Test 4 Practice Problems (with answers)
Bits of Chapters 31, 37 and 41
Version 4: 10 Dec 2024
(updated XX-2 answer to give photons/year instead of per second)

Here are a few practice problems to get started with. Some are from the homework and some are from old tests. The last page has the equation sheet that will be included with the test, along with relevant portions of the atomic mass tables.

Chapter 37 : Photoelectric (37-2), Bohr Atom (37-9,37-11)

1. (Done in class, so an easy warm-up question). What is the energy of photons (in joules and eV) of photons in the visible spectrum of 390 nm to 750 nm?

2. About 0.1 eV is required to break the 'hydrogen bond' in a protein molecule. Calculate the minimum frequency and maximum wavelength of a photon that can accomplish this.

3. The work function for barium is $W_o = 2.48\ eV$. When illuminated by white light (meaning the photons have wavelengths between $\lambda = 390\ nm$ and $750\ nm$) electrons with varying kinetic energies will be emitted.
   (a) What is the maximum kinetic energy of these ejected electrons? (Hint: to give the electron a maximum K, we need to hit this barium atom with the highest photon energy. Which of those two wavelengths would represent the higher energy photon?)
   (b) What is the longest wavelength that will still cause electrons to be ejected?

4. In a photoelectric-effect experiment it is observed that no current flows unless the wavelength is less than 650 nm (red). (a) What must the work function for the material be? (b) What stopping voltage will be required if we shine 450 nm (blue) light on the material?

5. How much energy is needed to ionize a hydrogen atom that is already in the $n = 3$ state? If this energy is being provided by a photon hitting the hydrogen atom, what wavelength photon would this represent? (Would it be visible?)

6. Suppose the ${}^{1}_{1}H$ atom in the previous problem (with its lone electron in the $n = 3$ state) is at rest. The electron drops down to the $n = 1$ state by emitting a photon in the $-X$ direction.
   (a) What wavelength photon will be emitted? (Will it be in the visible range?)
   (b) Momentum has to be conserved in this process, so what would the recoil velocity of the hydrogen atom be? (This part is from a section in Ch 31 that we'll do on the last day.)

Chapter 41 : Radioactivity (sections 2 through 6, 8) and attenuation/shielding

1. What is the maximum kinetic energy (in eV) of the electron emitted in the β decay of a neutron? At what speed (in m/s) would the electron be moving? (You'll find something faster than light, which isn't possible. In reality most of the energy is carried off by the neutrino.)

2. Most stable isotopes have more neutrons than protons in the nucleus. Carbon-11 has 6 protons but only 5 neutrons and is not stable. One possible decay path would be for the nucleus to eject a proton (one of the electrons would wander off also as a side effect), leaving behind a nucleus with 5 protons and 5 neutrons, which is a stable isotop of boron. Is this decay path possible for carbon-11? ${}^{11}_{6}C \rightarrow {}^{10}_{5}B + p + e$

3. Potassium-40 (${}^{40}_{19}K$) is radioactive, but only one of these decay paths occurs. Determine which one of these is 'allowed' and determine the energy released in the decay.

   (a) eject a neutron and become stable potassium-39 : ${}^{40}_{19}K \rightarrow {}^{39}_{19}K + n$

   (b) emit an electron (a neutron basically converting into a proton) becoming a stable isotope of calcium: ${}^{40}_{19}K \rightarrow {}^{40}_{20}Ca + \beta^-$

4. Suppose we have a 385 gram sample of pure natural carbon. The fraction of radioactive ${}^{14}_{6}C$ in this sample is 1.3×10^{-12}. What would the activity of this sample be, due to the radioactive decay of this isotope? (The atomic weight of generic 'carbon', which includes isotopes other than carbon-12, is 12.011 u, and the half-life of the ${}^{14}_{6}C$ isotope of carbon is 5730 years.)

5. Calculate the activity of a **pure** 8.7 μg sample of ${}^{32}_{15}P$ which has a half-life of 1.23×10^6 sec. **Pure** means it's the **only** isotope present. (In reality, it would probably just explode given all the heat the decaying atoms will generate...)

6. Suppose we have a (very tiny) sample of **pure** ${}^{7}_{4}Be$ (an isotope of beryllium) which has a half-life of $t_{1/2} = 53.22$ $days$. If we measure an activity of $200,000$ s^{-1}, what must the mass of this sample be? (Remember: it's all just that one isotope.)

7. We are standing 2 m from a radioactive sample with an activity of 0.2 μCi (where 1 Curie represents an activity of 3.7×10^{10} decays/second). (For the first three parts, ignore absorption.)

   (a) What would the intensity be at this distance? (I.e., how many particles per m^2 per second?).

   (b) How many clicks/sec would a Geiger counter pointed at the sample record, if the surface area of the business-end of the counter is 5 cm^2?

   (c) How far away would we need to move to reduce the intensity by a factor of 100?

   (d) We can also stay where we are and reduce the intensity by wearing some shielding material like lead. If we use a shielding material that is 1 mm thick, what would it's linear attenuation coefficient (and half-value layer thickness) need to be?

8. The attenuation coefficient for lead when exposed to the 4.3 MeV α radiation emitted by $^{238}_{92}U$ is about 265 cm^{-1}. What thickness of lead would be needed to reduce the radioactive intensity by a factor of 1000?

9. $^{192}_{77}Ir$ (iridium-192) decays by emitting a 296 keV gamma ray. The HVL ($x_{1/2}$) for lead for this γ energy is about 4.8 mm. What thickness of lead shielding would reduce the intensity by a factor of 100?

Chapter XX Topics: photon momentum, radiation pressure, matter waves

(Material from chapter 31 section 9, and chapter 37 sections 2, 3 and 7.)

1. A microwave oven, operating at 830 W produces electromagnetic radiation in the microwave range (hence the name of these gadgets) with a wavelength of $\lambda = 12.2\ cm$. How many of these microwave photons is the oven producing each second?

2. Sunlight reaching the top of the Earth's atmosphere has an intensity of about 1350 W/m^2. Estimate how many photons per square meter per second this represents. Take the average wavelength to be 550 nm. Roughly how many photons per **year** does the Sun emits? (The Sun is about $149 \times 10^6\ km$ from the Earth.)

3. A beam of red laser light (with $\lambda = 633\ nm$) hits a black wall and is fully absorbed. If this light exerts a force of $F = 7.5\ nN$ (that's nano-Newtons) on the wall: (a) how many photons per second are hitting the wall? (Hint: we don't know the area here, so think of this in terms of momentum. Remember $F = \Delta p/\Delta t$ so in one second, how much momentum was transferred to the wall? How much momentum does each photon carry?)
   (b) How much power (in Watts) must this laser be putting out?

4. When dealing with 'visible light' we often pick a wavelength of $\lambda = 550\ nm$ to represent it. How fast would an electron need to be moving to have that wavelength? What energy (in eV) does that represent? (I.e. across what voltage would we need to accelerate the electron to achieve this wavelength?)

5. What wavelength photon would have the same energy as a 145 $gram$ baseball moving at 27.0 m/s? Going the other way, what would the de Broglie wavelength of the baseball be?

6. (Silly example) A car with a mass of 1400 kg approaches a freeway underpass that is 12 m across. At what speed must the car be moving in order for it to have a wavelength such that is might somehow 'diffract' after passing through this 'single slit'? (Remember, from single-slit diffraction we had $\sin\theta = m\lambda/D$ so diffraction becomes an issue when $\lambda < D$, so at what speed would the car have a wavelength of, say, 12 m?)
   (You'll find a ludicrously small velocity here that isn't possible. Each atom in the vehicle will be randomly moving around far faster than that just due to the ambient temperature.)

7. Neutrons can be used in diffraction experiments to probe the lattice structure of crystalline solids. Since the neutron's wavelength needs to be on the order of the spacing between the individual atoms in the lattice (about 0.3 nm), what should the speed of the neutrons be? What would their energy be (in eV)?

Chapter 37 : Photoelectric effect, Bohr Atom : Answers

1. visible light energy : $5.09 \times 10^{-19}\ J$ to $2.65 \times 10^{-19}\ J$ or $3.18\ eV$ to $1.65\ eV$
2. $0.1\ eV$ photon : $\lambda = 12398.42\ nm$, $f = 2.418 \times 10^{13}\ Hz$
3. barium work function : $390\ nm$ photon has $E = 3.179\ eV$, yielding max electron $K = 0.6991\ eV$; max wavelength $499.9\ nm$ (just enough to overcome W).
4. photoelectric experiment: (a) $650\ nm$ implies W must be $1.907\ eV$; (b) $0.84776\ volts$ $450\ nm$ has $E = 2.7552\ eV$, leaving $0.84776\ eV$ for K of electron.
5. $1.512\ eV$ needed to remove electron in the $n = 3$ orbit; implies $\lambda = 820.14\ nm$ (infrared; not visible)
6. recoil : released photon $\lambda = 102.518\ nm$; photon momentum $6.463 \times 10^{-27}\ kg\ m/s$; implies atom recoil velocity of $3.862\ m/s$ (and energy of $K = \frac{1}{2}mv^2 = 7.79 \times 10^{-8}\ eV$ (ignorable); did this one in class)

Chapter 41 : Radioactivity and Attenuation/Shielding : Answers

1. Neutron decay : mass difference 8.4×10^{-4} u = 0.782 MeV = 1.2536×10^{-13} J; implies electron speed of 5.25×10^{8} m/s (faster than light). (In reality the neutrino carries off most of the energy instead of the electron.)

2. ${}^{11}_{6}C$ decay : $m_{before} = 11.0114326$ u; $m_{parts} = 11.0207615$ u (higher, so not possible)

3. ${}^{40}_{19}K$ decay paths
   (a) eject neutron: $m_{before} = 39.9639969$ u; $m_{parts} = 39.9723701$ u (higher, so not possible)
   (b) beta decay: $m_{before} = 39.9639969$ u; $m_{parts} = 39.9631386$ u (lower, so possible; energy released 0.79951 MeV

4. carbon activity : 96.186 s^{-1}
   number of generic 'carbon' atoms in sample: 1.930×10^{25};
   number of ${}^{14}_{6}C$ atoms in sample: 2.509×10^{13};
   decay constant: $\lambda = 3.83295609 \times 10^{-12}$ s^{-1}

5. pure ${}^{32}_{15}P$ activity : 9.234×10^{10} s^{-1}
   number of atom present: 1.6386×10^{17};
   decay constant: $\lambda = 5.635 \times 10^{-7}$ s^{-1}

6. pure ${}^{7}_{4}Be$: mass 9.31×10^{12} u or 1.546×10^{-14} kg
   given activity and decay constant of $\lambda = 1.5074 \times 10^{-7}$ s^{-1} implies $N = 1.32676 \times 10^{12}$ atoms present.

7. (a) intensity: 147.218 s^{-1} m^{-2};
   (b) geiger counter: 0.0736 $clicks/sec$ (about 4.4 $clicks/minute$);
   (c) distance needed: 20 m (10X farther away: $1/r^2$ effect);
   (d) μ needed: 4605.2 m^{-1} or 4.6052 mm^{-1}; HVL here is 0.0001505 m or 0.1505 mm

8. U-238 α shielding needed : 0.0261 cm = 0.261 mm (very thin layer)

9. Iridium γ shielding needed : 31.891 mm = 3.1891 cm = 0.031891 m (pretty thick)

Chapter XX : Photon momentum, radiation pressure, matter waves : Answers

1. microwave oven : 5.0975×10^{26} $photons/second$
   $830\ W = 830\ J/s$ being produced by the oven, and energy of each photon $1.628 \times 10^{-24}\ J$ or $1.0163 \times 10^{-5}\ eV$;

2. 3.29×10^{52} $photons/year$ (1.04×10^{45} $photons/sec$)
   energy per photon: $2.254\ eV = 3.6117 \times 10^{-19}\ J$
   intensity: $1350\ W/m^2 = 3.7378 \times 10^{21}\ photons\ s^{-1}\ m^{-2}$
   area will be a sphere with r being distance from Sun to Earth to get the total photons/second, then multiply by seconds in a year to get the final photons/year result.

3. red laser hitting wall: (a) photons needed: 7.1649×10^{18} $photons/sec$; (b) power: $2.24\ W$
   total Δp in one second: $7.5 \times 10^{-9}\ kg\ m/s$
   momentum of each photon: $1.04677 \times 10^{-27}\ kg\ m/s$
   photons needed: 7.1649×10^{18} $photons/sec$
   energy of each photon: $1.95868\ eV = 3.138 \times 10^{-19}\ J$

4. voltage needed: 4.972×10^{-6} $volts$
   desired wavelength 550 nm; required momentum: $1.2047 \times 10^{-27}\ kg\ m/s$
   electron speed implied: $1322.4\ m/s$; electron KE $7.96596 \times 10^{-25}\ J = 4.972 \times 10^{-6}\ eV$

5. Baseball $K = 52.852\ J$; photon needed: $\lambda = 3.7584 \times 10^{-27}\ m = 3.7584 \times 10^{-18}\ nm$
   baseball wavelength $\lambda = 1.692 \times 10^{-34}\ m = 1.6925 \times 10^{-25}\ nm$

6. car speed to diffract. Required $\lambda = 12\ m$
   implies $p = 5.52 \times 10^{-35}\ kg\ m/s$, so $v = 3.944 \times 10^{-38}\ m/s$

7. neutron diffraction: $\lambda = 0.3\ nm = 3 \times 10^{-10}\ m$
   requires $p = 2.2087 \times 10^{-24}\ kg\ m/s$; implies $v = 1318.71\ m/s$
   so $E = \frac{1}{2}mv^2 = 1.456 \times 10^{-21}\ J = 0.00909\ eV$

Important Units and Constants
Atomic weight (periodic table): *grams/mole*
$\mathbf{1\ amu = 1\ u = 1\ Da = 1.6605 \times 10^{-27}\ kg = 931.494893\ MeV/c^2}$
Avogadro's number: $N_a = 6.022 \times 10^{23}$ (1 mole)
Energy: $\mathbf{1\ eV = 1.602 \times 10^{-19}\ J}$ $\quad K = \frac{1}{2}mv^2$
Planck's constant: $h = 6.626 \times 10^{-34}\ J \cdot s = 4.1357 \times 10^{-15}\ eV \cdot s$
Charge unit: $e = 1.602 \times 10^{-19}\ C$
Electron mass $m_e = 9.11 \times 10^{-31}\ kg$
Permittivity: $\epsilon_o = 8.854 \times 10^{-12}\ C^2/(N\ m^2)$
Speed of light in vacuum: $v = c = 2.998 \times 10^8\ m/s$ $\quad$ ($c = 3 \times 10^8\ m/s$ is good enough)
Nanometer: $1\ nm\ =\ 10^{-9}\ m$

Bohr atom (Z protons, just 1 electron) :
$r_n = \frac{n^2 h^2 \epsilon_o}{\pi m_e Z e^2} = \frac{n^2}{Z} r_1$ where $r_1 = \frac{h^2 \epsilon_o}{\pi m_e e^2} = 0.0529\ nm$ (Bohr radius)
$E_n = -(\frac{e^4 m_e}{8\epsilon_o^2 h^2})\frac{Z^2}{n^2}$ (in J) $\quad E_n = -13.60569 Z^2/n^2$ (in eV)

Photoelectric effect : $E_\gamma = W_o + K$, stopping voltage: $K = (e)(V_s)$

Photon energy : $E = hf = hc/\lambda$ $\quad$ if λ in nm and E in eV then $E = (1239.84\ eV \cdot nm)/\lambda$
Photon momentum : $p = E/c = hf/c = h/\lambda$ $\quad$ Particle momentum: $p = mv$
Radiation Pressure : $P = F/A$ $\quad P = I/c$ if absorbed; $P = 2I/c$ if 100% reflected
Intensity in this context refers to power/area.

Radioactive decay : $N(t) = N_o e^{-\lambda t}$ $\quad t_{1/2} = ln(2)/\lambda$
Activity : $R(t) = |dN/dt| = \lambda N(t) = R_o e^{-\lambda t}$ $\quad R_o = \lambda N_o$
$1\ Curie = 1\ Ci = 3.7 \times 10^{10}\ decays/sec$
Radioactive attenuation : $I(x) = I_o e^{-\mu x}$ $\quad HVL = x_{1/2} = ln(2)/\mu$
Intensity in this context refers to activity/area.

Matter waves : de Broglie wavelength of a particle : $\lambda = h/p$ where $p = mv$

Miscellaneous
$1\ hour = 3600\ sec$ $\quad 1\ day = 86400\ s$ $\quad 1\ year = 3.156 \times 10^7\ s$
Pressure is force per area
Colors: Violet: 400 nm, Blue: 475 nm, Green: 525 nm, Yellow: 580 nm, Red: 700 nm
Ultraviolet: $\lambda < 390\ nm$ $\quad$ Visible: 390 nm to 750 nm $\quad$ Infrared: $\lambda > 750\ nm$
Frequency, wavelength and wave speed: $\lambda f = v$.
Typical densities: $\rho_{air} = 1.28\ kg/m^3$, $\rho_{water} = 1000\ kg/m^3$
Circle: circumference : $C = 2\pi r$ $\quad$ area : $A = \pi r^2$
Sphere: surface area : $S = 4\pi r^2$ $\quad$ volume : $V = \frac{4}{3}\pi r^3$
Arc-length: $s = r\theta$ (angle in radians)
Quadratic equation: if $ax^2 + bx + c = 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Isotope symbol: ${}^{A}_{Z}X$: element **X**, with **Z** protons and **A** nucleons (protons PLUS neutrons).

Masses in Kilograms, Atomic Mass Units, and MeV/c²			
Object	kg	u (Da)	MeV/c^2
1 u	1.6605×10^{-27}	1.0	931.494893
electron e, β^-, β^+	9.1094×10^{-31}	0.00054857991 .	0.510998950
proton: p^+	1.67262×10^{-27}	1.007276467	938.272088
neutron n	1.67493×10^{-27}	1.008664915	939.565420
alpha α	6.64466×10^{-27}	4.001506179	3727.379378
neutrino ν	~ 0	$\sim 0.$	$\sim 0.$
${}^{1}_{1}H$		1.007825032	938.78307
${}^{2}_{1}H$		2.014101778	1876.12393
${}^{3}_{1}H$		3.016049281	2809.43212
${}^{3}_{2}He$		3.016029322	2809.41353
${}^{4}_{2}He$		4.002603254	3728.40133
${}^{6}_{3}Li$		6.015123	
${}^{7}_{3}Li$		7.016003	
${}^{7}_{8}Li$		8.022486	
${}^{6}_{4}Be$		6.019726	
${}^{7}_{4}Be$		7.016929	
${}^{8}_{4}Be$		8.005305	
${}^{9}_{4}Be$		9.012183	
${}^{10}_{5}B$		10.012937	
${}^{11}_{5}B$		11.009305	
${}^{12}_{5}B$		12.014353	
${}^{11}_{6}C$		11.011433	
${}^{12}_{6}C$		12.000000	
${}^{13}_{6}C$		13.003355	
${}^{14}_{6}C$		14.003242	
${}^{32}_{15}P$		31.973908	
${}^{39}_{19}K$		38.963706	
${}^{40}_{19}K$		39.963998	
${}^{40}_{20}Ca$		39.962591	

Textbook Appendix G : Selected Isotopes				
Atomic Number (Z)	Element	Symbol	Mass Number (A)	Mass (u)
0	electron	e	0	0.000549
0	neutron	n	1	1.008665
1	proton	p	1	1.007276
1	hydrogen	${}^{1}_{1}H$	1	1.007825
	deuterium	${}^{2}_{1}H$	1	2.014102
	deuteron	D or d	1	2.013553
	tritium	${}^{3}_{1}H$	1	3.016049
	triton	T or t	1	3.015501
2	Helium	${}^{3}_{2}He$	3	3.016029
		${}^{4}_{2}He$	4	4.002603
3	Lithium	${}^{6}_{3}Li$	6	6.015123
		${}^{7}_{3}Li$	7	7.016003
4	Beryllium	${}^{7}_{4}Be$	7	7.016929
		${}^{9}_{4}Be$	9	9.012183
5	Boron	${}^{10}_{5}B$	10	10.012937
		${}^{11}_{5}B$	11	11.009305
6	Carbon	${}^{11}_{6}C$	11	11.011433
		${}^{12}_{6}C$	12	12.000000
		${}^{13}_{6}C$	13	13.003355
		${}^{14}_{6}C$	14	14.003242
7	Nitrogen	${}^{13}_{7}N$	13	13.005739
		${}^{14}_{7}N$	14	14.003074
		${}^{15}_{7}N$	15	15.000109
8	Oxygen	${}^{15}_{8}O$	15	15.003066
		${}^{16}_{8}O$	16	15.994915
		${}^{18}_{8}O$	18	17.999160
15	Phosphorous	${}^{31}_{15}P$	31	30.973762
		${}^{32}_{15}P$	32	31.973908
18	Argon	${}^{40}_{18}Ar$	40	38.963706
19	Potassium	${}^{39}_{19}K$	39	38.963706
		${}^{40}_{19}K$	40	39.963998
20	Calcium	${}^{40}_{20}Ca$	40	39.962591

Note: atomic weight is the mass (in grams) of 1 **mole** of that substance, where 1 mole represents $N_A = 6.02214179 \times 10^{23}$ atoms (called Avogadro's number).