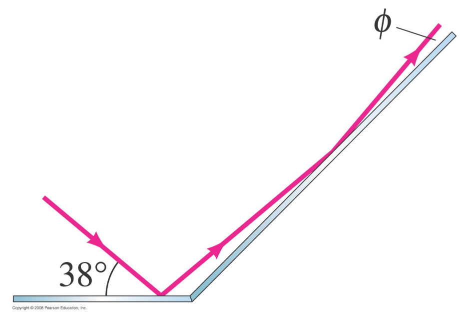
Test 2 Practice Problems
Version 3 : 29 Oct 2024
Chapters 32 and 33 (with numerical answers)

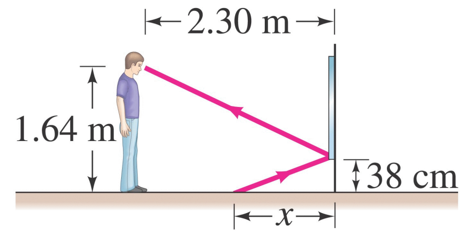
Here are a few practice problems to get started with. Some are from the homework and some are from old tests. The last page has the equation sheet that will be included with the test.

Chapter 32 : Light Reflection and Refraction

1. Two plane mirrors meet at a 135^o angle. If light rays strike one mirror at 38^o as shown in the figure, at what angle ϕ do they leave the second mirror? (Too easy for a test question, but a good 'propagating angles' exercise.)



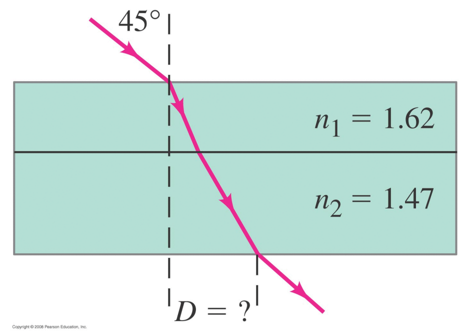
2. A person whose eyes are 1.64 m above the floor stands 2.30 m in front of a vertical plane mirror whose bottom edge is 38 cm above the floor. What is the horizontal distance x to the base of the wall supporting the mirror of the nearest point on the floor that can be seen reflected in the mirror? (Again, too easy for a test, but a good warmup for trig and propagating angle.)



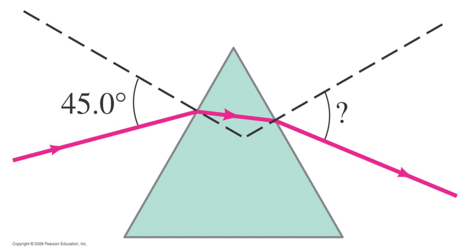
3. You look at yourself in a shiny 9.2 cm **diameter** Christmas tree ball. If your face is 25 cm away from the ball's front surface, where and how large is your image? Is it real or virtual? Upright or inverted? Draw a decent ray diagram for this scenario.

4. A 4.5 cm tall object is placed 26 cm in front of a spherical mirror. It is desired to produce a virtual image that is upright and 3.5 cm tall. (a) What type of mirror should be used (convex or concave)? (b) Where is the image located? (c) What is the focal length of the mirror? (d) What is the radius of curvature of the mirror? (e) Draw a decent ray diagram for this scenario.

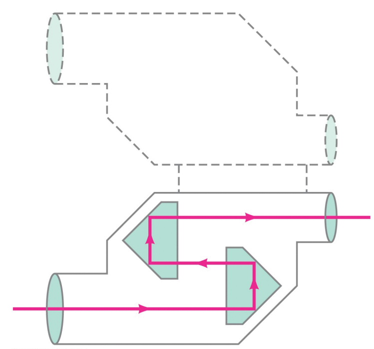
5. A light beam strikes a 2.0 cm thick piece of plastic with a refractive index of 1.62 at a 45^o angle. The plastic is on top of a 3.0 cm thick piece of glass for which $n = 1.47$. What is the distance D in the figure?



6. Light is incident on an equilateral glass prism at a 45^o angle to one face. (NOTE: those dashed lines are perpendicular to the corresponding faces of this prism.) Calculate the angle at which light emerges from the opposite face. Assume $n = 1.54$ for the prism.



7. (a) What is the minimum index of refraction for a glass prism to be used in these binoculars so that total internal reflection occurs at 45^o? (b) Will binoculars work under water ($n = 1.33$) if the index of refraction of the glass prism is 1.58? (c) What minimum n for the prism is needed is the prisms need to operate under water?



Chapter 33 : Lenses and Optical Instruments

1. We want to magnify reading material by a factor of 2.50 when the object (book, newspaper, etc) is placed 9 cm behind the lens. (Note: this is just the 'mathematical' magnification, not the apparent or angular magnification - we'll get to that!)

   What must the focal length of this lens be? What would its 'lens power' (in Diopters) be? Is this a converging or diverging lens? Where will the image form?

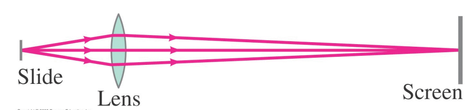
   Draw a ray diagram for this scenario.

   If we hold a book 30 cm away from our eyes (with the lens 9 cm in front of the book), what would the **apparent** (angular) magnification factor be?

2. How far from a 50 mm focal length lens must an object be placed if its image is to be magnified by 2.50 and be real? What if we want a virtual image with the same magnification?

3. A 1.75 m tall person is photographed with a camera that has a focal length of $f = 220\ mm$. The image that forms on the film (or image sensor) is 8.25 mm tall. How far away must the person have been from the camera? (HINT: this must be a **real** image since it's forming on the film or image sensor, so what sign does d_i have to be? What does that imply?)

4. In a slide or movie projector, the film acts as the object whose image is projected on a screen. If a 105 mm focal length lens is to project an image on a screen 6.50 m away, how far from the lens should the slide be? If the slide is 36 mm wide, how wide will the picture be on the screen?



5. Two 25 cm focal length converging lenses are placed 16.5 cm apart. An object is placed 35 cm in front of one lens. Where will the final image formed by the second lens be located? What is the total magnification factor?

6. A diverging lens with $f = -14\ cm$ is placed 12 cm to the right of a converging lens with a focal length of $f = +18\ cm$. An object is placed 33 cm to the left of the converging lens. (a) Where will the final image be located? (b) Will the image be real or virtual? Upright or inverted? What will the overall magnification factor be?

7. A double concave lens has surface radii of 33.4 cm and 28.8 cm. What is the focal length if $n = 1.58$? Remember the convention about the labelling and signs of the radius of curvature. The left side is side 1, the right side is side 2. Mark where each sides 'center of curvature' would be and use the 'double-convex' (bulging out on both sides) lens as our reference for R being positive. Is center 1 on the 'correct' side? Then R_1 is positive. If not, then R_1 is negative. Is center 2 on the 'correct' side? Then R_2 is positive. If not, then R_2 is negative. (So this is a separate process you have to do for each side (face) of the lens.)

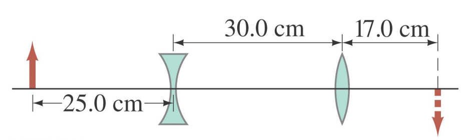
8. A prescription for a corrective lens calls for $+3.50$ $Diopters$. The lensmaker grinds the lens from a 'blank' that has $n = 1.56$ and the front surface of the lens is already convex with a radius of curvature of 30 cm. What does the radius of curvature of the other side need to be? (Does that make that side convex or concave? Sketch what this lens must look like.)

9. We need to design a lens that will operate entirely under water (i.e. both sides of the lens will be immersed in water, with $n = 1.33$). The lens will be made from glass with $n = 1.58$. An object 50 cm in front of the lens should create a real image 20 cm on the other side of the lens. If one surface of the lens is completely flat, what must the radius of curvature of the other side be? (Does that make that side convex or concave? Sketch what this lens must look like.)

10. A person struggles to read by holding a book at arm's length, a distance of 55 cm away. What power of reading glasses should be prescribed for her, assuming they will be placed 2.0 cm from the eye and she wants to read at the 'normal' near-point of 25 cm? (HINT: the goal here is to be able to hold the book 25 cm **from their eye** and create an image 55 cm **from their eye**, but d_i and d_o in the lens equation are measured **relative to the lens**, not the eye, so what are the actual d_i and d_o values needed here?)

11. A person has a far point of just 14 cm but wants to be able to see objects 'far away' clearly (like when driving, or watching a movie in a theater). What power glasses would allow this if the glasses will be located 2 cm from their eye? What power contact lenses, placed right on their eyes, would the person need?

12. A small object is 25 cm from a diverging lens as shown in the figure. A converging lens with a focal length of $f = 12$ cm is 30 cm to the right of the diverging lens. The two-lens system forms a **real**, **inverted** image 17 cm to the right of the converging lens. What must be the focal length of the diverging lens?



(We'll do this one in class as an example. HINT: work backwards here. We know where the 2nd lens is forming an image, and we know its focal length, so that tells us where 'object' must have been located. That 'object' for the 2nd lens is actually the **image** location for the first lens. That means we know where the first lens is forming its image, and we know where the original object was located, so that tells us what the first lens's focal length must be.)

Light reflection and refraction

- Spherical Mirror : $f = \frac{1}{2}R$ (Concave mirror $R > 0$; convex $R < 0$)
- Mirror Equation : $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ Variants: $f = \frac{d_o d_i}{d_o + d_i}$ $d_o = \frac{d_i f}{d_i - f}$ $d_i = \frac{d_o f}{d_o - f}$
- Magnification : $m = h_i/h_o = -d_i/d_o$

Indices of Refraction	
Material	$n = c/v$
Vacuum	1.0000...
Air (STP)	1.0003
Water	1.33
Ethyl alcohol	1.36
Glass (fused quartz)	1.46
Glass (crown glass)	1.52
Glass (light flint)	1.58
Lucite, plexiglas	1.51
Sodium chloride	1.52
Diamond	2.42

Snell's Law

- general : $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$
- EM waves : $n \equiv c/v$ so $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Total Internal Reflection (ray moving from higher n to lower n) $\sin\theta_c = n_{lower}/n_{higher}$

Lenses and Optical Instruments

Lens Equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ Variants: $f = \frac{d_o d_i}{d_o + d_i}$ $d_o = \frac{d_i f}{d_i - f}$ $d_i = \frac{d_o f}{d_o - f}$

Magnification $m = h_i/h_o = -d_i/d_o$

Lens power (diopters) : $P = 1/f$ (with f measured in meters)

Lensmaker's equation : lens material n surrounding material n_o

- $\frac{1}{f} = \frac{n - n_o}{n_o}(\frac{1}{R_1} + \frac{1}{R_2})$
- in air ($n_o = 1.000..$) : $\frac{1}{f} = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$

Angular size : $\theta \approx (size)/(distance)$ (from $s = r\theta$, with θ in radians)

Apparent, or angular magnification : $M = \theta_{image}/\theta_{object}$

Random bits

$v = \lambda/T = \lambda f = \omega/k$ $k = 2\pi/\lambda$ $\omega = 2\pi/T$

Speed of light (in vacuum) : $3 \times 10^8\ m/s$
Speed of sound in air (STP) : 343 m/s
Speed of sound in pure water (STP) : 1500 m/s

NOTE: you'll need to know about ray diagrams and sign rules. See book.

Chapter 32 : Light Reflection and Refraction (answers)

1. **Two mirrors at angle** : final exit angle is $\phi = 7^o$
2. **Person in front of mirror** : $x = 0.69\ m$ (Draw a horizontal line that passes through the bottom of the mirror and note the two similar triangles present.)
3. **Christmas tree ball** : $f = R/2$ and here $|R| = 4.6\ cm$ and convex, so $R = +4.6\ cm$; $f = R/2 = +2.3\ cm$. Image: $d_i = -2.1\ cm$, $m = +0.084$ (upright and virtual) (CORRECTED : original version had 0.84).
4. **Object and spherical mirror design** : Virtual so $d_i < 0$ which means $m = -d_i/d_o$ will be positive: $m = h_i/h_o = 3.5/4.5 = +0.777...$; leads to $d_i = -20.222..\ cm$ and $f = -91\ cm$ (exact value; negative, so convex). $f = R/2$ so $R = -182\ cm$ (exactly).
5. **Two layers of glass** : Use Snell's law to find the angles at each interface; find the length of the 'base' of each triangle formed in the two layers. Overall $D = 0.9702795 + 1.646018 = 2.6163\ cm$.
6. **Equilateral triangle Prism** : Snell's law at each interface, with a bit of angle-propagation to move information from the left to the right side. 'Equilateral triangle' so known 60^o angle at each of the three corners of the triangle. Sum of interior angles in a triangle is 180^o; for a quadrilateral it's 360^o.
   Exit angle (relative to normal) will be 56.22479^o.
7. **Porro prism in air and water** : (a) In air, will need $n > 1.4142$. (b) Under water, the surrounding material has $n = 1.33$ and Snell's law yields a viable refracted ray solution, so the prism won't work under water. (c) In order to make it work, n_{glass} will need to be 1.881.. or higher.

Chapter 33 : Lenses and Optical Instruments (answers)

1. **Book magnifier** : We'll want the image to be upright, so $m = h_i/h_o$ will be positive, implying that d_i will be negative (a virtual image). Using $m = +2.5$, find $d_i = -22.50\ cm$, leading to $f = +15.0\ cm = +0.15\ m$ (exact) so $P = 1/f = +6.67\ D$. $f > 0$ so a converting lens. Image forms 22.5 cm behind the lens (same side as object).

   Book held 30 cm from eye (which means the lens is 21 cm from the eye): $\theta_{obj} = h_o/30$ and $\theta_{img} = (size\ of\ image)/(distance\ to\ image) = (2.50)(h_o)/(\mathbf{21} + 22.5) = h_o/17.4$. Angular magnification: $M = \theta_{img}/\theta_{obj} = 1.724$. (Still magnified, but definitely not 2.5 times larger.)
2. **Lens creating real vs virtual image** : Real image case: real, so $d_i > 0$ which means $m = -d_i/d_o$ will be negative, so $m = -2.50$ (inverted image) and $d_i = -md_o = +2.50d_o$. Use lens equation with that substitution to find $d_o = 1.4f = 70\ mm$ (2 cm beyond the focal length).
   Virtual image case: virtual, so $d_i < 0$ meaning $m = +2.50$ (upright image) so $d_i = -2.50d_o$. This time, lens equation yields $d_o = 0.60f = 30\ mm$ (2 cm inside the focal length).
3. **Person being photographed** : Real image required here, so $d_i > 0$ which will make $m < 0$ so here $m = h_i/h_o = -(0.825\ cm)/(175\ cm) = -0.004714286$. $m = -d_i/d_o$ so $d_o = -d_i/m = +212.1212..d_i$. Making that substition in the lens equation and using $f = 22\ cm$: $d_i = (\frac{213.1212...}{212.1212...})(22\ cm) = 22.1037\ cm$ and $d_o = 4689\ cm$ or $46.89\ m$ away from the lens.

4. **Slide Projector** : (I've converted everything to centimeters here.) $f = 10.5\ cm$ and $d_i = +650\ cm$ (real image) which yields $d_o = 10.672\ cm$ (just $1.72\ mm$ outside the focal length of the lens). The slide has $h_o = 3.6\ cm$ and $m = h_i/h_o$ but also $m = -d_i/d_o = -650/10.672 = -60.907$ so ultimately $h_i = (-)219.3\ cm$ or $2.19\ m$ (and inverted relative to the slide).

5. **Two Converging Lenses** : First lens would create an image at $d_i = 87.5\ cm$ with $m_1 = -2.50$. Lens 2 treats this as an object at $d_o = -(87.5 - 16.5) = -71\ cm$, creating a final image at $d_o = +18.49\ cm$ with $m_2 = +0.260422...$
   Final image then is $18.49\ cm$ past the 2nd lens (real), with $m = m_1 m_2 = -0.6511$ (so inverted).

6. **Combination of converting and diverging lenses** : Order here is: object, converging lens, diverging lens (eye further over to the right).
   First (converting with $f = +18\ cm$) lens would create an image at $d_i = +39.6\ cm$ with $m_1 = -1.20$
   Second (diverging with $f = -14\ cm$) lens treat that as an object at $d_o = -(39.6 - 12) = -27.6\ cm$ creating the final image at $d_i = -28.4\ cm$ (that far to the left of the 2nd lens; which would be $16.4\ cm$ to the left of the original lens; either way a virtual image), with $m = -1.029$ so the overall magnification would be $(-1.20)(-1.029) = +1.235$. Virtual; upright; slightly magnified.

7. **Lensmaker's equation** : Both sides of this lens are curved the 'wrong way' relative to the reference picture so $R_1 = -33.4\ cm$ and $R_2 = -28.8\ cm$. Lensmaker's equation yields $f = -26.66\ cm$ (a diverging lens).

8. **Lensmaker's equation : designing a lens** : $P = 1/f$ with f measured in meters, so let's use meters throughout here. The Lensmaker's equation involved $1/f$ so basically: $P = (n-1)(\frac{1}{R_1} + \frac{1}{R_2})$ with $P = 3.50$ and $R_1 = +0.30\ m$ (that side is convex - bulging out like the reference picture for signs, so a positive radius of curvature). Solving, $R_2 = 0.34286\ m$ or $34.286\ cm$ (positive, so also convex: bulging out on this side too).

9. **Lensmaker's equation : underwater lens** : Underwater lens: $d_o = +50\ cm$ is creating a real image at $d_i = +20\ cm$ so lens equation yields $f = 14.2857\ cm$ for this lens. For the Lensmaker's equation, we have $n = 1.58$ and $n_o = 1.33$ and we also know that one side is flat, so let's say R_2 is infinity. Solving, we find $R_1 = +2.685\ cm$ (must be a small lens to have that much curvature and still be relatively thin).

10. **Far-sighted correction** : Eyeglass version : Object is 25 cm from the eye, so 23 cm from the lens ($d_o = +23\ cm$). Image will be virtual, 55 cm from the eye or 53 cm behind the lens ($d_i = -53\ cm$). Lens equation yields $f = +40.633\ cm = +0.40633\ m$ so $P = 1/f = +2.46\ D$.
    Contact lens version : skip the 2 cm correction step, so here we have $d_o = +25\ cm$ and $d_i = -55\ cm$ yielding $f = +45.833\ cm = 0.45833\ m$ so $P = 1/f = +2.18\ D$.

11. **Near-sighted correction** : Eye glass version : object is 'far away' (basically $d_o = \infty$) and we need to create an image $14\ cm$ from their eye. The glasses will be $2\ cm$ from their eye, so this will be a virtual image $12\ cm$ behind the lens: $d_i = -12\ cm$. Lens equation yields a needed lens focal length of $f = -12\ cm = -0.12\ m$ or $P = 1/f = -8.33\ D$.
    Contact lens version: same argument but without the 2 cm adjustment; leads to $f = -14\ cm = -0.14\ m$ or $P = -7.14\ D$. (Enough different they'd need different prescriptions for glasses vs contacts.)

12. **Two lenses** : Work backwards here. Looking at the converging lens, it has focal length of $f = +12\ cm$ and is producing an image at $d_i = +17\ cm$. It's 'object' must have been located (using the

lens equation) at $d_o = 40.8\ cm$. That came out positive, so it's on the correct side of that lens, i.e. $40.8\ cm$ to the left of the 2nd lens. That puts it $40.8 - 30 = 10.8\ cm$ to the left of the first lens. We need the first lens to take an object at $d_o = 25\ cm$ and create an image that's $10.8\ cm$ to it's left (i.e. on the 'wrong' side, so the first lens is creating a virtual image at $d_i = -10.8\ cm$). Using these values, we find $f = -19\ cm$ for the first lens.