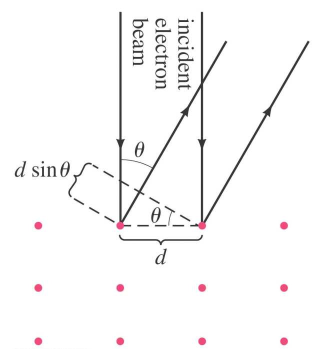
In the case of photons, their momentum is $p = E/c = hf/c = h/\lambda$ and we discussed this in the context of **radiation pressure** in chapter 31.

It had been observed that beams of electrons (known to be particles of matter) falling on materials with a regular crystal pattern can yield the same sort of intensity patterns as light passing through a diffraction grating does. In Quantum Mechanics, even matter can have wave-like properties, with a wavelength of $\boxed{\lambda = h/p}$ where now $\boxed{p = mv}$ is the normal form of momentum.

Electron Diffraction

In 'electron diffraction', beams of electrons falling on a thin film of material will 'reflect' off the atoms (separated by about 0.2 nm). If this were light, reflecting off tiny reflective surfaces (like a CDROM), we'd treat this as a diffraction-grating type problem, and would find that constructive and destructive interference would be happening with the light at various different wavelengths, creating the rainbow patterns we see.

If the electrons just behaved like normal matter, they should bounce off the atoms in random directions and create a blur of electrons reflecting back in all directions. That is not what we see, though.
With tiny particles of matter, like electrons, we actually see the same diffraction phenomenon occurring, with the electrons behaving like waves with a wavelength of $\boxed{\lambda = h/p}$ (called the **de Broglie wavelength**) where $p = mv$ is the momentum of the electron.
If we accelerate electrons across a 100 V potential, at what angles will we see strong reflections?



First, we'll need the momentum of the electrons. They'll have an energy of 100 eV after crossing this potential difference, representing an energy in joules of: $E = (100\ eV) \times \frac{1.609\times10^{-19}\ J}{1\ eV} = 1.609 \times 10^{-17}\ J$.

That represents a kinetic energy of $K = \frac{1}{2}mv^2 = 1.609 \times 10^{-17}\ J$ and using the electron mass of $m = 9.11 \times 10^{-31}\ kg$ we find $v = 5.943 \times 10^6\ m/s$ (which is far enough below the speed of light we can ignore relativity...).

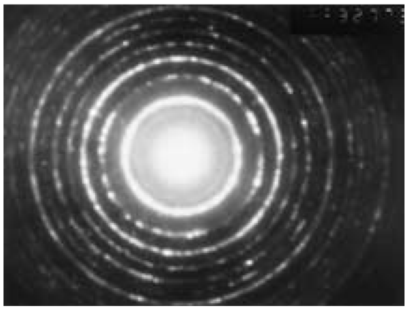
The wavelength of this electron then will be $\lambda = h/p = h/(mv) = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(5.943\times10^{6})} = 1.22 \times 10^{-10}\ m$ or about $\boxed{\lambda = 0.122\ nm}$.

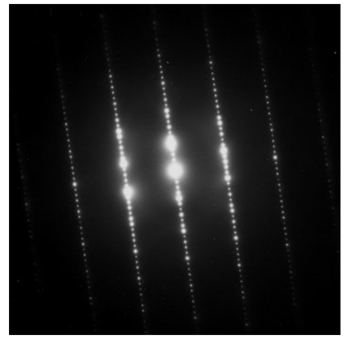
The wavelength is just a bit smaller than the spacing between the reflection points (basically the diameter of the atoms in the material) and we'll have constructive interference where $d\sin\theta = m\lambda$ so here $\sin\theta = m\lambda/d = (m)(0.122\ nm)/(0.2\ nm) = (m)(0.61)$.

$m = 0$ yields a solution, so some of the electrons will be reflected straight back. There are other solutions though, at $m = \pm1$, which yield solutions of $\theta = \pm 37.6^o$. We will also get a strong signal of electrons reflecting at these angles.

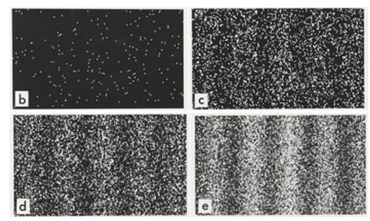
Instead of just randomly scattering off the atoms as particles would do, the electrons instead are behaving like waves with a wavelength here of 0.122 nm scattering off the atoms and interfering.

Here are a couple of random pictures from actual electron diffraction studies. The resulting interference patterns ultimately reveal details about how the atoms are arranged in the material.





Finally the classic **double slit** experiment has been done with electrons as well, and the resulting interference pattern again confirms that they're behaving more like waves than particles.



Baseball Diffraction

(a) What would the de Broglie wavelength of a 90 mph baseball be?

$v = 90\frac{miles}{hr} \times \frac{1609\ m}{1\ mile} \times \frac{1\ hr}{3600\ s} = 40.225\ m/s.$

Baseball mass is about 145 grams, or $m = 0.145\ kg$.

Momentum: $p = mv = (0.145\ kg)(40.225\ m/s) = 5.93\ kg\ m/s$.

Wavelength: $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{5.93} = 1.12 \times 10^{-34}\ m$.

That's many orders of magnitude smaller than even the size of a single proton, so we don't realistically expect to see any quantum effects on the flight of this baseball.

(b) If this baseball passes through a circular hole 10 cm across, what angular spread will this introduce?

Recall from the resolution material we covered in chapter 35 that a wave passing through a circular aperture gets spread out by an angle of $\theta = 1.22\lambda/D$. Our baseball was initially travelling in a straight line but after passing through this aperture, it's wave nature has introduced an uncertainty in it's direction of $\theta = (1.22)(1.12 \times 10^{-34}\ m)/(0.1\ m) = 1.4 \times 10^{-33}\ radian$ or about $8 \times 10^{-32}\ deg$.

If we threw the ball repeatedly and perfectly, the position where it hits the wall will technically vary by this amount. For example, 10 meters after passing through the hole, its position would be spread out (from $s = r\theta$) by about $1.4 \times 10^{-32}\ meters$ which is vastly smaller than the diameter of even a single proton.

Clearly nothing we'll notice up at OUR scale, but the **nanotechnology** field is encroaching on this quantum realm where oddball effects like this can't be ignored.