Physics 2233 : Examples from last bits covered in the course

Electromagnetic Waves (chapter 31; section 8)

In the wave model of EM phenomena, the waves represents electric and magnetic fields varying at a usually very high frequency, travelling at the speed of light in the medium.

If the intensity of the waves is I (watts per square meter), the underlying E and B field strengths in a vacuum can be found to be: $I = \frac{1}{2}\epsilon_o c E_o^2 = \frac{1}{2}\frac{c}{\mu_o}B_o^2$ (Note that $E = cB$.)

where $\epsilon_o = 8.85 \times 10^{-12} C^2/N \cdot m^2$ and $\mu_o = 4\pi \times 10^{-7} T \cdot m/A$

(If travelling through a material, ϵ_o and μ_o are replaced with the ϵ and μ of that material.)

Photons (chapter 31; section 9)

In the particle model for light and other electromagnetic phenomena, the energy is being carried by tiny massless particles called photons.

Each photon carries an energy of $E = hf = hc/\lambda$ where f is the usual frequency, λ is the wavelength, and $h = 6.63 \times 10^{-34}\ J \cdot s$ (called Planck's constant).

Each photon also carries a regular linear momentum of $p = E/c = hf/c = h/\lambda$

Since it carries momentum (even though it's massless), it can transfer this momentum to physical objects during 'collisions'. If I represents the intensity ($watts/m^2$) of the EM wave, then this transfer of momentum represents a pressure (called the **radiation pressure**) of :

- $P = I/c$ (if the photons are completely absorbed)
- $P = 2I/c$ (if the photons are perfectly reflected back)

Note: here P represents **pressure**, not power, so $P = F/A$ (force/area).

Matter Waves (chapter 37, section 7)

Borrowing from the relationship above, quantum mechanics shows that moving physical particles have an effective wavelength of $\lambda = h/p$ where $p = mv$ is the linear momentum of the object.

Particles can display wave-like phenomena (such as diffraction) as a result.

Game Lag

Electrical signals (such as information traveling through the internet) travel at nearly the speed of light. A game server is located 2000 km away. How much latency will this introduce into the game?

When I hit a key, this information must travel from my computer to the server and then the response needs to travel from the server back to my computer. The signal is thus traveling $2 \times 2000\ km$ or $4000\ km$ or $4 \times 10^6\ m$.

$d = vt$ so this signal, traveling at the speed of light, represents a time delay of $t = d/v = d/c = (4 \times 10^6\ m)/(3 \times 10^8\ m/s) = 0.0133\ s$ or about $13.3\ ms$ (NOTE: electrical signals travelling in wires travel at about $0.9c$ so using this slightly slower speed would increase the lag by about 10 percent, making it more like $15\ ms$.)

If the signal has to travel via one of the geostationary satellites ('satellite internet'), the time delay will be much worse. These satellites orbit at roughly $35,800\ km$ above the surface of the earth so the signal must travel from your computer to the satellite, from the satellite to the server, server to satellite, and finally satellite to your computer. In doing so it's traversed four times the orbital height of the satellite, or $4 \times 35,800\ km = 143,200\ km$. In this case the time delay becomes $t = d/c = (1.43 \times 10^8\ m)/(3 \times 10^8\ m/s) = 0.477\ s$ or $477\ ms$ (making multiplayer games over satellite internet pretty unpleasant). The real-world lags for these systems is typically closer to $600\ ms$.

In recent years, several companies have places large constellations of communications satellites in **low earth orbit**, so the distance from you to the nearest satellite to the server and back is considerably shorter, resulting in latencies around $25\ ms$ to $40\ ms$, making them viable for gaming (although they're wreaking havoc on ground-based astronomy...)

Radiation Pressure : Solar Sail

At the orbital distance of the earth from the sun, the sun's power represents an intensity of about 1400 W/m^2. This yields a radiation pressure that is sufficient to affect satellites with large arrays of solar cells to power the satellites. This effect is usually undesirable since it gradually pushes the satellites out of their designated orbits, but can also be exploited to push interplanetary probes.

Suppose we have a large square 'sail', **1 km** along each side, made of an extremely thin and light material. We orient the sail so that it points towards the sun, collecting as much momentum as possible. What acceleration would this provide if the overall mass of the satellite and sail is 1000 kg, and the satellite were presently located at the same distance from the sun that the earth is? How long would it take for this satellite to accelerate to 17 km/s which is roughly the fastest any interplanetary probe travels in deep space, away from the gravitational effects of the planets?

We might as well use this sail to generate the power for our spaceship, so let's assume the Sun's photons are completely absorbed by the sail.

Radiation pressure for fully absorbed energy is $P =< I > /c$ and here we have $< I >= 1400\ W/m^2$ giving us a pressure of $P = (1400)/(3 \times 10^8) = 4.67 \times 10^{-6}\ N/m^2$.

$P = F/A$ so the force on the object will be $F = PA = (4.67 \times 10^{-6})(1000\ m)^2 = 4.67\ N$.

$F = ma$ so $a = F/m = (4.67\ N)/(1000\ kg) = 0.00467\ m/s^2$.

How long will it take to reach 17.1 km/s or 17100 m/s?

$v = at$ so $t = v/a = (17100)/(0.00467) = 3.67 \times 10^6\ s$ or only about 42 $days$.

This idea has been tested on a small scale several times, and actually used in practice with the IKAROS probe to Venus in 2010.

Wikipedia has an extensive article on this concept: `https://en.wikipedia.org/wiki/Solar_sail`

MSU Campus Radio Station

WMSV operates at a frequency of 91.1 MHz with a power output of $14,000$ $watt$.

(a) What is the wavelength of this EM wave?

$v = \lambda/T = \lambda f$ so $\lambda = v/f$ with v being the speed of light in air. This is close enough to c that we can use $\lambda = c/f = (3 \times 10^8\ m/s)/(91.1 \times 10^6\ s^{-1}) = 3.293\ m$.

(b) What is the intensity at a distance of 2 km from the antenna?

The intensity I (or S) is the power per area, in this case the area being the area of a sphere with a radius of 2000 m. Technically this is the average intensity so $< I >= power/(4\pi r^2) = (14000)/(4\pi 2000^2) = 0.000278\ W/m^2$.

(c) What is the amplitude of the electric and magnetic fields in the EM wave at this distance?

The average intensity is related to the amplitude of the electric field by: $< I >= \frac{1}{2}\epsilon_o c E_o^2$ so $E_o = \sqrt{2 < I > /(\epsilon_o c)}$ or:

$$E_o = \sqrt{\frac{(2)(0.000278)}{(8.85 \times 10^{-12})(3 \times 10^8)}} = 0.458\ V/m.$$

$B_o = E_o/c = 1.52 \times 10^{-9}\ T$

(d) What is the radiation pressure at this distance for waves that are fully absorbed?

In the case of fully absorbed waves, the pressure is related to the intensity by $P =< I > /c$ so here $P = (0.000278)/(3 \times 10^8) = 9.3 \times 10^{-13}\ N/m^2$.

(e) Suppose we have a 30 cm by 30 cm piece of aluminum foil (which completely reflects the waves) hanging vertically in the presence of these EM waves. Is the radiation pressure enough to cause the sheet of foil to deflect from the vertical to any observable degree?

Aluminum foil is about 0.016 mm thick and has a density of 2.7 gm/cm^3 so this little piece of foil has a mass of $m = (2.7\ gm/cm^3)(30\ cm)(30\ cm)(0.0016\ cm) = 3.9\ gm$ or $m = 3.9 \times 10^{-3}\ kg$. The force of gravity would be $F_g = mg = 3.8 \times 10^{-2}\ N$.

The radiation pressure will result in a force of $F = (pressure)(area) = (2)(9.3 \times 10^{-13}\ N/m^2)(0.3\ m)^2 = 1.68 \times 10^{-13}\ N$. (Note we've doubled the pressure since in this case the EM waves are reflected instead of absorbed.)

That's 11 orders of magnitude smaller, so any deflection would be infinitesimal.

Microwave Oven

A microwave oven has conducting metal panels on its sides (highly protected from being touched). A very high frequency EM source generates standing EM waves in this cavity. The frequencies are chosen to be highly absorbed by molecules in food, heating them up.

From PH2223, we know that at a conductor, the component of $\vec{E}$ parallel to the conductor has to be zero. We can rig this by creating standing EM waves such that $E = 0$ at these locations. This is similar to what we did with standing waves on a string and results in the same equation: some number of half-wavelenghts has to exactly fit between the two sides: $n(\frac{\lambda}{2}) = L$ where L say is the distance between the plates on the left and right sides of the oven. Thus these standing waves must have wavelenghts of $\lambda_n = (2L)/n$. We can relate this to frequency: $c = \lambda f$ for EM waves though, so the frequencies of these standing waves will be $f_n = n\frac{c}{2L}$.

A wavelength of 12.2 cm is highly absorbed by food. What frequency does this represent?

$f = c/\lambda = (3 \times 10^8\ m/s)/(0.122\ m) = 2.46\ GHz$

That is unfortunate since old cordless phones and things like DSL modems also operate near this frequency, and using a microwave oven would disrupt those.

These are standing waves, so the (say) width of the chamber has to be a multiple of half the wavelength, or some multiple of $12.2\ cm/2 = 6.1\ cm$. (If there are plates in the top and bottom, they would also be separated by some multiple of 6.1 cm.)

The peaks of these standing waves will be 6.1 cm apart. Since the intensity depends on the square of the amplitude of the wave, this results in uneven intensity, with hot and cold zones (hence the rotating platform in many microwave ovens).

Light-bulb Photons

A 100 W lightbulb will emit light over a wide spectrum of frequencies and wavelengths, but assume it's emitting at a wavelength of exactly $\lambda = 550\ nm$.

(a) How much energy does each photon carry?

A single photon carries an energy of $E = hf = hc/\lambda$ and here $\lambda = 550\ nm = 550 \times 10^{-9}\ m$ so:

$E = (6.63 \times 10^{-34}\ J \cdot s)(3 \times 10^{8}\ m/s)/(550 \times 10^{-9}\ m) = 3.62 \times 10^{-19}\ J.$

Energies this small are often quotes in units of **electron-volts** (the energy involved in accelerating something with a charge of $1e$ across a potential of 1 $volt$, so 1 $eV = 1.609 \times 10^{-19}\ J$.

Here then, we have: $E = (3.62 \times 10^{-19}\ J) \times \frac{1\ eV}{1.609 \times 10^{-19}\ J} = 2.25\ eV$.

(This is enough energy to eject electrons from some atoms, creating a current in what is called the **photo-electric effect** which was seen in Lab 10.)

(b) How many photons/second is the lightbulb emitting?

The bulb has an intensity of 100 W or 100 J/s, so in one second the bulb emits 100 J of energy. If each photon is only carrying an energy of $3.62 \times 10^{-19}\ J$, how many (N) do we need to represent that much energy?

$(N)(3.62 \times 10^{-19}\ J) = 100\ J$ so $N = 2.76 \times 10^{20}$.

The light bulb is emitting that many photons every second.

Light Bulb Radiation Pressure

Extending the previous example, if we're standing 1 m away from the light bulb and hold out our hand, how much force should we feel? (Assume the photons are entirely reflected by your hand. That isn't true, of course, since some are reflected and make their way to our eyes so we can actually see our hand, and some are absorbed and warm up our hands, but here let's assume they're entirely reflected.)

There are two approaches we can take here: one we covered, and one we did not.

Method we covered : radiation pressure (P) for fully reflected EM waves is $P = 2I/c$, where I is the intensity at the given location. Here, we have a 100 W bulb with it's light spread around a sphere of radius 1 m, representing an area of $S = 4\pi r^2 = (4)(\pi)(1)^2 = 12.6\ m^2$.

The intensity here then is $I = (100\ W)/(12.6\ m^2) = 7.94\ W/m^2$.

The radiation pressure for fully reflected light should be $P = 2I/c = (2)(7.94)/(3 \times 10^8) = 5.29 \times 10^{-8}\ N/m^2$.

This is force per area though, so we'll need to multiply by the area of our palm. Assume it's roughly a square 7 cm (0.07 m) on each side, so the area would be about $5 \times 10^{-3}\ m^2$.

$P = F/A$ so $F = PA = (5.29 \times 10^{-8}\ N/m^2)(5 \times 10^{-3}\ m^2) = 2.64 \times 10^{-10}\ N$.

Method we did not cover : Each second, the bulb is emitting a huge number of photons, each carrying some tiny amount of momentum which is being transferred to our hand. This thus represents a change in momentum of our hand occurring in a given time interval (one second, say), which is a force (recall $F_{avg} = \Delta p/\Delta t$ from Physics I). This gives us another path to compute the force that our hand should feel, so let's try this approach:

From the first example we found a huge number of photons are emitted per second by the bulb. Each carries some momentum: $p = hf/c = h/\lambda = (6.63 \times 10^{-34}\ J \cdot s)/(550 \times 10^{-9}\ m)$ or $p = 1.205 \times 10^{-27} J\ s/m$ but joules are a measure of energy so their fundamental units are $kg\ m^2/s^2$ so working through the units we end up with just $kg\ m/s$ which is the usual metric units for momentum. $p = 1.205 \times 10^{-27}\ kg\ m/s$. That's the momentum that each photon has when it 'collides' with our hand. They're being completely reflected so the change in momentum of our hand is twice that in the opposite direction, so **each photon** is introducing a Δp of $2.4 \times 10^{-27}\ kg\ m/s$ to our hand.

How many are landing on our hand in one second?

Well, in one second, we found that the bulb emits 2.76×10^{20} photons but they're spread around in all directions, so at a distance of 1 m from the bulb, they're spread around over an area of $S = 4\pi r^2 = 12.6\ m^2$. We found above that the palm of our hand has an area of about $5 \times 10^{-3}\ m^2$ so the **fraction** of these photons that fall on our hand would be $\frac{5 \times 10^{-3}\ m^2}{12.6\ m^2}$ or about 4×10^{-4}.

Of the 2.76×10^{20} photons the bulb is emitting, only $(2.76 \times 10^{20}) \times (4 \times 10^{-4}) = 1.1 \times 10^{17}$ of them land on our hand in one second.

Each is creating a Δp on our hand of $2.4 \times 10^{-27}\ kg\ m/s$ so the total Δp on our hand (in one second) would be: $(2.4 \times 10^{-27}\ kg\ m/s) \times (1.1 \times 10^{17}) = 2.64 \times 10^{-10}\ kg\ m/s$.

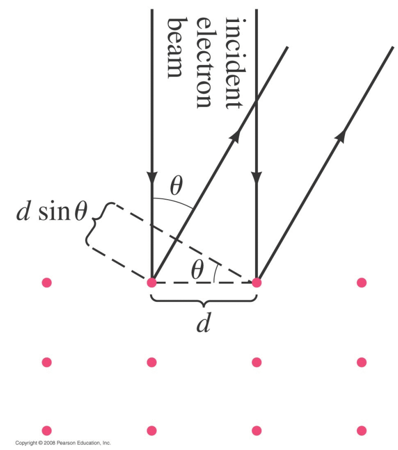
That much momentum change is happening each second and $F_{avg} = \Delta p/\Delta t$ so our hand is feeling a force of $F = 2.6 \times 10^{-10}\ kg\ m/s^2 = 2.64 \times 10^{-10}\ N$.

(Same result, and in either case way too weak to feel.)

Matter Waves: electron beam diffraction

In 'electron diffraction', beams of electrons falling on a thin film of material will 'reflect' off the atoms (separated by about 0.2 nm). If this were light, reflecting off tiny reflective surfaces (like a CDROM), we'd treat this as a diffraction-grating type problem, and would find that constructive and destructive interference would be happening with the light at various different wavelengths, creating the rainbow patterns we see.

If the electrons just behaved like normal matter, they should bounce off the atoms in random directions and create a blur of electrons reflecting back in all directions. That is not what we see, though.
With tiny particles of matter, like electrons, we actually see the same diffraction phenomenon occurring, with the electrons behaving like waves with a wavelength of $\lambda = h/p$ where $p = mv$ is the momentum of the electron.
If we accelerate electrons across a 100 V potential, at what angles will we see strong reflections?



First, we'll need the momentum of the electrons. They'll have an energy of 100 eV after crossing this potential difference, representing an energy in joules of: $E = (100\ eV) \times \frac{1.609\times10^{-19}\ J}{1\ eV} = 1.609 \times 10^{-17}\ J$.

That represents a kinetic energy of $K = \frac{1}{2}mv^2 = 1.609 \times 10^{-17}\ J$ and using the electron mass of $m = 9.11 \times 10^{-31}\ kg$ we find $v = 5.943 \times 10^6\ m/s$ (which is far enough below the speed of light we can ignore relativity).

The **wavelength** of this electron then will be $\lambda = h/p = h/(mv) = \frac{6.63\times10^{-34}}{(9.11\times10^{-31})(5.943\times10^6)} = 1.22 \times 10^{-10}\ m$ or about 0.122 nm. The electron moving this fast has wave-like properties represented by that wavelength.

(NOTE: electron microscopes exploit this to be able to 'see' scales much smaller than can be seen using visible light, which has wavelengths several thousands of times larger.)

The wavelength is just a bit smaller than the spacing between the reflection points and we'll have constructive interference where $d \sin\theta = m\lambda$ so here $\sin\theta = m\lambda/d = (m)(0.122\ nm)/(0.2\ nm) = (m)(0.61)$ so it looks like there is only one solution, at $m = 1$ (well, and $m = -1$). Some electrons will also be reflected straight back, the $m = 0$ case, but apparently we'll also see a strong signal where the electrons reflected back at $\theta = \pm 37.6^o$, instead of electrons being scattered back at random angles.